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M.K.Volkov, V.N.Pervushin

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THE ELECTROMAGNETIC FORM FACTOR OF THE PION



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INTRODUCTION.

In the present paper our aim is to show how it is possible to calculate the electromegnetic form factor of the pion in a theory of the chiral type $^{2/}$ on the basis of the method suggested by one of the authors (M.V.) for describing nonpolynomial lagrangians. This method makes it possible to find the contribution of one-loop diagrams in the pion form factor. These diagrams describe the form factor behaviour in the low-energy region.

Attemps to describe the chiral-invariant lagrangians by means of the superpropagator method $^{1/}$ were first undertaken by a number of authors $^{3/}$ in 1970 and 1971. It seems to us that the most interesting studies in this line were performed by Lehmann in 1972 which were devoted to the description of the low-energy correlations for pion-pion scattering $^{4/}$. The obtained results are in satisfactory agreement with experimental data.

Lehmann has considered the case of massless pions which is characteristic of chiral-invariant theories. However, as far as at low energies the pion mass is rather essential, we aim at the calculation of the pion form factor for the case of massive pions. In so doing, we start from the lagrangian which, in the limit $m_r=0$ and after the electromagnetic field has been switched out, coincides exactly with the chiral-invariant lagrangian in the exponential representation $\frac{2}{r}$. For the expressions corresponding to the one-loop diagrams we are dealing with the dependence of the chiralinvariant lagrangian on the choice of some or other representation is noneasential. It affects only the terms $\frac{d}{dn} \left[\frac{l_n}{n} C(n) \right]$, where C(n)is the expansion coefficient for the chiral-invariant lagrangian

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in the pion field powers. The latter dependence is of the logarithmic type. We use the exponential representation which has some advantages compared with others ^{/4/}. In the framework in which we perform our calculations such an approach seems to us to be quite justified.

The pion radius and the pion form factor in the threshold region are calculated. As far as we are dealing with the massive pions, we are able to deduce for the form factor an expression from which it is possible immediately to calculate the pion P wave length. The calculations show that the main contribution to the pion radius comes from triangular baryon diagrams.

Next, more speculative calculations show that the position of the ρ -meson resonance and the δ'_i phase behaviour are in agreement with experiment.

2. The Pion Form Factor.

The classic chiral-invariant lagrangian can be written in the following form (2,4/+) $d(x) = \frac{F_{\pi}^{2}}{L} Sp[d]_{\mu} exp[i\frac{\vec{r}\cdot\vec{F}(x)}{F_{\mu}}]d^{\mu}exp[-i\frac{\vec{r}\cdot\vec{F}(x)}{F_{\mu}}]f] + i\vec{V}(x)\delta V(x) -$

- M. W(x) exp[-x, TV(x)] W(x)

⁺)Here the pseudovector current is not taken into account. In calculating the triangular spinor diagrams we take into account the contribution from it by means of renormalization of the coupling constants in pion-nucleon vertices (the Goldberger-Treiman relations). A similar procedure is used by Lehmann ^{/4/}.

(1)

Here $\vec{\varphi}(x)$ is the pion field, $\psi(x)$ - the baryon field, $\xi = \mathcal{Y}_{\mathcal{F}_{x}}$ the pion decay constant. The part of the Lagrangian responsible for pion-pion interactions can be rewritten as

$$\mathcal{L}_{\rm FF}(x) = \frac{1}{2} \left[\left(2, \vec{\varphi} \right)^2 - \frac{\left(\vec{\varphi} \cdot 2, \vec{\varphi} \right)^2}{\left(\vec{\varphi} \cdot 2 \right)^2} \right] \left[\frac{S_{\rm in}^2 \left(\sqrt{\frac{2}{\beta_{\rm in}}} \right)}{\left(\vec{\varphi} \cdot 2 \right)^2 \left(\frac{2}{\beta_{\rm in}} \right)^2} - 1 \right]. \tag{2}$$

We introduce here in a gauge invariant way an interaction with: electromagnetic field $\mathcal{A}_{A}(x)$

$$\vec{P} = (\vec{Y}_1, \vec{Y}_2, \vec{Y}_3) = \left(\frac{\vec{Y}^{*} + \vec{Y}}{\vec{v}_2}, \frac{\vec{\varphi}^{*} - \vec{\varphi}}{i\sqrt{z}}, \vec{Y}_0\right)$$
(3)

 $2\varphi \rightarrow (2 + ieA_{\mu})\varphi \quad , \quad 2\varphi^{*} \rightarrow (2 - ieA_{\mu})\varphi^{*}$

or

where $t'_{j_j} = t'_{j_j}$ is the antisymmetric tensor. Then we arrive at a Lagrangian which describes the pion-photon interaction

$$\mathcal{L}_{\mathbf{x},\mathbf{A}}(\mathbf{x}) = \mathbf{e} \left[\left(\Psi_{2}^{2} \partial^{A} \Psi_{1}^{c} - \Psi_{1}^{c} \partial^{A} \Psi_{2}^{c} \right) \mathcal{A}_{\mathbf{x}}^{c} + \frac{\mathbf{e}}{2} \left(\Psi_{1}^{2} + \Psi_{2}^{2} \right) \mathcal{A}_{\mathbf{x}}^{2} \right] \frac{S.n^{2} \left[\left(\overline{\Psi_{2}^{2}} - \Psi_{2}^{2} \right) \mathcal{A}_{\mathbf{x}}^{2} \right]}{\overline{\Psi_{2}^{2}} \mathcal{A}_{\mathbf{x}}^{2}} . \tag{4}$$

By expanding Lagrangians (2) and (4) in powers of the field $\vec{\varphi}'$ and introducing normal ordering of these fields we pass to the formulation of the perturbation theory of quantized field. We are interested in the $\vec{\varphi}_{r}^{2}$ approximation, which is quite applicable to the description of the pion form factor behaviour for small values of the squared photon momentum. As regards the diagrams with pion-baryon vertices, following Lehmann, we calculate them in the lowest order in $\vec{\varphi}_{r}^{-1}$. In this case higherorder effects are assumed to be taken into account, at least partially, by normalizing the pion-nucleon vertices $^{14/2}$.

+)Contrary to the Jehmann calculation for low-energy J-T

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Afterwards the Lagrangian responsible for the pion-nucleon part of the interaction can be written in the lowest order in F_{π}^{-1}

$$\mathcal{L}_{\mathcal{T}N}(\kappa) = \frac{17\pi}{F_{\pi}} g_{\pi} : \overline{\mathcal{V}}(\kappa) \mathcal{E}_{\pi} \overline{\mathcal{E}} \overline{\mathcal{P}}(\kappa) \mathcal{V}(\kappa):$$
(5)

where $g_{f} \simeq 1.26$. Or

$$\mathcal{L}_{\mathcal{IN}}(x) = G \cdot \overline{\mathcal{V}}(x) \, \mathcal{E}_{S} \, \overline{\mathcal{E}}^{\, \vec{p}}(x) \, \mathcal{V}(k) \tag{5}$$

where $G_{4F}^2 = 14.7$ - is the strong interaction coupling constant. The interaction of spinors with the electromagnetic field

The interaction of spinors with the electromagnetic field is introduced in the ordinary way

$$\mathcal{I}_{\psi A}(x) = -e \,\overline{\psi}(x) \,\frac{1+c_0}{2} \, g^{A} \, \psi(x) \, \widehat{\mathcal{A}}_{\mu}(x) \,. \tag{6}$$

Now we show how it is possible to calculate the pion form factor in the $\binom{\ell}{f_r^2}$ approximation by means of lagrangians (2), (4) and (5) and (6).

The matrix element corresponding to the pion form factor can be written in the form

$$\langle \pi^{+} | S' | \pi^{+} \rangle = -ie \frac{P_{\alpha} A_{\alpha}(q)}{(2\pi)^{3} 2 \sqrt{\omega_{1} \omega_{2}}} \int_{-\infty}^{\infty} (q)_{\beta}$$
 (7)

where $p = P_1 + P_2$, $f = P_1 - P_2$, $P_1(\omega_r)$ - is the momentum (energy) of the outgoing pion, $P_2(\omega_r)$ is the momentum (energy) of the incident pion.

scattering, we calculate the interactions of pions not only with nucleons, but also with Σ . And Ξ , baryons. These interactions contribute noticeably to the pion root-mean-square redius. The authors are grateful to D.V.Shirkov who has pointed to this fact.

$$\prod_{\mathcal{M}_{\mathcal{V}}}(q) = g_{\mathcal{M}_{\mathcal{V}}} + \prod_{\mathcal{M}_{\mathcal{V}}}^{(\kappa)}(q) + \prod_{\mathcal{M}_{\mathcal{V}}}^{(\beta)}(q) + \Delta_{\mathcal{M}_{\mathcal{V}}}.$$
 (8)

Here $\mathcal{G}_{\mu\nu}$ is the Bohrn term, $/\overline{f}_{\mu\nu}^{(K)}$ - the contribution of the pion loop in the $e'/_{F_{\mu}^{2}}$ approximation to the form factor, $/\overline{f}_{\mu\mu\nu}^{(B)}$ the contribution of the baryon triangular diagrams in the same approximation, and $\Delta_{\mu\nu}$ the contributions of the remaining terms higher in powers of (F_{μ}^{-1}) which we do not take into account for small q^{2} .

3. Calculations of 17/48 .

Let us consider the diagram of Fig. 1(a)



Eig. 1.

The integral corresponding to this diagram is quadratically divergent. To derive an ultimate expression for this part of the form factor we use the superpropagator method.

We consider an infinite series of two-vertex diagrams with an arbitrary number of internal pion lines in each of them. These diagrams are obtained by taking the T product of the legrangians (2) and (4). The superpropagator method makes it possible to calculate the final expression for all the diagrams. Then we

can extract from the expression obtained the part that concerns diagram (a) of interest for us

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$$\langle \pi^{+}| \int dx dy \Big(d_{\pi\pi^{-}}(x), d_{\pi\pi^{-}}(y) \Big) | \pi^{+} \rangle = -ie \frac{P^{A} \hat{A}(q)}{(2\pi)^{3} 2 f_{44}(u)} \tilde{\Pi}_{\mu^{+}}^{(m)}(q) \quad (9)$$

$$\tilde{\Pi}_{\mu^{+}}^{(\pi)}(\pi) \qquad \text{is the form factor commenceding to the$$

diagram (b).

$$\overline{\Pi}_{\mu\nu}^{(\mathbf{r})}(q) = \int d^{t}_{x} \, \mathcal{L}^{sq}\left[\dot{g}_{x} \, \Delta^{s}(\mu) \, \dot{g}_{x} \, \Delta^{s}(\mu) - \Delta^{s}(\mu) \, \dot{g}_{x} \, Q_{y} \, \Delta^{s}(\mu) \right] \mathcal{L}(x) \tag{10}$$

$$(f'(x) = i \frac{4}{3f_{T}^{2}} \sum_{0}^{\infty} C(n) \left[-i \Delta^{2}(x) \right]^{2n}$$
(11)

$$C(n) = \left(\frac{4}{F_{p}^{2}}\right)^{\frac{2n}{(2n+2)(2n+4)}} \frac{4(2n+3)}{(2n+2)(2n+4)\Gamma(2n+3)}$$
(12)

 $\Delta^{'}(x) = i \langle T/(\varphi(x) \varphi(\sigma) \rangle \rangle_{o} , \qquad \Gamma(h) \quad \text{is the gamma function.} \\ \text{As far as we are interested in the one-loop approximation,} \\ \text{we need diagrams with four and more internal lines with the only } \\ \text{sim to regularize the one-loop diagram (a). Therefore in our } \\ \text{further calculations the propagators in the square brackets are } \\ \text{assumed to be massive, and the propagators in (11) are massless } \\ \text{one, keeping only the main festures of the massive propagators}^{5/}. \\ \\ \text{We replace the sum in eq. (11) by the Mellin integral and integrate } \\ \text{over } d^{4}x \quad \text{. Then we perform a series expansion of the Bessel } \\ \\ \text{function appearing due to integrating over the angular variables } \\ \text{and integrate over } d^{4}r \quad \text{. As a result, we are led to the follow-ing expression } \\ \end{array}$

$$\overline{\Pi}_{\mu\nu}^{(T_{\tau})}(q) = \int dz \, ct_{g \, \tau \, z} \, c(z) / \overline{\Pi}_{\mu\nu}(q, z) \,, \qquad (13)$$

where the contour λ goes around clockwise the real positive axis and the origin, and

$$C(z) = \frac{2}{3} \left(\frac{m_T}{2\pi f_T}\right)^{4z+2} \frac{(2z+3)}{(2z+2)(2z+4)f^2(2z+3)}$$
(14)

$$\Pi_{\mu\nu}(q, z) = i \left(g_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right)_{1}^{\infty} \left(\frac{q^2}{m_{\nu}^2} \right)^{\kappa} \frac{\Gamma(\kappa - 2z + 3) \Gamma(\kappa - 2z + 1) \Gamma(\kappa - 2z) \Gamma(\kappa - 2z + 2)}{(\kappa - 1)! (\kappa + 2)! \Gamma'(2\kappa - 4z + 2)} + (15) + \Delta_{\mu\nu}^{\xi}(q, z)$$

$$\Delta_{\mu\nu}^{\mathcal{E}}(q_{\bar{z}}) = -i3q_{\mu\nu} \left\{ \frac{\bar{z}}{\bar{z}+\epsilon} \right\}_{0}^{2} \left(\frac{q_{\bar{z}}}{m_{\mu}^{2}} \right)^{k} \frac{\Gamma'(k\cdot2\bar{z}+2)\Gamma'(k\cdot2\bar{z}+1)\Gamma'(k\cdot2\bar{z})\Gamma'(k\cdot2\bar{z}-1)}{\kappa!(\kappa+2)!\Gamma'(\kappa-2\bar{z}+1)\Gamma'(\kappa-2\bar{z}-1)}.$$
(16)

Here $\Delta_{\mu\nu}^{\ell}(t^{2})$ is a gauge-noninvariant term which does not contribute to the one-loop approximation, if, following Salam, we introduce a term $\left(\frac{2}{\ell+\ell}\right)^{2}$ and make a transition to the limit $\ell=0$ at the end of the calculationa. For a discussion of the same procedure applied to describing nonpolynomial lagrangians with rediretives in the case of maasless particles see paper of Salam et al. $^{\ell}(\ell+)$

To calculate $\int_{\mu_V}^{\pi} (q)$ it is enough in the integral (13) to take a residue only at the point Z=0 . As a result, we get after summing the series over the powers of $\binom{q^2}{m_r^2}$ $\int_{\mu_V}^{\pi} (q) = \frac{1}{3} \left(\frac{m_\pi}{2\pi_V} \right)^2 \left(\frac{q}{g_{-2}} - \frac{q}{g_{-1}} \right) \left\{ \frac{q^2}{g_{m_r^2}} \frac{f_{2}}{12} - 3c - \frac{f_m}{m_r} \left(\frac{m_\pi}{2\pi_V} \right)^2 \right\} -$ $-1 + \frac{q^2}{3m_r^2} + \frac{q^2}{4m_r^2} \left(\frac{4m_q^2}{q^2} - 1 \right)^{\frac{2}{2}} \arctan \left(\frac{q}{g_{-2}} - 1 \right)^{-\frac{3}{2}}$ (17)

The term in the square brackets contributes to the rootmean square radius of the pion. This contribution is found to be $(f_{\mu\nu\nu}^{(r)}(q))$ without recourse to the Salam procedure little affects the ultimate results for the not so large

$$\left\langle \tau^2 \right\rangle_{T} = 0.065 f^2 \tag{18}$$

in the Fermi units . The remaining part is proportional to $q_{m_{p}}^{2}$ at low energies. When $q^{2} \ge 4m_{p}^{2}$ the form factor becomes complex since the point $q^{2} \ge 4m_{p}^{2}$ is the beginning of the cut in the q^{2} plane. The wave length in the Born approximation is the imaginary part $\int_{m_{v}}^{m_{v}} (q)$ divided by $\left(\frac{q^{2}}{\xi} - m_{e}^{2}\right)^{\frac{2}{2}}$ at the point $q^{\frac{2}{m}} - 4m_{p}^{\frac{2}{m}}$ and is equal to $0.031m_{p}^{-1}$, which is in good agreement with the experimental values $q_{v}^{2} = (q\,036\%)^{\frac{2}{m_{v}}}$

Here we indicate another interacting feature of eq. (17). When the pion loop is regularized by ordinary methods, for example, by means of the Pauli-Villars regularization, after a transition to the limit $m_{\pi}=0$ there can arise infrared divergences. Nothing of the kind can happen in the expression (17), where after the transition to the limit the squared pion mass under the sign of logarithm is simply replaced by 9^2 . This follows from the fact that we are dealing with the whole set of an infinite number of twovertex diagrams with any number of intermediate pions rather than with a single diagram.

pion form factor.

4. Calculation of $\Pi_{AV}^{(B)}(9^2)$

Now we pass to the calculation of the contribution of the baryon loops to the pion form factor. We show that this contribution defines the value of the root-mean-square radius of the pion.

In the approximation $\frac{\gamma_{r}^{2}}{r}$, we are interested in, the contribution to the pion form factor comes from the following disgrams (Fig. 2). The account of these diagrams



Fig. 2.

introduces an essential correction to the g^2 term in eq.(17). Terms with higher powers of g^2 are nonessential at low energies and have small coefficients, so that they will be neglected in what follows.

By the example of the nucleon diagram (c) we recall briefly how one celculates triangular diagrams of the kind. In the same way as, e.g., in the monograph of Schveber^{/8/}, we consider the following set of the diagrams



Fig.3.

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In virtue of the Yord identity, the divergences in the integrals of the corresponding diagrams $C_{,,C'}$, C'', C''', C'''', C'''' are mutually compensated. The constant terms cancel. The remaining integrals are slready finite and can be calculated in a usual way.

Using Lagrangians (5) and (6) and following the above procedure we derive for $T_{A_{n}}^{(6)}(q)$ the following expressions

$$\prod_{\mu\nu}^{(c)}(q) = \frac{g_{\mu\nu}}{6(2\pi)^2} \frac{G^2}{M_{\mu}^2} q^2 + O\left(\frac{(g^2)}{M_{\mu}^2}\right)^2.$$
(19)

Inserting the value of the strong coupling constant $G'_{4\pi}$ =14.7 in eq. (19) we obtain for the nucleon-loop contribution to the pion mean-square radius the following value

$$\left\langle \tau^{2}\right\rangle_{N} = 0, 2\,\mathcal{C}6\,\mathcal{F}^{2}. \tag{20}$$

This noticeably exceeds the pion loop contribution.

Now we have only to estimate similar contributions of the baryon diagrams (d), (e) and (f). The expressions corresponding to these diagrams have the form analogous to eq. (19), but with other values of the constant G and the mass M. It follows from the \mathcal{SU}_3 invariant theory that all these constants are expressed in terms of two constants \mathcal{G}_{pr} and \mathcal{G}_{p} $\frac{19}{}$.

$$g_{\pi,\mu,\mu} = \frac{1}{2} \left(g_{b} + g_{F} \right) = G , \quad g_{\pi,FFF} = -g_{F} , \quad (21)$$

$$g_{\pi,FFA} = \frac{1}{\sqrt{3}} g_{ab} , \quad g_{\pi,FFF} = \frac{1}{2} \left(g_{b} - g_{F} \right) .$$

The process $\Sigma^{t} \rightarrow \Lambda + e^{t} + \nu'$ yields for the ratio $\mathcal{D}_{\mathcal{J}_{t}}$ the value ≈ 2 , ref. /10/. Using this ratio and eqs. (21) all the coupling constants can be expressed via the strong pionnucleon interaction constant G'

$$g_{\pi \Sigma \Sigma} = -\frac{2}{3} G', \quad g_{\pi \Sigma A} = -\frac{4}{3\sqrt{3}} G, \quad g_{\pi \Sigma \Sigma} = \frac{1}{3} G. \quad (22)$$
As a result, for $(\prod_{\mu\nu}^{(B)}(q))$ we get
$$(\prod_{\mu\nu}^{(B)}(q)) = \frac{g_{\mu\nu}}{6(2\pi)^2} \left(\frac{G'^2}{M_{\mu\nu}^2}\right) \left[\frac{1}{4} + \frac{4}{9} \left(\frac{M_{\mu\nu}}{M_{\mu\nu}}\right)^2 + \frac{16}{22} \left(\frac{M_{\mu\nu}}{M_{\mu\nu\mu}}\right)^2 + \frac{1}{9} \left(\frac{M_{\mu\nu}}{M_{\mu\nu}}\right)^2 \right] q_{\pm}^2 \quad (23)$$

$$\cong g_{\mu\nu} = \frac{q^2}{6(2\pi)^2} \frac{G'^2}{M_{\mu\nu}^2} = \frac{1}{23} \frac{q^2}{M_{\mu\nu}^2} q_{\pm}^2$$

From where we find

<2'> ~ 0,356 f2.

By adding this value to the contribution of the pion loop we finally obtain for the pion-mean-square radius the following value $^{+)}$

$$\sqrt{\langle z^2 \rangle} \simeq 0,65 f. \qquad (24)$$

This value is in good agreement with the available experimental data /11/.

*) We notice that if eq. (5) is used for the pion-nucleon vertex then the root mean-square radius slightly decreases $\sqrt{\langle z^2 \rangle} \sim Q^{62f}$

5. Discussion of the Results.

The diagrams considered give the main contribution to the pion form factor. As we have seen, the baryon triangular diagrams define almost completely the coefficient for the 9^{ℓ} - term while the pion loop gives the correct energy description of the form factor in the threshold region. The account of the K- meson loop little affects the coefficient for 9^2 since, as the calculations show, the contribution of it is much smaller than even a small contribution of the pion loop. The form factor behaviour in the threshold region is also little affected by the K- meson contribution. Therefore we do not take into account the K meson diagrams. Inserting eqs. (17) and (23) to (8) we get for the form factor the ultimate expression

$$\begin{aligned} F_{T}^{r}(q) &= 1 + \frac{1}{3} \left(\frac{m_{\pi}}{2\pi r_{\pi}^{2}} \right)^{2} \left\{ -1 + \frac{q^{2}}{m_{\pi}^{2}} \left[0, 61 + 1, 53 \right]^{2} + \frac{q^{2}}{9 m_{\pi}^{4}} \left(\frac{4m_{\pi}^{2}}{q^{2}} - 1 \right)^{3/2} - \alpha \cos \left(\frac{q}{9} \left(\frac{4m_{\pi}^{2}}{q^{2}} - 1 \right)^{-\frac{1}{2}} \right)^{2} \right\}. \end{aligned}$$
(25)

The first figure in the square brackets corresponds to the pion loop, the second one to the baryon diagrams. This formula well describes the pion form factor behaviour in the region of small

 q^2 up to the production threshold for two pions as well as in the threshold region. It leads to the pion radius being in good agreement with experiment (eq. 24) and gives a reasonable value for the acattering wave length q_r^2 calculated by the formulas which are valid for small q^2

$$\xi_{g} \delta_{i}^{\prime} = \frac{I_{m} f_{r}(q)}{Re f_{r}(q)} , \quad \delta_{i}^{\prime} = a_{i}^{\prime} \left(\frac{q^{2}}{4} - m_{\pi}^{2}\right)^{\frac{1}{2}}, \quad (26)$$

where δ_{l}^{t} is the $\pi\pi$ - scattering phase in the state $I = \mathcal{F}$ =1.

The absolute values of the pion form factor calculated by eq. (25) in the threshold energy region of 9^2 are also in good agreement with experimental data recently obtained at Dubna^{/12/}. The corresponding values of the form factor are given in Table I.

9 ² /4m ²	0,85	1,1	1;45
F, ^{exa} (q)	1,10±0,07	1,14±0,06	1,3050,07
F. Thear	1,12	1,16	1,22

A less strict result is the following one. If we sum up a chain of diagrams consisting of pion loops then in the obtained expression there can be observed a β - resonance at an energy of about 950 NeV. A similar result has been obtained by Lehmann when analysing the $\pi\pi$ scattering β wave $^{/4/}$. However, the main contribution to the β - meson resonance is found by him to come from the Born term and the nucleon diagrams,

CONCLUSION

The calculations performed show that the pion root-meansquare radius is almost completely defined by the contribution of the baryon triangular diagrams to the pion form factor. The pion

loop, similarly to the k'-meson loops, give a small contribution to the q^4 term of the form factor. At the same time the account of the pion loop is very important for the correct description of the threshold description of the form factor. The set of diagrams in Figs. 1 and 2 defines thereby completely the form factor behaviour in the threshold region.

The question associated with the estimation of the contributions of diagrams of higher orders in $(f_{\mu}^{-\gamma})$ to the form factor remains still open. The estimation of individual diagrams shows that for small g^2 these contributions are not of much improtance. However, this question needs undoubtedly be studied more thoroughly.

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