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RANDOM PROCESS APPROACH TO MULTIPARTICLE PRODUCTION AT HIGH ENERGIES

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RANDOM PROCESS APPROACH TO MULTIPARTICLE PRODUCTION AT HIGH ENERGIES

Душутин Н.К., Мальцев В.М. **E2 - 7276**

Приближение случайных процессов для множественного рождения частиц при высоких энергиях

Множественное рождение частиц при высоких энергиях изучается в приближении случайных процессов. Работа содержит основные результаты, полученные в этом приближении. (Обзор).

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Random Process Approach to Multiparticle Production at High Energies

A review of. some fundamental results obtained in models, which use an approximation of random processes for multiparticle production, is given.

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Much attention is given to the problem of multi-particle production in hadron interaction at high energies [1] and at present a great number of models is suggested and studied together with experimental data both in the range of accelerator and cosmic-ray energies [2].

A number of produced charged particles is one of the most important and directly observable characteristics. But we knew little about this quantity: it is known, that the average multiplicity grows slowly with the energy (presumably, as $\ln S$ or S^k, where S is equal to initial energy in lab. system and K is equal to or less than 1/4), but sometimes we observe the events, which differ from the average distribution [3-5].

Presently, we get the opportunity to study in detail the multiplicity distribution and the data about the existence of high correlations between secondary particles.

From theoretical point of view the cross sections of inclusive processes, i.e. the following form

$$A + B \to C + D + \ldots + \text{anything}$$

where only one part of secondary particles is identified and measured, are described in terms of single- or multi-particle momentum distribution functions, or, equally, by the correlation functions. Thus, the multiplicity distribution in definite region of phase-space allows one to evaluate the momentum distribution and the correlation functions integrated over this region.

This multiplicity distribution was considered from the point of view of correlations by some authors [6] and especially by Mueller, who suggested a good scheme - namely, the generating funotional method.

In the present paper we want to suggest another approach which differs from the Mueller method and allows one to get the correlation functions. Moreover, the physical basis of our approach seems to be more visual.

We call it - " jet approaoh", though the most important is the description of random prooess interaction. Such an approach is somewhat not very new; it was used by: a) Farry [7] for the description of the travelling of high energy electrons through matter; b) it was used for the desoription of nuclear prooesses (so oalled Master Equation Approaoh) [8] ; o) Fujiwara and Kitazoe [9] for construoting a jet model (and we have ohosen this name for our approaoh).

But in present paper we would like to show the possibilities of this method for the description of inclusive processes whioh speoifically differ from the processes considered by the authors, and in more general way than by Fujiwara and Kitazoe [9] .

In Section I we will consider the basic statements of this approach, show how to write down the Chapmen-Kolmogorov equations for the interaction and their corresponding initial conditions, and how to pass over to the momentum distribution equations, i.e. to the definition of correlation funotions.

In Section II we will discuss the solutions of these equations for different types of elementary processes (production, generation and annihilation).

In Section III the correlation functions are considered. At first, we will consider the restriotions on their behaviour, which follow from analyticity and unitarity of the soattering amplitude, and also the behaviour, satisfying scale invarianoe then we will proceed to the correlation form for each of distributions, obtained in previous section.

In Section IV the conolusions are studied and discussed in oomparison with experiment.

I. Basic statements of jet-approach

The most significant in jet approaoh is the following: the interaotion process consists of a set of "elementary" processes, eaoh of which occurs independently and has definite charaoteristic probability.

The following system of the Chapmen-Kolmogorov equation oan be written down for n-particle production probability $P_n^a(t)$ at a time t (a is a sort of particles).

$$\dot{P}_n^a(t)=\sum_{-\ell}^{k}\left[\lambda_i(n-i|t)P_{n-i}^a(t)-\lambda_i(n-i+1|t)P_{n-i+1}^a(t)\right] \quad (1.1)$$

where $\lambda_i(n-i|t)$ is transition probability from the state with $n-i$ partioles to n particle state. The summing is defined by the maximum number of partioles of a kind produced in elementary process. Transition probabilities depend both on elementary prooess probabilities per unit of time and on a number of intermediate particles. So, if we have such prooesses as

$$
\begin{array}{lll}
a \to a + a & \text{per unit of time probability} & g_1^a \\
a \to b + \bar{b} & \text{— — ——} & g_0^a \\
b \to b + a & \text{— — ——} & g_b^a \\
c \to c + a & \text{— — ——} & g_c^a
\end{array}
\tag{1.2}
$$

then we have four transition probabilities, defined as follows

$$
\begin{aligned}
\lambda_1(n-1|t) &= g_1^a (n-1) + g_b^a \sum_{k=0}^{\infty} k \cdot P_k^b(t) + g_c^a \sum_{m=0}^{\infty} m \cdot P_m^c(t), \\
\lambda_1(n|t) &= g_1^a n + g_b^a \sum_{k=0}^{\infty} k\, P_k^b(t) + g_c^a \sum_{m=0}^{\infty} m\, P_m^c(t), \\
\lambda_{-1}(n+1|t) &= -g_0^a (n+1), \\
\lambda_{-1}(n|t) &= -g_0^a n
\end{aligned}
\tag{1.3}
$$

And if we have more kinds of particles, we can define other transition probabilities. In order to make (1.3) complete and closed it is necessary to use the equations for multiplicity distributions of other particles, which can be produced as a result of an interaction. The boundary conditions on this system are connected with the initial particle taken into account. We can consider the initial particles independently of secondaries. Then we write

$$
P_n^a(0) = \delta_{0n} = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}
\tag{1.4}
$$

this follows from the fact that at the moment of interaction the secondary particles do not exist yet. Adding to (1.3) the following terms, connected with the secondary particle generation

by the initial particles, we get

$$
\sum_{k} \mu_k^a \left(P_{n-k}^a(t) - P_{n-k+1}^a(t) \right)
\tag{1.5}
$$

where μ_k^a is per unit of time probability for the process

$$
\text{initial particles} \longrightarrow k\,a \;+\; \text{anything}
\tag{1.6}
$$

If the initial and secondary particles are of the same sort (e.g. a sort), then $P_n^a(\tau)$ is the production probability as a result of interaction, i.e. at the moment $t=\tau$ $(n+1)$ (or $n+2$) a sort of particles and the average "true" number of a sort of particles differ from a number of secondary particles (i.e. by 1 or 2) calculated from $N_a = \sum_{n=1}^{\infty} n\, P_n^a(\tau)$. The system (1.1) for the a sort of particles and the same systems for b, c, ... sorts of particles are highly connected, because in $\lambda_i(n-i|t)$ transition probabilities of one sort of particles we have the average numbers of other particles. To simplify the solution, it is advisable to make some propositions about the behaviour of average particles, and then to make bootstrap i.e. to calculate these numbers from the equations

$$
\begin{aligned}
N_a &= \sum_{n=1}^{\infty} n\, P_n^a(N_a, N_b, \dots) \\
N_b &= \sum_{k=1}^{\infty} k\, P_k^b(N_a, N_b, \dots) \\
&\quad \dots \qquad \dots
\end{aligned}
\tag{1.7}
$$

The general character of the behaviour and the shape of multiplicity distribution weakly depend on the behaviour of average multiplicities and, thus, we can suppose that almost always transition probabilities do not depend on time (except some special cases (e.g. Feynman-gas) when other suppositions are available).

Such an assumption is equivalent to the following: all processes occur virtually during the time of interaction, and then produced particles turn into physical particles. Thus, to define the concrete form of the model it is necessary to know:

1) production channels (the sort of produced particles, at least);

2) the number of final particles, produced in elementary reactions (one-particle, two-particle, etc. productions);

3) time dependence of the average number of particles ($N_a(t)$ = Const – is more convenient).

The first two statements are more important.

It is not necessary to solve equaitons for multiplicity distribution in order to calculate the correlation functions, because you can calculate them according to factorial moments, the equations for which can be obtained from (1.1). To get the equation for the i -th factorial moment $\alpha_{[i]} = \langle n(n-1)\dots(n-i+1)\rangle$ we must multiply both sides of the equation for $P_n^a(t)$ by $n(n-1)\dots(n-i+1)$ and sum them from 0 to infinity. As a result, we get the linear differential equaitons

$$\dot{\alpha}_{[i]} = -f(g_0^a, g_1^a, \dots)\,\alpha_{[i]} + \mathcal{F}(g_0^a, g_1^a, \dots g_b^a, g_c^a, \dots N_a, N_b, \dots) \qquad (1.8)$$

where $\mathcal{F}(\{g\},\{N\})$ is some function (most often, Const) and the coefficient of $\alpha_{[i]}$ is connected both with the creation and annihilation processes only.

II. Some sorts of elementary processes and distributions following from them.

Elementary processes can be chosen in the following way:

1) creation processes, i.e. processes in which one particle of some sort turns into several particles of the same sort;

$a \to a + a$ probability per unit of time g_1^a (2.1a)

$a \to a + 2a$ probability per unit of time g_2^a (2.1b)

2) annihilation processes, i.e. the processes opposite creation, in which one or several particles of "a" sort are absorbed

$\left.\begin{matrix} a \to b + \bar{b} \\ a + b \to c + d \end{matrix}\right\}$ probability per unit of time g_0^a (2.2a)

$a + a \to c' + d'$ probability per unit of time g_0^{2a} (2.2b)

3) generation processes, i.e. in which one or several a -sort particles are produced with the help of another sort of particles.

$b \to b + a$ probability per unit of time g_b^a (2.3a)

$b \to b + 2a$ probability per unit of time g_b^{2a} (2.3b)

We can also consider the contributions of several processes. It seems reasonable to consider only one-particle production and annihilation processes (i.e. in which only one-particle is produced); for annihilation processes this approximation is quite available owing to the small probability of simultaneous double particle absorption (2.2b) but in creation processes this approximation is justified because of the simplicity since the inclusion of one-particle creation gives satisfactory results (from the point of view of correlations).

Now, let us consider in detail the distributions which satisfy different sorts of elementary processes.

If we have creation processes with only one-particle production, and the initial and secondary particles are of the same sort, then (1.1) can be written down as follows:

$$\dot{P}_n^a(t) = -g_1^a\left[(n+2)P_n^a(t) - (n+1)P_{n-1}^a(t)\right]. \quad (2.4)$$

The solution of this system for physical particles (i.e. at the moment of $t=\tau$) is

$$P_n^a(\tau) = (n+1)e^{-2g_1^a\tau}\left(1-e^{-g_1^a\tau}\right)^n \quad (2.5)$$

If we suppose, that in the initial state we have only one particle, then we replace $(n+2)$ and $(n+1)$ in (2.4) by $(n+1)$ and n, respectively, and we come to the distribution, obtained by Fujiwara and Kitazoe for one pure pion jet.

We note, that the requirement of jet uninteractions, used by Fujiwara and Kitazoe to get the distribution in 2 jets, is not necessary, because the equation system for 2 initial particles can be always written down (moreover, if the 2 jet formation accurs in the same volume, then the assumption about the independent production cannot be justified

from the point of view of jet approach). Let us emphasize once more the need of adjusting boundary conditions and the system of equations for multiplicity distribution. To illustrate this statement, we suppose that if there is no particle in the initial state then (2.4) with boundary conditions (1.4) does not have any solution at all.

Let us write the average multiplicity for solution (2.5)

$$N_a = 2\left(e^{g_1^a\tau} - 1\right) \quad (2.6)$$

the dispersion in this case is

$$D^2 = 2e^{g_1^a\tau}\left(e^{g_1^a\tau} - 1\right) \quad (2.7)$$

A consideration of the pure creation processes is interesting only from the academical point of view, because physical analogue for such processes does not exist. So, we will consider the case, when both the creation and annihilation processes are possible. We can consider annihilation processes in several ways:

firstly, annihilation processes are the same as creation processes but having the negative probability. Then the factor g_1^a, is replaced by $g_1^a - g_0^a$ in (2.4) and solutions can be written in the form of (2.5), (2.6), (2.7), with the same change in arguments.

If we present the annihilation processes as independent ones, then we have the following system (one-particle in the initial state) for one-particle annihilation and creation processes:

$$\dot{P}_n^a(t) = -(n+1)(g_1^a+g_0^a)P_n^a(t) + ng_1^a P_{n-1}^a(t) + (n+2)g_0^a P_{n+1}^a(t). \quad (2.8)$$

Its solution can be written as

$$P^a_n(\tau)=\left(1-\frac{g^a_0}{g^a_1}\right)\left(1-e^{-g^a_1\tau+g^a_0\tau}\right)^n\left(1-\frac{g^a_0}{g^a_1}e^{-g^a_1\tau+g^a_0\tau}\right)^{-n-1} \qquad (2.9)$$

which coincides with (2.5) if we replace the argument

$$\left(1-e^{-g^a_1\tau}\right) \quad \text{by} \quad \left(1-e^{-g^a_1\tau+g^a_0\tau}\right)\Big/\left(1-\frac{g^a_0}{g^a_1}e^{-g^a_1\tau+g^a_0\tau}\right)$$

and take the same number of inital particles. When production probability is equal to annihilation probability, (2.8) takes the following form:

$$P^a_n(\tau)=\frac{(g^a_1\tau)^n}{(1+g^a_1\tau)^{n+1}} \qquad (2.10)$$

We can consider annihilation processes, using another method. Let us assume that elementary annihilation processes, owing to their small probability, are realized only from the ground state. In this case we have

$$\dot{P}^a_n(t)=-\left[(n+1)g^a_1+ng^a_0\right]P^a_n(t)+ng^a_1P^a_{n-1}(t) \qquad (2.11)$$

Then we obtain

$$P^a_n(\tau)=\left(Y-\frac{g^a_1}{g^a_0}\right)^{\frac{g^a_1}{g^a_0-g^a_1}}\left(Y-1\right)^{\frac{g^a_0}{g^a_0-g^a_1}}\left(\frac{g^a_1}{g^a_0}Y\right)^n \qquad (2.12)$$

where

$$Y=\left(1-\frac{g^a_1}{g^a_0}\right)\left\{\frac{1-e^{(g^a_0-g^a_1)\tau}}{1-\frac{g^a_1}{g^a_0}e^{(g^a_0-g^a_1)\tau}}\right\}$$

It also coincides with (2.5) if the corresponding change of the argument is made. Thus, applying any method of consideration, the general form of multiplicity distribution does not change; it remains equal to geometric distribution, as for creation processes, but has different arguments. It is interesting to point out, that if we replace g^a_1 by $(g^a_1-g^a_0)$ and assume annihilation priority over $g^a_0>g^a_1$ then some numbers of secondary particles are forbidden (as having negative production probability). In our case this represents mathematical apparatus costs, though this fact can be used in resonance description.

Let us proceed to generation processes. If in elementary processes it is possible to produce only one particle of a sort with effective probability $a_a(t)$ (i.e. by all other possible particles), defined as:

$$a_a(t)=g^a_b\sum_n nP^b_n(t)+g^a_c\sum_m mP^c_m(t)+\dots \qquad (2.13)$$

then for the normalized multiplicity distribution we have:

$$\dot{P}^a_n(t)=-a_a(t)\left[P^a_n(t)-P^a_{n-1}(t)\right] \qquad (2.14)$$

As a solution of this equation we have

$$P^a_n(\tau)=e^{-N_a}\frac{(N_a)^n}{n!} \qquad (2.15)$$

where N_a is an average number of particles

$$N_a=\int_0^\tau a_a(t)\,dt \qquad (2.16)$$

Such a distribution is characteristic of thermodynamical models, and, thus, the Fujiwara and Kitazoe[9] requirements of nonequilibrium are not necessary.

If we assume that 2-particle generation processes exist with the effective probability $a_{2a}(t)$, defined as $a_a(t)$; then we come to the following system

$$\dot{P}_n^a(t) = -a_a(t)\left[P_n^a(t) - P_{n-1}^a(t)\right] - a_{2a}(t)\left[P_{n-1}^a(t) - P_{n-2}^a(t)\right] \quad (2.17)$$

Its solution is

$$P_n^a(\tau) = (n!)^{-1} e^{\frac{1}{2}(N_a^2 - A) - N_a} \left(\frac{N_a^2 - A}{2}\right)^{\frac{n}{2}} H_n\left(\frac{N_a^2 - A + N_a}{\sqrt{-2N_a^2 - 2A}}\right) \quad (2.18)$$

where $H_n(x)$ are the Hermit polynomials N_a -the average number of particles and A are defined by the following relations

$$N_a = \int_0^\tau \left[a_a(t) + a_{2a}(t)\right] dt = f_1$$

$$A = 2\int_0^\tau N_a(t)\left[a_a(t) + a_{2a}(t)\right] dt + 2\int_0^\tau a_{2a}(t)\, dt = f_2 - f_1^2 \quad (2.19)$$

This coincides with the distribution for Feynman-gas[10] (the second equalities determine the connection with the Mueller correlation functions[6]). There is nothing unexpected in our coincidence, because binary-interaction, characteristic for Feynman-gas (short-range forces), is equivalent to the pair particle production.

Now we will consider the top case of generation processes,

when the production of any number of particles, up to " n " simultaneously is possible. In this case n effective probabilities enter the equations for multiplicity distributions:

$$\dot{P}_n^a(t) = -\sum_{k=1}^{n} a_{ka}(t)\left[P_{n-k+1}^a(t) - P_{n-k}^a(t)\right] \quad (2.20)$$

Let k particle production probability $a_{ka}(t)$ be connected with $k-1$ particle production probability in the following form:

$$a_a(t) = F_1(t)$$

$$a_{2a}(t) = F_2(t) \quad (2.21)$$

$$a_{ka}(t) = a_{(k-1)a}(t) - F_2\, \frac{e \cdot \Gamma(n-k;1)}{(k-1)!\,(n-k-1)!}, \quad k \geq 3$$

where $F_{1,2}(t)$ are some functions, e is the base of natural logarithms; $\Gamma(\alpha, x)$ is an uncomplete gamma function. Though this connection is somewhat specific, its consequences result in experimentally observed behaviour of the correlation functions. Using the connection in (2.20), we obtain:

$$\dot{P}_n^a(t) = -F_1(t)\left[P_n^a(t) - P_{n-1}^a(t)\right] - \sum_{i=0}^{n} P_{n-i}^a(t)\,(-1)^i \sum_{\nu=i}^{n} F_2(t) \binom{\nu}{i} \frac{1}{\nu!} \quad (2.22)$$

The solution is

$$P_n^a(\tau) = e^{2N_a(N_a - 1)} \sum_{k=0}^{n} \frac{\left[N_a(N_a - \frac{3}{2})\right]^k}{k!} \cdot \prod_{i=0}^{k-1} \sum_{m=0}^{n-i-1} (-1)^m e\, \frac{\Gamma(n-m-1;1)}{m!\,(n-m-1)!} \quad (2.23)$$

where

$$N_a = \int_0^{\tau} \frac{F_2(t)}{F_1(t) - \frac{3}{2}}\, dt \tag{2.24}$$

is the average number of particles. This distribution oan be approximated more simply

$$P_n^a(\tau) \approx \frac{e^{2N_a(N_a-1)}}{n!} \cdot \frac{\left[N_a\left(N_a - \frac{3}{2}\right)\right]^n - 1}{N_a^2 - \frac{3}{2}N_a - 1} \tag{2.25}$$

which coincides in principal terms with the Czyzewski and Rybicki[11] empirical distribution. Now we will consider combined distributions. If there exist one-particle creation and one-particle generation prooesses, then we can represent the normalized multiplicity distribution as follows:

$$\dot{P}_n^a(t) = -g_1^a\left[n P_n^a(t) - (n-1) P_{n-1}^a(t)\right] - a_a(t)\left[P_n^a(t) - P_{n-1}^a(t)\right] \tag{2.26}$$

The solution

$$P_n^a(\tau) = \frac{a_a \ldots \left[a_a + (n-1) g_1^a\right]}{(g_1^a)^n\, n!}\, e^{-\int_0^{\tau} a_a(t)\,dt} \left(1 - e^{-g_1^a \tau}\right)^n \tag{2.27}$$

coincides by form with the Fujiwara and Kitazoe[9] distribution. If the simultaneous generation of 2 particles is possible, then we have

$$\dot{P}_n^a(t) = -g_1^a\left[n P_n^a(t) - (n-1) P_{n-1}^a(t)\right] - a_a(t)\left[P_n^a(t) - P_{n-1}^a(t)\right] - a_{2a}(t)\left[P_{n-1}^a(t) - P_{n-2}^a(t)\right] \tag{2.28}$$

Its solution is

$$P_n^a(\tau) = \exp\left\{-\int_0^{\tau}\left[2a_a(t) + a_{2a}(t)\right]dt\right\} \frac{Y^n}{n!} \cdot \frac{\partial^n}{\partial Y^n} \exp\left\{\int_0^{\tau}\left[a_a(t) - \frac{1}{2}a_{2a}(t)\right]dt\right\}, \quad Y = 1 - e^{-g_1^a \tau} \tag{2.29}$$

We will not consider the combination with annihilation prooesses as they lead to a shift of the argument in (2.28) (2.29) distributions only but do not change their forms. A combination of creation and generation processes of 3 or more particles does not seem to be reasonable, as it does not give any important things, but makes the distribution more complete.

To sum up:

1) There exist 3 sorts of elementary processes: generation, creation and annihilation.
2) Creation processes lead to the geometrical distribution; while generation processes lead to more sharp distribution (Poisson, Feynman-gas).
3) Three new types of distributions are found: two for combined processes (creation + one-particle generation and creation+ two-partiole generation) and one for maximum generation. The combinations with annihilation processes do not lead to changes in a distribution form (only its argument is changed).

Later we will consider the obtained distributions from the point of view of their fitness to the desoription of experimentally observed situation.

III. Correlations in models of jet approach.

To characterize the correlations we will use correlation parameters, which are calculated simply (e.g.[4])

$$\rho_1 = \frac{1}{\sigma_{tot}} \int \frac{d\sigma}{dy}\, dy = \langle n \rangle$$

$$\rho_2 = \frac{1}{\sigma_{tot}} \int \frac{d^2\sigma}{dy_1 dy_2}\, dy_1 dy_2 - \left(\frac{1}{\sigma_{tot}} \int \frac{d\sigma}{dy}\, dy \right)^2 = \langle n(n-1) \rangle - \langle n \rangle^2 \qquad (3.1)$$

$$\rho_3 = \frac{1}{\sigma_{tot}} \int \frac{d^3\sigma}{dy_1 dy_2 dy_3}\, dy_1 dy_2 dy_3 - 3 \int \frac{d^2\sigma}{dy_1 dy_2}\, dy_1 dy_2 \cdot \frac{1}{\sigma^2_{tot}} \cdot \int \frac{d\sigma}{dy}\, dy - \left(\frac{1}{\sigma_{tot}} \int \frac{d\sigma}{dy}\, dy \right)^3 = \langle n(n-1)(n-2) \rangle - 3\langle n(n-1) \rangle \langle n \rangle - \langle n \rangle^3$$

From experimental data, obtained recently, high-order correlations are considered to be well-established (higher than four, at least)[4]. To prove this, there are some theoretical speculations [12], which follow as from the momentum conservation laws, so from dynamical effects (Pomeron exchange non-factorisability). Another characteristic feature of correlation functions is their sign alteration with the energy increase which follows from the conservation laws of four momenta[2]. Besides, correlation parameters, (i.e. the correlation functions integrated over phase space) higher than second order, have approximately the same magnitude, but alter in sign.

Let us consider some restrictions on correlation functions. Using the Froissart theorem[13]

$$\sigma_{tot}(s) < \ln^2\left(\frac{s}{s_0}\right) \qquad (3.2)$$

and the inequality

$$\rho_1(s_1) = \frac{1}{\sigma_{tot}} \int \frac{d\sigma}{dy_1}\, dy_1 \le \frac{1}{2}\left(\frac{d\sigma}{dy}\right) \frac{1}{\sigma_{tot}} \int_0^Y dy = \frac{1}{2}\left(\frac{d\sigma}{dy}\right) \frac{Y}{\sigma_{tot}}, \qquad (3.3)$$

where

$$y_i = \ln\left(\frac{s_i}{m_i^2}\right), \quad Y = \ln\left(\frac{s}{m^2}\right) \qquad (3.4)$$

are rapidities of i -th secondary and projectile particles, we have

$$\rho_1(s_1) < \frac{s_1}{s}\, n \,. \qquad (3.5)$$

The factor n appears because of the identity of particles. Inequality (3.5) coincides with the result, obtained by Logunov et al.[14]

Similarly, for the second correlation function we get

$$\rho_2(s) \le \frac{s_1 s_2}{s^2}\left(1 - \ln\frac{s}{s_0}\right)\frac{n(n-1)}{2} - \frac{s_1^2}{s^2} n^2 = \rho_1(s_1)\rho_1(s_2)\left(1 - \ln\frac{s}{s_0}\right) - \rho_1^2(s_1) \,. \qquad (3.6)$$

For estimation we assume that both particles have equal energy, then

$$\rho_2(s) \le -A\rho_1^2(s) \qquad (3.7)$$

where the coefficient A is approximately equal to unity. The restrictions on the correlation functions of higher order are weaker, than experimentally observed amplitude constancy of the correlation functions, so we will not consider them.

The condition that KNO scaling does exist can be written in the form:

$$\sqrt{\frac{D^2}{\langle n\rangle^2}} = \sqrt{1+\frac{\rho_2}{\rho_1^2}} = Const. \qquad (3.8)$$

Now we proceed to correlations for different distributions, obtained earlier.

For (2.5) distribution we may write down the correlation functions of any order

$$\rho_1 = 2\left[e^{g_1^a \tau} - 1\right]$$

$$\rho_2 = 2\left[e^{g_1^a \tau} - 1\right]^2 = \frac{1}{2}\rho_1^2$$

$$\rho_3 = 4\left[e^{g_1^a \tau} - 1\right]^3 = \frac{1}{2}\rho_1^3 \qquad (3.9)$$

$$\rho_m = \frac{2}{e^2}(m-1)\,\Gamma(m-2;-2)\left[e^{g_1^a \tau} - 1\right]^m$$

Scaling is fulfilled and scaling constant is equal to

$$\sqrt{\frac{D^2}{\langle n\rangle^2}} \simeq 1.7 \qquad (3.10)$$

But this distribution has some drawbacks: the behaviour of correlation functions does not agree with the experimentally observed one and does not satisfy the restriction (3.7).

The substitution of g_1^a by $g_1^a - g_0^a$ (an account of annihilation processes) in (3.9) does not change essentially the situation.

For (2.9) distribution the form of correlation functions is analogous:

$$\rho_1 = \left(\frac{g_0^a}{g_1^a - g_0^a}\right)\left[e^{(g_1^a - g_0^a)\tau} - 1\right]$$

$$\rho_2 = \left(\frac{g_0^a}{g_1^a - g_0^a}\right)\left[e^{(g_1^a - g_0^a)\tau} - 1\right]^2 = \frac{g_1^a - g_0^a}{g_0^a}\,\rho_1^2 \qquad (3.11)$$

$$\rho_m = \frac{(m-1)}{e^2}\left(\frac{g_0^a}{g_1^a - g_0^a}\right)\Gamma(m-2;-2)\left[e^{(g_1^a - g_0^a)\tau} - 1\right]^m$$

For the scaling constant we have

$$\sqrt{\frac{D^2}{\langle n\rangle^2}} = \sqrt{\frac{g_1^a}{g_0^a}}. \qquad (3.12)$$

Though the correlation functions of given distribution can be put in an agreement with experimental data and the condition (3.7), the agreement is possible in the lower part of spectra only (i.e. for momenta less than 20–30 GeV/s).

It is known, that correlation functions for Poisson distribution equal zero, though the scaling is fulfilled.

For Feynman-gas (2.18) distribution correlation parameters are

$$\rho_1 = \int_0^\tau [a_a(t) + a_{2a}(t)]\,dt$$

$$\rho_2 = 2\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt \cdot \int_0^t [a_a(t_1) + a_{2a}(t_1)]\,dt_1 - \left\{\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt\right\}^2 + 2\int_0^\tau a_{2a}(t)\,dt$$

$$\rho_3 = 6\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt \int_0^t [a_a(t_1) + a_{2a}(t_1)]\,dt_1 \cdot \int_0^{t_1} [a_a(t_2) + a_{2a}(t_2)]\,dt_2 + 6\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt \cdot \int_0^t a_{2a}(t_1)\,dt_1 + 6\int_0^\tau a_{2a}(t)\,dt \int_0^t [a_a(t_1) + a_{2a}(t_1)]\,dt_1 - \left\{\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt\right\}^3 - 6\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt \left\{\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt \int_0^t [a_a(t_1) + a_{2a}(t_1)]\,dt_1\right\} - 6\int_0^\tau [a_a(t) + a_{2a}(t)]\,dt \left\{\int_0^\tau a_{2a}(t)\,dt\right\} \quad (3.13)$$

When $a_a(t)$ and $a_{2a}(t)$ are polynomials in t, the expressions are simplified

$$\rho_1 = \int_0^\tau [a_a(t) + a_{2a}(t)]\,dt$$

$$\rho_2 = 2\int_0^\tau a_{2a}(t)\,dt \quad (3.14)$$

$$\rho_3 = 0, \quad \rho_{m>3} = 0.$$

The distribution has two independent parameters, so the behaviour of the first and the second correlation parameters can be agreed both with restriction (3.7) and with the scaling but as it was pointed out, the distribution has the lack of correlation functions higher than the second order.

The correlation parameters (2.23) can be presented as follows

$$\rho_1 = N_a = \int_0^\tau \frac{F_2(t)}{F_1(t) - \frac{3}{2}}\,dt$$

$$\rho_2 = \rho_1^2 - \frac{3}{2}\rho_1 \quad (3.15)$$

$$\rho_m = (-1)^m \rho_2$$

This agrees with experimentally observed behaviour of correlation parameters. The value $\left[D^2/\langle n\rangle^2\right]^{1/2}$ is equal to

$$\sqrt{\frac{D^2}{\langle n\rangle^2}} = \sqrt{2 - \frac{3}{2}\rho_1^{-1}} \quad (3.16)$$

i.e. the scaling is reached in assymptotics only but the value of the scaling constant does not coincide with that, obtained by Koba,Nielsen and Olesen for their generating functional /6/. Condition (3.7) for this model also holds.

For distribution (2.27) the correlation functions are as follows:

$$\rho_1 = \frac{g_a}{g_1^a}\left(e^{g_1^a \tau} - 1\right), \quad g_a = \int_0^\tau a_a(t)\,dt,$$

$$\rho_2 = \frac{2g_a}{g_1^a}\left(e^{g_1^a \tau} - 1\right)^2 \qquad (3.17)$$

$$\rho_m = \frac{g_a}{g_1^a}(m-1)!\left(e^{g_1^a \tau} - 1\right)^m \sum_{\ell=0}^{m-3}\left(-\frac{g_a}{g_1^a}\right)^\ell \frac{1}{\ell!}$$

They agree with experimental data only with annihilation processes taken into account and under condition that annihilation probability is larger, than creation probability only in the region of not high energies. Next section will be dedicated to (2.27)-sort of distribution, which describes processes of diffractive dissociation kind.

The existence of the scaling for (2.27) can be illustrated as follows

$$\sqrt{\frac{\mathcal{D}^2}{\langle n\rangle^2}} = \sqrt{1 + \frac{g_1^a}{g_a}} = \mathrm{Const} \qquad (3.18)$$

Condition (3.7) is also fulfilled. As it is known, diffractive dissociation processes give small contribution with energy increase, so the behaviour of correlation parameters (3.17) can be agreed with experimental ones at high energies. For

(2.28) distribution there exist correlation parameters, which differing from 0 in any order, are too unwieldy; so for the first three equations we have:

$$\rho_1 = \frac{f_1}{g_1^a}\left[e^{g_1^a \tau} - 1\right]$$

$$\rho_2 = (\rho_1)^2 + \frac{f_2}{2(g_1^a)^2} e^{2g_1^a \tau} - \frac{f_2 \tau}{g_1^a} - \frac{f_2}{2(g_1^a)^2} \qquad (3.19)$$

$$\rho_3 = -(\rho_1)^3\left[1 + \frac{2}{3}\left(\frac{g_1^a}{f_1}\right)^2 + \frac{2}{3}\frac{g_1^a f_2}{(f_1)^3} + \frac{10}{3}\frac{g_1^a}{f_1}\right] - \rho_1\rho_2 - e^{2g_1^a\tau}\left[\frac{f_2}{g_1^a} + 8\left(\frac{f_1}{g_1^a}\right)^2\right] + e^{g_1^a \tau}\left[\frac{2f_2}{(g_1^a)^2} - \left(\frac{f_1}{g_1^a}\right)^2\right] - \frac{f_1 f_2}{(g_1^a)^2} + \frac{13}{3}\frac{f_1 f_2 \tau}{(g_1^a)^2},$$

$$f_1 = \frac{1}{\tau}\int_0^\tau \left[a_a(t) + a_{2a}(t)\right]dt, \quad f_2 = \frac{1}{\tau^2}\int_0^\tau a_{2a}(t)\,dt.$$

From (3.19) it is clearly seen, that the correlation functions have correct sign alternation.

Three free distribution parameters allow us to coordinate the behaviour of correlation parameters with experimentlally observed ones.

The scaling condition

$$\sqrt{\frac{\mathcal{D}^2}{\langle n\rangle^2}} = \left[2 + \frac{f_2}{2f_1^2} + \frac{f_2}{2f_1 g_1^a} - \frac{f_2}{2(g_1^a)^2}\cdot\frac{1 - g_1^a\tau}{\langle n\rangle^2}\right]^{1/2} \qquad (3.20)$$

is fulfilled assymptotically at high energies. (3.7) is also fulfilled. So, we conclude:

1) Experimentally observed existence of the correlation functions up to n -order satisfies the distributions, which include production(annihilation) processes and sometimes generation processes (when the simultaneous production of any number of particles is possible).

2) Distribution for pure creation processes gives correlation functions which differ from 0 in any order, but their behaviour cannot agree with experimentally observed distribution.

3) Combined distributions of creation processes with annihilation and creation processes (fragmentation), with one-particle generation give correlation parameters which agree with experimentally observed behaviour in the region of not very high energies (up to 20 GeV/c). The scaling for such a distribution exists.

4) The top case of generation processes, combined creation processes and two-particle generation give a consistent behaviour of correlation parameters in the whole energy interval.

• Here the scaling is achieved only asymptotically.

IV. Comparison with experiment and discussion.

To compare distributions, obtained earlier, with experiment it is necessary to attach some physical and practical meaning to probabilities of "elementary processes". We will use the approximation by physical processes, i.e. elementary processes, probability $a \to a + a$ is assumed to be equal to the probability of physical processes $a + B \to a + a + C$ (where B and C are target particles before and after interaction correspondingly).

$$g_1^a \tau = \frac{\sigma(a+B \to a+a+C)}{\sigma_{tot}(aB)}$$

$$g_b^a \tau = \frac{\sigma(b+B \to b+a+C)}{\sigma_{tot}(bB)} \qquad (4.1)$$

Though this method allows us to calculate all the unknown distribution parameters, its applicability is not quite substantiated. So, when possible, it is necessary to calculate elementary process probabilities from such equations:

$$P_0^a(\tau) = \frac{\sigma_{el}(AB)}{\sigma_{tot}(AB)} \qquad (4.2)$$

where A and B are initial particles. The given relation (at first used by Fujiwara and Kitazoe[9]) satisfies any sorts of elementary processes. The only drawback of such an approach is that we can write down as many relations (4.2) as sorts of particles obtained after the interaction, while in models there are often much more parameters.

One more approximation may be proposed for the elementary process probabilities. From Regge analysis we get, that these probabilities must be linear functions of rapidity[+)], i.e.

$$g_1^a \tau = \alpha_1 Y + \beta_1$$
$$\int_0^{\tau} a_a(t)\,dt = \alpha_2 Y + \beta_2 \qquad \text{etc., where } Y = \ln\left(\frac{s}{m_A m_B}\right). \tag{4.3}$$

Though the number of parameters doubles, it allows us to calculate the approximate energy behaviour of the distribution. The average multiplicity dependence on energy for all sorts of distributions is close to that experimentally observed:

distr.(2.6), (2.8)	$N_a \sim s^{\alpha}$	
distr.(2.10), (2.15) (2.18), (2.25)	$N_a \sim \ln\left(\frac{s}{s_0}\right)$	(4.4)
distr. (2.12)	$N_a \sim s^{\alpha} \ln\left(\frac{s}{s_0}\right)$	
distr.(2.27), (2.29)	$N_a \sim \left[\left(\frac{s}{s_0}\right)^{-\gamma}+1\right] \dfrac{\ln\left(\frac{s}{s_0}\right)}{\ln\left(\frac{s}{s'}\right)}$	

where $0 \le \alpha \le 0.5$, $0 \le \gamma \le 1$

+) Rapidity for the first time was used by Chernikov /18/ for the analysis of events in the momentum space.

All the given distributions do not differ much. The distributions, which include creation and annihilation processes, are somewhat wider than Poisson distributions (i.e. they have more low and more wide maximum). The distributions which take account of several particle generation simultaneously, are narrower than Poisson distribution (the maximum is more sharp), moreover, the distribution becomes narrower with the increase of simultaneously generated particles. However, even in the limiting case (when simultaneous generation of any number of particles is possible) the distribution is not so narrow that we cannot define using any calculations, what distribution is in the best agreement with experiment.

There is one more possibility to compare the prediction with experiment. Excluding unknown parameters, we have obtained certain sum rules which connect topological cross-sections. These sum rules for interactions, including various kinds of elementary processes, can be represented as follows:

annihilation and creation processes
(distributions (2.6), (2.8), (2.10)

$$\sigma_{n-1} \cdot \sigma_{n+1} = \sigma_n^2 \tag{4.5}$$

one-particle generation (distribution (2.15))

$$\frac{(n^2-1)}{n^2}\,\sigma_{n-1} \cdot \sigma_{n+1} = \sigma_n^2 \tag{4.6}$$

two-particle generation (distribution (2.18))

$$\frac{n^2}{(n-1)}\cdot\frac{\sigma_n^{\ 2}}{\sigma_{n-1}\sigma_{n+1}}+\frac{(n+1)(n-2)}{n}\cdot\frac{\sigma_{n-2}\cdot\sigma_{n+1}}{\sigma_n\,\sigma_{n-1}}+$$

$$+\frac{(n+2)(n-1)}{(n+1)}\cdot\frac{\sigma_{n+2}\,\sigma_{n-1}}{\sigma_n\cdot\sigma_{n+1}}=(2n+1)+\frac{n^2-4}{n+1}\cdot\frac{\sigma_{n+2}\,\sigma_{n-2}}{\sigma_{n+1}\,\sigma_{n-1}} \tag{4.7}$$

the limiting case of generation

(distribution (2.25))

$$\frac{(n+1)\sigma_{n+1}-\sigma_n}{n\sigma_n-\sigma_{n-1}}=\frac{n\sigma_n-\sigma_{n-1}}{(n-1)\sigma_n-\sigma_{n-2}} \tag{4.8}$$

the combination of creation prooesses and one-particle generation

$$\frac{(n+1)}{n}\cdot\frac{\sigma_{n+1}}{\sigma_n}+\frac{(n-1)}{(n-2)}\frac{\sigma_{n-1}}{\sigma_{n-2}}=\frac{2n}{(n-1)}\frac{\sigma_n}{\sigma_{n-1}} \tag{4.9}$$

When creation probability becomes small, the last rule turns into (4.6), when generation probability is small, it turns into the following

$$\frac{(n^2-1)}{n^2}\cdot\frac{\sigma_{n+1}}{\sigma_n}=\frac{n}{(n-1)}\cdot\frac{\sigma_n}{\sigma_{n-1}} \tag{4.10}$$

where σ_n is any sort of n -particle production cross-seotion. Besides, the well known Regge sum rules /15/ are fulfilled for all distributions:

$$\sigma(AA)\cdot\sigma(BB)=\sigma^2(AB) \tag{4.11}$$

where A and B are initial particles. The fulfilment of this rule is connected with factorization of elementary process probabilities over different sorts of incident particles.

Short conclusions

1) The behaviour of the average multiplicity for all distributions agrees with the experimentally observed one.

2) Regge sum rules are fulfilled for all distributions.

3) Some detailed calculations are needed to compare multiplicity distributions with experiment and to choose the most adequate one.

V. Conclusions

In the present paper we used some results, obtained in previous studies /17/ (where some statements are concretized and some models for certain sorts of interactions are studied, e.g. NN- interactions with consideration of concrete types of elementary processes).

In the given paper we wanted to show the possibilities of the jet approach (i.e. the random process approximation) but these possibilities are much wider.

It would be interesting to research the possibility to use conditional multiplicity distribution for the description of inclusive processes, i.e. such functions, which will simultaneously describe the distributions of two or more different sorts of particles. The possibility to research the correlations between particles of different sorts (though the usual distribution functions, which were used in the given paper, are available for correlation research between one sort of particles only) is very important. The first steps along such a research have been already made (a model of $\bar{N}N$ -annihilation into pions, kaons and hyperons has been obtained where the distribution of charged and neutral pions is described by the same function) [17].

The second approach is the utilization of the random vector, but not scalar distribution function. In this case the elementary process probability is a random quantity. So the multiplicity distribution can be represented as the Veneziano amplitude. [17]

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