ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 7320

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1973

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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Submitted *to Physics Letters*

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The traditional way of investigating the UV asymptotic behaviour in quantum field theory is to sum up the main logarithmic terms. This summation first performed in spinor electrodynamics[I] leads to the well-known ghost-pole trouble. This difficulty is common for a wide class of field models, e.g. for the two-coupling constant model of pion-nucleon interaction[2].

The attempts to improve the situation in quantum electrodynamics by summing the next logarithmic terms have failed because the corrections have the same sign[3]. As a result, it became a general belief that the UV behaviour of field models is completely defined by the main logarithmic terms.

In recent interesting papers[4,5] it was discovered that the massless Yang-Mills theory possesses a remarkable UV behaviour, quite different from the above-mentioned ghost-pole situation. In this model the ICC tends to zero

$$\bar{g}^2(L) = \frac{g^2}{1 + cg^2 L}\,, \quad c > 0 \qquad \text{(I)}$$

as $L = \ln(-k^2/\lambda^2) \to \infty$. However, the massless Yang-Mills theory cannot be considered as a realistic one because of untractable infrared singularities. The authors of paper[4] considered also spontaneously broken gauge theory[6,7] which is free of infrared troubles. These models inevitably include the interaction of the massive gauge field $\vec{B}_\mu$ with the scalar fields σ and $\vec{\varphi}$, and also quartic self-interaction

$$\mathcal{L} = -h(\sigma^2+\vec{\varphi}^2)^2 , \quad h = \frac{g^2 m_\sigma^2}{32 M_B^2} . \tag{2}$$

This interaction is repulsive and, as is well known[2], in the main logarithmic approximation leads to the ghost-pole trouble

$$\bar{h}(L) = \frac{h}{1-ahL} , \quad a>0 \tag{3}$$

Trying to save the situation the authors of paper[4] investigated a number of gauge groups. Unfortunately they have not succeeded in finding any acceptable model and have been forced to hope on some hypothetical dynamical mechanism of symmetry breaking.

We have investigated a much simpler possibility. In the framework of the standard Kibble model based on the SU(2) group we considered the UV behaviour taking into account the second logarithmic terms $h\,(hL)^n$. It appears that these terms change drastically the situation. Instead of the ghost-pole situation we get the finite value of the asymptotic ICC:

$$\lim_{L\to\infty} \bar{h}(L) = h_\infty . \tag{4}$$

The Lie equations for ICC in the theory with quartic self-interactions looks like[8]

$$\frac{d\bar{h}(L)}{dL} = \varphi(\bar{h}), \tag{5}$$

where $\varphi(h)$ is of the form

$$\varphi(h) = ah^2 - bh^3 \quad , \ a>0, \ b>0. \tag{6}$$

The second term originates from the h^2L contributions to the self-energy and four-vertex.

The crucial point is the relative sign of the first and second terms. In the theory described by the Lagrangian (2), contrary to electrodynamics, the signs are different. As a result, the solution of the Lie equation (5) is of the form

$$\bar{h}(L) = \frac{h}{1 - ahL + \frac{h}{h_\infty} \ln\left\{\frac{h_\infty - h}{h_\infty - \bar{h}(L)} \cdot \frac{\bar{h}(L)}{h}\right\}} . \tag{7}$$

ICC $\bar{h}(L)$ defined by eq.(7) has the critical value $h_\infty = a/b$. Such a behaviour is unprecedental in quantum field theory, although for a long time many people dreamed about this possibility[9] as it leads to the scaling and Regge-type asymptotics of the amplitudes. For the Lagrangian (2)

$$h_\infty = \frac{4\pi^2}{13} . \tag{8}$$

In the spontaneously broken gauge theory, instead of (5), we have the system

$$\frac{d\bar{h}(L)}{dL} = a\bar{h}^2 - b\bar{h}^3 + \alpha(\bar{g}^2)^2 + \beta(\bar{g}^2)h, \qquad (9)$$

$$\frac{d\bar{g}^2}{dL} = -c(\bar{g}^2)^2 + d(\bar{g}^2)^2\bar{h} + \gamma(\bar{g}^2)^3, \quad c>0. \qquad (10)$$

Here the constants a and b are the same as in (6). For sufficiently small physical values

$$\bar{h}(0) = h \ , \ \bar{g}^2(0) = g^2$$

the terms $\sim \alpha, \beta, \gamma$ are nonessential in the UV limit. This means that asymptotically $\bar{h}(L)$ is given as before by eq. (7) and tends to the limiting value (8). The estimation of the coefficient d shows that in the model under consideration $|d h_\infty| \ll c$. Therefore $\bar{g}^2(L)$ is asymptotically given by (I).

Thus, in the Kibble model the ICC $\bar{g}^2$ asymptotically tends to zero and ICC $\bar{h}$ tends to the finite limit h_∞. The numerical value (8) should not be taken too seriously because the account of the next logarithmic terms may change it. Strictly speaking, the r.h.s. of eq. (9) is an infinite series in h. We cannot say anything about the convergence of this series but the important fact is that there exists the mechanism of compensation which leads to the finite h_∞.

In this short note we presented the results for the simplest massive gauge theory. The mechanism described here is quite general and works as well in a wide class of more realistic theories including the interaction with additional particles (hyperons, pseudoscalar mesons, etc.) and based on more general gauge groups. It opens the gates for the construction of realistic models of weak and strong interactions without UV troubles.

The details of calculations and the analysis of the highest logarithmic terms in eq.(9) will be published elsewhere.

We are grateful to N.N.Bogolubov, A.V.Efremov, B.V.Medvedev and A.N.Tavkhelidze for the useful discussions and helpful comments.

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Received by Publishing Department
on July 12, 1973.