# ОБЬЕАИНЕННЫЙ ИНСТИТУТ <br> ЯАЕРНЫХ <br> ИССАЕАОВАНИЙ 

АУБНА

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THE ELECTROMAGNETIC
FORM FACTOR OF THE PION

# E2. 7283 

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Submilted to $\boldsymbol{\text { 月 }}$

## INTRODUCTION.

In the present paper our aim is to show how it is possible to calculate the electromegnetic form factor of the pion in a theory of the chiral type $/ 2 /$ on the basis of the method suggest ed by one of the authors (M.V.) for describing nonpolynomial lagrangians. This method makes it possible to find the contribution of one-loop diagrams in the pion form factor, These diagrams describe the form factor behaviour in the low-energy region.

Attemps to describe the chiral-invariant lagrangians by aeans of the superpropagator method $/ 1 /$ were first undertaken by a number of authors $/ 3 /$ in 1970 and 1971. It seems to us that the most interesting studies in this line were performed by Lehnann in 1972 which were devoted to the description of the low-energy correlations for pion-pion scattering /4/. The obtained results are in satisfactory agreement with experimental dato.

Lehmann has considered the case of massless pions which is characteristic of chiral-invariant theories. llowever, as far ar at low energies the pion mass is rather essential, we nim at the calculation of the pion form factor for the case of massive pions. In so doing, we atart from the lagrangian which, in the limit $m_{s}=0$ and after the electromagrietic field has been switched out,coincides exactly with the chiral-invariant lagrangian in the exponential representation /2/. For the expressions corresponding to the ont-loop diagrams we are dealing with the dependence of the chiralinvariant lagrangian on the choice of some or other representation is noneasential. It affects only the terms $\frac{d}{d n}[\ln C(n)]$, where $(n)$ is the expansion coefricient for the chirel-invarisnt lagrangian
in the pion field powers. The latter dependence is of the logarithric type. We use the exponential representation which has some advantages compared with others /4/. In the framework in which we perform our calculations such an approach seems to us to be quite justified.

The pion radius and the pion form factor in the threshold region are calcclated. As far as we are dealing with the masaive pions, we are able to deduce for the form factor an expression from which it is passible.imediately to calculate the pion $P_{\text {- }}$ wave length. The calculations show that the mein contribution to the fion xatiua comes from triangular baryon diagrams.

Next, more speculstive celculations show thet the position of the . $\rho$-meson resonance and the $\delta_{i}^{\prime}$ phase behaviour are in gereement with experiment.

## 2. The Pion Form Factor.

- The elassic chiral-invariant lagrangian can be written in the following form $/ 2,4 /+$ ) $\mathcal{L}(x)=\frac{F_{r}^{2}}{4} S_{P}\left\{\partial_{\mu} \exp \left[i \frac{\vec{r} \vec{\varphi}(x)}{\vec{F}_{F}}\right] \partial^{\mu} \exp \left[-i \frac{\vec{F} \vec{\varphi}(x)}{\vec{F}}\right]\right\}+i \vec{\psi}(x) \hat{\delta} \varphi(x)-$

$$
\begin{equation*}
-M_{N} \ddot{\psi}(x) \exp \left[-\gamma_{s} \cdot \frac{\vec{T} \vec{\varphi}(x)}{F}\right] \psi(x) \tag{1}
\end{equation*}
$$

${ }^{+}$Here the pseudovector current is not taken into account. In calculating the triangular spinor diagrams we take into account the contribution from it by means of renormalization of the soupling conatants in pion-nucleon vertices (ths GoldbergerTreiman relations). A similar procedure is used by. Lehmenn $/ 4 /$.

Here $\vec{\varphi}(x)$ is the pion field, $\psi(x)$ - the baryon field, $F=y Z_{m+1}$. the pion decay constant. The part of the Lagrangian responsible for pion-pion interactions can be rewritten as

$$
\begin{equation*}
\mathcal{L}_{\pi k}(x)=\frac{1}{2}\left[\left(\sum_{n} \vec{\varphi}\right)^{2}-\frac{\left(\vec{\varphi} \dot{v}_{n} \vec{\varphi}\right)^{2}}{\vec{\varphi}^{2}}\right]\left[\frac{\sin ^{2}\left(\sqrt{\vec{f}^{2}}\right)}{\overrightarrow{\vec{f}}^{2} / \pi_{n}^{2}}-1\right] \tag{2}
\end{equation*}
$$

We introduce here in a gauge invariant way an interaction witt: electromagnetic field $A_{4}(x)$

$$
\begin{align*}
& \vec{\varphi}=\left(\varphi_{1}, \dot{\varphi_{2}}, \varphi_{3}\right)=\left(\frac{\varphi^{\prime}+\varphi}{\sqrt{2}}, \frac{\varphi^{3}-\varphi}{i \sqrt{2}}, \varphi_{0}\right)  \tag{3}\\
& \partial_{\mu} \varphi \rightarrow\left(\partial_{\mu}+i \varphi A_{A}\right) \varphi, \partial_{\mu} \varphi^{\prime \prime} \rightarrow\left(\partial_{\mu}-i \dot{\theta}_{4}\right) \varphi^{*}
\end{align*}
$$

or

$$
\partial_{\mu} t_{i} \rightarrow\left(\partial_{\mu} d_{j}+e \Delta A_{\mu} t_{i j}\right) f_{j}^{\prime}
$$

where $t_{j_{j j}}=\mathcal{\epsilon}_{j, j}$ is the antisymmetric tensor. Then we arrive at a Lagrangian which describes the pion-photon intersection

By expanding Lagrangtans (2) and (4) in powers of the field $\vec{\varphi} \quad$ and introducing normal ordering of these fields we pass to the formulation of the perturbation theory of quantized field. We are interested in the $2 / \xi^{2}$ approximation, which is quite applicable to the description of the pion form factor behaviour for small values of the squared photon momentum. As regards the diagrams with pion-beryon vertices, following Lehmann, we calculate them in the lowest order in $\frac{e}{F_{F}}$. In this sase higherorder effects are assumed to be taken into account, et least partialby, by normalizing the rion-nucieon vertices $/ 4 /$. ${ }^{4}$ ).
${ }^{+5}$ Contrary tr the iehmann calculation for bow-enerpy $\bar{\prime}$ - 5

Afterwards the Lagrangian responsible for the pion-nucleon part of the interaction can be written in the lowest order in $F_{\pi}^{-f}$

$$
\begin{equation*}
\mathcal{L}_{\pi N}(x)=\frac{\pi}{F_{N}} g_{A}: \vec{\psi}(x) \gamma_{s} \vec{x} \vec{\varphi}_{(x)} \dot{\psi}(x) \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
& g_{F}=1.26 . \text { or } \\
& \mathcal{L}_{\vec{A} N}(x)=G\left(\vec{\psi}(x) \gamma_{5} \vec{C} \overrightarrow{\rho_{(x)}} \psi(x)\right. \tag{5}
\end{align*}
$$

where $G^{2} / 4 \pi=14.7$ - is the strong interaction coupling constant. The interaction of spinors with ths electromagnetic field is introduced in the ordinery way

$$
\begin{equation*}
\mathcal{L}_{\psi A}(x)=-e \bar{\psi}(x) \frac{1+i_{H}}{2} \gamma^{\mu} \psi(x) A_{\mu}(x) \tag{6}
\end{equation*}
$$

Now we show how it is possible to calculate the pion form factor in the ( $/ / /_{\mathrm{T}}$ ) approximation by means of lagrangians (2), (4) and (5) and (6).

The matrix element corresoonding to the pion form factor can be written in the form

$$
\begin{equation*}
\left.\langle\pi||S| 5^{+}\right\rangle=-i e \frac{p_{\mu} A_{\mu}(q)}{(2 \pi)^{3} 2 \sqrt{\omega_{1} \omega_{2}}} \nabla_{\mu \nu}(q), \tag{7}
\end{equation*}
$$

where $p=\rho_{1}+P_{2}, F_{=} \rho_{1}-\rho_{2} \quad, P_{1}\left(\omega_{1}\right)$ - is the momentua (energy) of the outgoing pion, $\rho_{2}\left(\omega_{2}\right)$ is the momentum (energi) of the incident pion.
scattering, we calculate the interactions of pions not only with nucleons, but also with $\Sigma$ Aand $\Xi$ baryons. These interactions contribute noticeably to the pion root-mean-square. redius. The . authora are grateful to D.V.Shirkov who has pointed to shis fact.

$$
\begin{equation*}
\Pi_{\mu \nu}(q)=g_{\mu \nu}+\Pi_{\mu \nu}^{(\pi)}(q)+\Pi_{\mu \nu}^{(s)}(q)+\Delta_{\mu \nu} . \tag{8}
\end{equation*}
$$

Here $g_{\mu \nu}$ is the Bohrn term, $/ /_{\mu v}^{\text {(i) }}$ - the contribution of the pion loop in the $e / \sigma_{\pi}^{2}$ approximation to the form factor, $\Pi_{\mu \nu}^{(8)}$ the contribution of the baryon triangular diagrams in the same approximation, and $\Delta_{\mu \nu}$ the contributions of the remaining terms higher in powers of $\left(F^{-1}\right)$ which we do not toke into account for small $q^{2}$.
3. Calculations of $/ 7_{\mu y}^{(5)}$.

Let use consider the diagram of Fig. $I(a)$


Fig. 1.

The integral corresponding to this diagram is quadratically divergent. To derive an ultimate expression for this part of the form factor we lie the superpropagator method.

We consider an infinite series of two-vertex diagrams with an arbitrary number of internal pion lines in each of them.Theae diagrams are obtained by taitrig the $T$ product of the legrangins (2) ard (4). The superpropagator method makes it possible to calculate the final expression for all the diagrams. Then we
can extract from the expression obtained the part that concerns diagram (a) of interest for us

$$
\begin{aligned}
& \left\langle\pi+1 \int d^{4} d^{4} y\left(\mathcal{L}_{x_{2}}(x), \mathcal{L}_{x-A}(x)\right) / / x^{+}\right\rangle=-i e \frac{p^{\mu} A^{v}(q)}{\left(2 x^{3} 2 \sqrt{4 \omega_{2}}\right.} \bar{\Pi}_{\mu v}^{(n)}(9) \\
& \overline{7}_{\mu v}^{(x)} \quad \text { is the form factor corresponding to the }
\end{aligned}
$$

diagram (b).

$$
\begin{gather*}
\overrightarrow{7}_{\mu \nu}^{(n)}(q)=\int d_{x}^{4} e^{i q x}\left[\partial^{c}(x) \partial^{c}(x)-\Delta^{c}(x) 2 Q_{\nu} \Delta^{c}(x)\right] G(x)  \tag{10}\\
G(x)=i \frac{4}{3 r^{2}} \sum_{0}^{\infty} c(n)\left[-i \Delta^{c}(x)\right]^{2 n}  \tag{11}\\
C(n)=\left(\frac{4}{\sigma^{2}}\right)^{2 n} \frac{4(2 n+3)}{(2 n+2)(2 n+4) r(2 n+3)} \tag{12}
\end{gather*}
$$

$\left.\Delta^{c}(x)=i\langle T / \varphi(x) \varphi(0))\right\rangle_{0}, \quad \Gamma(n)$ is the gamma function.
As far as we are interested in the one-loop approximation, we need diagrams with four and more internal lines with the only aim to regularize the one-loop diagram (a). Therefore in our further calculations the propagators in the square brackets are assumed to be massive, and the propagators in (11) are massless one, keeping only the main features of the massive propagators $/ 5 /$. We replace the sum in eq. (11) by the mellon integral and integrate over $d^{4} x$. Then we perform a series expansion of the Bessel function appearing due to integrating over the angular variables and integrate over $d \tau$. As a result, we are led to the following expression
where the contour $\mathcal{L}$ goes around clockwise the real positive axis and the origin, and

$$
\begin{equation*}
C(z)=\frac{2}{3}\left(\frac{m_{3}}{2 \pi F_{\pi}}\right)^{4 z+2} \frac{(2 z+3)}{(2 z+2)(2 z+4) \Gamma(2 z+3)} \tag{14}
\end{equation*}
$$

$$
\begin{aligned}
& +\Delta_{i v}^{\varepsilon}(q, z) \\
& \Delta_{\mu \nu}^{\varepsilon}(q, z)=-i 3 g_{\mu \nu} q\left(\frac{z}{z+\epsilon}\right)^{2} \sum_{0}^{\infty}\left(\frac{q^{2}}{m_{n}^{2}}\right)^{k} \frac{\Gamma(k-2 z+2) \Gamma(k-2 z+1) \Gamma(k-2 z) \Gamma\left(k-z_{z}-1\right)}{k!(k+2)!\Gamma(2 k-4 z+2)} \cdot(16)
\end{aligned}
$$

Here $\Delta_{\mu \nu}^{\varepsilon}(t, z)$ is a gauge-noninvariant term which does not contribute to the one-loop approximation, if, following Salam, we introduce a term $\left(\frac{Z}{z+\epsilon}\right)^{2}$ and make a transition to the limit $\varepsilon=0$ at the end of the calculationa. For a diacussion of the same procedure applied to deacribing nonpolynomial lagrangians with rediretives in the case of masaless particles see paper of Salam et al. ${ }^{/ 6 /+)}$

To calculate $\Pi_{\mu \nu}^{*}(q)$ it is enough in the integral (10) to take a reaidue only at the point $z=0$. AB a result, we get aftar summing the series over the powers of ( $4^{2} / \mathrm{m}_{x}^{2}$ )

$$
\begin{align*}
& \prod_{-\nu}^{(n)}(\hat{r})=\frac{1}{3}\left(\frac{m_{r}}{2 \pi k_{1}}\right)^{2}\left(g_{0}-\frac{89}{9^{2}}\right)\left\{\frac{9^{2}}{8 m_{2}^{2}}\left[\frac{13}{12}-3 c-\ln \left(\frac{m_{3}}{2 \pi r_{x}}\right)^{2}\right]-\right. \\
& -1+\frac{q^{2}}{3 m_{r}^{2}}+\frac{q^{2}}{4 m_{2}^{2}}\left(\frac{4 m_{2}^{2}}{q^{2}}-1\right)^{\frac{1}{2}} \operatorname{arctg}\left(\frac{4 m_{x}^{2}}{q^{2}}-1\right)^{-\frac{1}{2}} \text {. } \tag{17}
\end{align*}
$$

The term in the aquare brackets contributes to the rootnean square radius of the pion. This contribution is found to be $\bar{H}_{\text {Note that }}$ the calculation of $\Gamma_{\mu \nu}^{(a)}(g) \quad$ without recourse to the Salam procedure little affects the ultinate results for the
not so large

$$
\begin{equation*}
\left\langle\tau^{2}\right\rangle_{\pi}=0,065 f^{2} \tag{18}
\end{equation*}
$$

in the Fersi units - The remaining part is proportional to $9^{4} / \mathrm{mp}$, at low energies. When $9^{2} \geqslant 4 \mathrm{~m}_{\pi}^{2} \quad$ the form factor becomes complex since the point $q^{2}=4 \mathrm{~m}_{\pi}^{2}$ is the beginning of the cut in the $q^{2}$ plane. The wave length in the Born approximation is the imaginary nart $\quad 7_{\mu \nu}^{m}(q) \quad$ divided by $\left(\frac{q^{2}}{\varphi}-m_{r}^{2}\right)^{\frac{3}{2}}$ at the point $9^{2} 4 \mathrm{~m}^{2}$ and is equal to $0.031 \mathrm{~m}^{-1}$, which is in good agreement with the experimental valuea $\quad a^{\prime}=10036 .^{\circ}$ 20,02 max $_{\text {m }} / 7 /$.

Here we indieate enotikn :-is? easting foat:ure of eq. (17). When the pion loop ia rezularized by ardinury methods, for example, by neans of the Pauli-Villars regularization, after a transition to the 1init $m_{r}=0$ there can arise infrared divergences. Nothing of the kind aan happen in the expreasion (17), where after the transition to the limit the squared pion mass under the sign of logarithm ia simply replaced by $q^{2}$. This follows from the frat that we are dealing with the whole set of an infinite number of twovertex diagrame with any number of intarmediate pions rather than with a single diagram.
pion form factor.

## (8) <br> 4. Calculation of $\Pi_{\mu \nu}^{(8)}\left(9^{2}\right)$

Now we pass to the calculation of the contribution of the baryon loops to the pion form factor. We show the this contribution defines the value of the root-mean-square radius of the pion.

In the approximation $e / /^{2}$,we are interested in, the contribution to the pion form factor comes from the following diagrams ( Fig. 2). The account of these diagrams

(c)

(d)

(e)


Fig. 2.
introduces an essential correction to the $q^{2}$ term in eq. (17). Terms with higher powers of $9^{2}$ are nonessential at low energies and have small coefficients, so that they will be neglected in what follows.

By the example of the nucleon diagram (c) we recall
briefly how one calculates triangular diagrams of the kind. In the same way as, egg., in the monograph of Schveber $/ 8 /$, we consider the following set of the diagrams

(c)


Fig. 3.

In virtue of the yord identity, the divergences in the integrala of the corresponding diagrams $C, C^{\prime}, C^{\prime \prime}, C^{\prime \prime \prime}, c^{\prime \prime}$ are mutually compensated. The constant terms cancel. The remaining integrals are already finite and can be calculated in a uaual way.

Using Lagrangians (5) and (6) and following the above procedure we derive for $\Pi_{\mu_{4}}^{\left({ }^{(C)}\right.}(9)$. the following expressions

$$
\begin{equation*}
\Pi_{\mu \nu}^{(c)}(q)=\frac{g_{\mu \nu}}{6(2 \pi)^{2}} \frac{G^{2}}{M_{\mu}^{2}} q^{2}+O\left(\left(\frac{q^{2}}{M_{\mu}^{2}}\right)^{2}\right) \tag{19}
\end{equation*}
$$

Inserting the value of the atrong coupling constant $G^{2} / 4 \pi \quad=14.7$ in eq. (19) we obtain for the nucleon-loop contribution to the pion mean-square radius the following value

$$
\begin{equation*}
\left\langle r^{2}\right\rangle_{N}=0,206 f^{2} \tag{20}
\end{equation*}
$$

This noticeably exceeds the pion loop contribution.
Now we have only to estimate similar contributions of the baryon diagrans ( $d$ ), ( $(0)$ and ( $f$ ). The expressions corresponding to these diagrama have the form analogous to eq. (19), but with other values of the constant $G$ and the mase $M$. It follows from the ${ }^{S} U_{3}$ invariant theory that all these constanta are expressed in terms of two constants $g_{f}$ and $g_{D} / 9 /$.

$$
\begin{array}{ll}
g_{\pi N N}=\frac{1}{2}\left(g_{D}+g_{F}\right)=G & , g_{\pi_{\Sigma \Sigma}}=-g_{F},  \tag{21}\\
g_{\pi \Sigma N}=\frac{1}{\sqrt{3}} g_{D}, & g_{\pi E B}=\frac{1}{2}\left(g_{D}-g_{f}\right) .
\end{array}
$$

The procesa $\Sigma^{ \pm} \rightarrow \Lambda+e^{ \pm}+b^{\prime} \quad$ yields for the ratio $g / g_{f}$ the value $\approx 2$, ref. / $10 /$. Using this ratio and eqs. (21) all the coupling constants can be expressed via the strong pionnucleon interaction constant $G$
$g_{\pi \Sigma \Sigma}=-\frac{2}{3} G, g_{\pi \Sigma A}=-\frac{4}{3 \sqrt{3}} G, g_{\pi \Sigma \Sigma}=\frac{1}{3} G$.
Aa a result, for $\Pi_{\mu \nu}^{(s)}(q)$ we get

$$
\begin{aligned}
& \Pi_{\mu_{0}}^{(\theta)}(q)= \frac{g_{\mu_{p}}}{6(2 \pi)^{2}} \\
&\left(\frac{G^{2}}{M_{N}^{2}}\right) / 1+\frac{4}{g}\left(\frac{M_{2}}{M_{2}}\right)^{2}+\frac{16}{22}\left(\frac{M_{\mu}}{M_{M_{R}}}\right)^{2}+\frac{1}{g}\left(\frac{M_{N}}{M_{B}}\right)^{2} 7 q^{2}= \\
& \simeq g_{\mu u} \frac{q^{2}}{6(2 \pi)^{2}} \frac{G^{2}}{M_{N}^{2}} 1,73
\end{aligned}
$$

From where we find

$$
\left\langle r^{2}\right\rangle_{B} \simeq 0,356 f^{2}
$$

By adding this value to the contribution of the pion loop we finally obtain for the pion-mean-square radius the following value ${ }^{+ \text {) }}$

$$
\begin{equation*}
\sqrt{\left\langle z^{2}\right\rangle} \simeq 0,65 \mathrm{f} . \tag{24}
\end{equation*}
$$

This value is in good agreement with the available experimental deta/11/.

[^0]
## 5. Discusgion of the Results.

The diagrams considered give the main contribution to the pion form factor. As we have seen, the baryon triangular diagrame define alnost completely the coefficient for the $\boldsymbol{q}^{\prime \prime}$ - torm while the pion loop gives the correct energy description of the form factor in the threahold region. The account of the $K$ - meson loop Iittle affects the coefficient for $\boldsymbol{q}^{2}$ since, as the calculations show, the contribusion of it is much smaller then even a small contribution of the pion loop. The form factor behaviour in the threshold region is also little affected by the $K$-meson contribution. Therefore we do not take into account the $K=$ meson diegrams. Inserting eqs. (17) and (23) to (8) we get for the form factor the ultimate expression

$$
\begin{align*}
& F_{T}(q)=1+\frac{1}{3}\left(\frac{m_{\pi}}{2 \pi F_{\pi}^{2}}\right)^{2}\left\{-1+\frac{q^{2}}{m_{\pi}^{2}}[0,61+1,53]+\right.  \tag{25}\\
& \left.+\frac{q^{2}}{4 m_{4}^{2}}\left(\frac{4 m_{4}^{2}}{q^{2}}-1\right)^{3 / 2} \text { acctg }\left(\frac{4 m_{\pi}^{2}}{q^{2}}-1\right)^{-1 / 2}\right\}
\end{align*}
$$

The first figure in the squere brackets corresponds to the pion loop, the second one to the baryon diagrans. This formula well describes the pion form factor behaviour in the region of amall $q^{2} \quad u p$ to the production threshold for two pions as well as in the threshold region. It leads to the pion radius being in good agreenent with experiment (eq. 24) and gives a reasonable value for the acattering wave length $q_{p}^{\prime}$ calculated by the formulas which are valid for small $q^{2}$

$$
\begin{equation*}
\operatorname{tg} \delta_{1}^{\prime}=\frac{\operatorname{Im} F_{1}(q)}{R_{e} F_{2}(q)}, \quad \delta_{1}^{\prime}=a_{1}^{\prime}\left(\frac{q^{2}}{4}-m_{\pi}^{2}\right)^{3 / 2}, \tag{26}
\end{equation*}
$$

where $\delta_{1}^{t}$ is the $J T-$ scattering phase in the state $I=y \quad=1$ 。

The absolute values of the pion form factor calculated by eq. (25) in the threshold energy region of $q^{2}$ are also in good agreement with experimental aata recentiy obtained at Dubna/l2/ The corresponding values of the form factor are given in Table I.

| $9^{2} / 4 m_{r}^{2}$ | 0,85 | 1,1 | 1,45 |
| :---: | :---: | :---: | :---: |
| $F_{\pi}^{\text {exc }}(9)$ | $1,10 \pm 0,07$ | $1,14 \pm 0,06$ | $1,30 \pm 0,67$ |
| $F_{\pi}^{\text {7heor }}$ | 1,12 | 1,16 | 1,22 |

A less atrict result is the following one. If we sum up a chain of diagrams consisting of pion loops then in the obtained expression there can be observed a $\rho$ - resonance $a \dot{i}$ an energy of about 950 MeV . A similar rasult has been obtained by Lehmann when analysing the $\mathbb{T}_{5}$ scattering $P$ wave /4/. However, the main contribution to the $\rho$ - meson resonance is found by him to come from the Born term and the nucleon diagrams.

## CONCLUSION

The calculations performed show that the pion root-meansquare radius is almost completely defined by the contribution of the baryon triangular diagrams to the pion fora factor. The pion

1000, similerly to the $K$-meson loops, give a amall contribution to the $g^{2}$ term of the form factor. At the same time the account of the pion loop is very important for the correct deecription of the threshold description of the form factor. The set of diagrams in Figs. 1 and 2 defines thereby completely the form factor behnviour in the threshold region.

The question associated with the eatimation of the contributions of diagrams of higher orders in $\left(\xi^{-}\right)$to the form factor remains still open. The estimation of individual diagrems shows that for small $q^{2}$ these contributions are not of wuch improtance. However, this question neede undoubtedly be studied more thoroughly.

The authors express their deep gratitude to D.I.Blokhinteev, V.A.bfremov, V.A.Heshcheriakov, Nguen Van Hieu,L.L.Nemenov, V.V.Serebriakoy and D.V.Shirkov for useful discussions.

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Received by Publishing Department on July 3, 1973.


[^0]:    +) We notice that if eq. (5) is used for the pion-nucleon vertex then the root mean-square radius slightly decreases $\sqrt{\left\langle r^{2}\right\rangle} \sim 0,68 f$

