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1. Introduction

In the present paper we shall obtain a set of sum rules for the virtual Compton scattering amplitudes on the basis of an investigation of the vector current anticommutator matrix elements and of the corresponding structure functions with "wrong" (i.e. opposite to the usual ones) crossing properties. Our main purpose is to derive and to discuss a class of new relations proceeding from the dynamical assumption that the leading light-cone singularity of the local current-hadron interaction amplitudes may be abstracted from the quark free field theory model. In order to clarify our terminology, we shall say that a sum rule has a "right" signature when the basic object in its derivation is a causal amplitude which is usually defined by a T-product or a retarded commutator of currents. The absorptive part of such an amplitude is expressed through a current commutator, fulfilling the microcausality condition. It is easy to realize that the crossing relations yield a trivial vanishing of the even (odd) moment energy integrals of odd (even) amplitudes. Hence it is impossible to get, from dispersion relations sum rules like, for example, the bremsstrahlung-weighted sum rule for the photoabsorption total cross section, known in quantum mechanics /1/. The derivation of such a type of sum rules requires model assumptions about the current operator structure in an infinite momentum frame /2-4/. Since the sum rules of "wrong" signature present a real interest, especially for studying hadron electroproduction processes, /5/ it justifies our attempt to propose and discuss a more general approach to their derivation.

We applied in this work a sum rule derivation technique proposed by Dicus, Jackiw and Teplitz (further DJT) in Ref. [6] DJT have shown in their paper [6] that the use of light-cone current commutators for deriving sum rules of "right signature" for the virtual Compton scattering amplitudes is more general and advantageous than the "old" method of the equal-time current commutator algebra in an infinite momentum frame. Contrary to DJT we shall not consider the commutator but the current anticommutator restricted to a light-like hyperplane $x^0 + x^3 = 0$. Only the leading light-cone singularity of the current anticommutator can be expressed in terms of known bilocal operators. So, instead of sum rules valid for arbitrary fixed q^2, we are led in this case to asymptotic ones, which must hold, in principle, only in the limit of $-q^2 \to \infty$. However, one would expect the obtained sum rules do not only present an academic interest and they may provide a really useful information in comparison with experiments for not too large values of q^2. This is suggested by a well-known empirical fact, namely the very successful average description (i.e. in the sense of finite energy sum rules) of the experimental data on inelastic $e-p$ scattering by some universal scale-invariant function for relatively small $q^2 (-q^2 \gtrsim 1\, Gev^2)$ [7]. If the appearance of this "precocious" scaling in the sum rules of "right signature" turns out to be closely linked to the smooth behaviour of the bilocal form factors near the light-cone and if, as one can think, causality does not play here any essential role, then we are encouraged to expect our sum rules to be valid in the region: $-q^2 \gtrsim 1\, Gev^2$. The assumption of smooth behaviour ensures the integrals of the bilocal form factors, which enter the sum rules, will be q^2 -independent. We are here concerned with such a specific q^2 -dependence, which might occur in principle in our case, but would automatically drop down in the current commutator case by virtue of its assumed causal structure. Thus our more general approach for deriving electroproduction sum rules of "wrong signature" clearly shows they can be applied for testing more critically the smooth behaviour dynamical assumption made for the bilocal form factors in the vicinity of the light-cone. And finally, one can exploit the additional information about the existence of a local limit for some bilocal operators for studying the high energy asymptotic behaviour of the structure functions.

A more detailed analysis of these questions will be given in the next sections.

2. Derivation of the Sum Rules

In this section we shall follow as far as possible, the notations adopted by DJT [6] for defining the kinematical structure of the virtual Compton scattering diagonal matrix element. The only difference is that instead of a commutator we shall study the anticommutator matrix element of two conserved vector currents between fermion states with 4-momentum p and spin s:

$$A^{\mu\nu}_{ab}(p,q) = \int d^4x \exp(iqx) \langle p,s \,|\{ V^{\mu}_a(x), V^{\nu}_b(0)\}|\, p,s\rangle , \qquad (1)$$

where the spin state is described by the vector

$s^{\alpha} = i\,\bar{u}(p)\,\gamma^{\alpha}\gamma_5\, u(p)$ and a and b are the isotopic spin indices.

By virtue of the general invariance principles the amplitude (1) has the tensor structure:

$$\begin{aligned} A^{\mu\nu}_{ab}(p,q) &= (-g^{\mu\nu} + q^{\mu}q^{\nu}/q^2)\, A^{ab}_L(\nu, q^2) + \\ &+ [\, p^{\mu}p^{\nu} - \frac{\nu}{q^2}(p^{\mu}q^{\nu} + p^{\nu}q^{\mu}) + \frac{\nu^2}{q^2} g^{\mu\nu}]\, A^{ab}_2(\nu, q^2) \qquad (2) \\ &+ i\epsilon^{\mu\nu\alpha\beta} s_{\alpha} q_{\beta} A^{ab}_3(\nu, q^2) + i(qs)\,\epsilon^{\mu\nu\alpha\beta} p_{\alpha} q_{\beta} A^{ab}_4(\nu, q^2), \end{aligned}$$

where $\nu = (pq)$. The structure functions A^{ab}_i may be decomposed into parts symmetric $(A^{(ab)})$ and antisymmetric $(A_i^{[ab]})$ with respect to the isotopic spin indices:

$$A_i^{ab} = A_i^{(ab)} + i A_i^{[ab]}, \quad i = L, 2, 3, 4. \tag{3}$$

The following crossing-symmetry relations then emerge from Eqs. (1)-(3):

$$\begin{aligned} A_i^{(ab)}(\nu, q^2) &= A_i^{(ab)}(-\nu, q^2), \quad i = L, 2, 3 \\ A_4^{(ab)}(\nu, q^2) &= -A_4^{(ab)}(-\nu, q^2). \end{aligned} \tag{4}$$

Inverse relations hold for the $A_i^{[ab]}$, i.e. $A_4^{[ab]}$ is a symmetric function of ν, etc. It results from definition (1) that for $\nu > 0$ $A_{ab}^{\mu\nu}(p,q)$ is proportional to the absorptive part of the causal forward Compton scattering amplitude, i.e.:

$$A_i^{ab}(\nu, q^2) = W_i^{ab}(\nu, q^2), \quad \nu > 0, \tag{5}$$

where $W_i(\nu, q^2)$ are the conventional structure functions defined in Ref. /6/. Note that the symmetry properties of the functions $W_i(\nu, q^2)$ and $A_i(\nu, q^2)$ with respect to the substitution $\nu \to -\nu$ are opposite. By means of a dimensional analysis procedure the functions $W_i(q^2, \nu)$ can be expected to obey the scaling laws (automodel behaviour) /8/ :

$$W_L(\nu, q^2) = -\frac{1}{2\omega} F_L(\omega), \quad \nu W_2(\nu, q^2) = F_2(\omega),$$

$$\nu W_3(\nu, q^2) = F_3(\omega), \quad \nu^2 W_4(\nu, q^2) = F_4(\omega)$$

in the Bjorken region: $-q^2 \to \infty$, $\nu \to \infty$ with $\omega = -q^2/2\nu$ fixed.

We now apply the general DJT method [6] and arrive at the basic equality:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} dq^- A^{\mu\nu}_{ab}(p,q)|_{q^+=0} = \int d\xi \exp(-iq_\perp x_\perp) \langle p,s| \times \tag{6}$$

$$\times \{ V^\mu_a(x), V^\nu_b(0) \} \delta(x^+) |p,s\rangle ,$$

where

$$q^{\pm} = \frac{q^0 \pm q^3}{\sqrt{2}}, \qquad q_\perp = (q_1, q_2).$$

The different components $(\mu,\nu = +,-,1,2)$ of the left-hand side of Eq. (6) can be transformed into integrals of the structure functions $W^{ab}_i(\nu,q^2)$ by using Eqs. (4) and (5). We have listed these integrals in table 1. The evaluation of the right-hand side of Eq. (6) requires to specify the current anticommutator on the light-like hyperplane $x^+=0$.

3. Current Anticommutator Near the Light-Cone in the Quark Free Field Theory Model.

According to the hypothesis of Fritzsch and Gell-Mann [9] we assume that *the quark free field theory model* may be used for abstracting the null-plane current algebra structure. In such a model, the vector current operator is given by:

$$V^\mu_a(x) = :\bar\psi(x)\gamma^\mu \frac{1}{2}\lambda_a \psi(x): , \tag{7}$$

where : : denotes the normal field operator product.

The free current anticommutator then reads:

$$\{ V^\mu_a(x), V^\nu_b(0) \} = :\bar\psi(x)\gamma^\mu \frac{1}{2}\lambda_a S^{(1)}(x)\gamma^\nu \frac{1}{2}\lambda_b \psi(0): +$$

$$+ :\bar{\psi}(0)\gamma^{\nu}\frac{1}{2}\lambda_b S^{(1)}(-x)\gamma^{\mu}\frac{1}{2}\lambda_a\psi(x):$$
$$+2:\bar{\psi}(x)\gamma^{\mu}\frac{1}{2}\lambda_a\psi(x)\bar{\psi}(0)\gamma^{\nu}\frac{1}{2}\lambda_b\psi(0):+\dots, \tag{8}$$

where we did not write the singular C -number part which shall play no role in the following,

$$S^{(1)}(x)=(i\gamma^{\mu}\partial_{\mu}+m)D^{(1)}(x)$$
$$iD^{(1)}(x)=D^{(+)}(x)-D^{(-)}(x) \tag{9}$$

(Here and in what follows we adopt for the singular functions the notations and definitions of Bogolubov and Shirkov [10]).

The idea of light-cone dominance, i.e. that the leading light-cone singularity of the current product plays a dominant role in the Bjorken limit, suggests us to neglect in the following the unknown contribution of the four-operator bilocal product which does not involve singularities on the cone. It is just this approximation which formally gives our sum rules the meaning of asymptotic relations.

Note that the four-operator products automatically cancel in the commutator case.

Taking into account the above mentioned approximation we shall generalize formula (8) and postulate the following null-plane vector current anticommutator (it is understood that only the operator part of the anticommutator is included):

$$\{V_a^{\mu}(x), V_b^{\nu}(0)\}_{x^+=0} \hat{=} -\partial_{\alpha}\{D^{(1)}(x)[f_{abc}\, s^{\mu\nu\alpha\beta} v_{c\beta}(x|0)+$$
$$+\epsilon^{\mu\nu\alpha\beta}\bar{a}_{c\beta}(x|0))+d_{abc}(s^{\mu\nu\alpha\beta}\bar{v}_{c\beta}(x|0)-\epsilon^{\mu\nu\alpha\beta}a_{c\beta}(x|0))]\}+$$

$$+ 2 g^{\mu a} D^{(1)}(x) \partial_a [f_{abc} v^{\nu}_{c}(x|0) + d_{abc} \bar{v}^{\nu}_{c}(x|0)], \qquad (10)$$

where $v^{\mu}_{a}(x|y)$ and $a^{\mu}_{a}(x|y)$ ($\bar{v}^{\mu}_{a}(x|y)$, $\bar{a}^{\mu}_{a}(x|y)$) are the hermitian (antihermitian) parts of the bilocal currents $V^{\mu}_{a}(x|y)$ and $A^{\mu}_{a}(x|y)$ which read in the quark free field theory:

$$V^{\mu}_{a}(x|y) = :\bar{\psi}(x)\gamma^{\mu}\frac{1}{2}\lambda_a \psi(y):$$

$$A^{\mu}_{a}(x|y) = i:\bar{\psi}(x)\gamma^{\mu}\gamma_5\frac{1}{2}\lambda_a \psi(y): \qquad (11)$$

f_{abc} and d_{abc} are the structure constants of the λ -matrix commutators and anticommutators ,

$$s^{\mu\nu a\beta} = g^{\mu a} g^{\nu\beta} + g^{\mu\beta} g^{\nu a} - g^{\mu\nu} g^{a\beta},$$

$\epsilon^{\mu\nu a\beta}$ is the fully antisymmetric tensor and the symbol $\hat{=}$ denotes the approximate character of Eq. (10) due to the fact that we neglected the bilocal operator of canonical dimension $d=6$ (i.e. the four- operator product in the fermion free field theory model). Notice we got the precise form of Eq. (10) by retaining the quark mass which enters Eq. (9) defining the singular functions $S^{(1)}(x)$. It was eliminated from the final expression by using the free field Dirac equation.

Hence, Eq. (10) differs from the free massless quark model results of Fritzsch and Gell-Mann /9/. The less singular mass term contribution in $S^{(1)}(x)$ is needed to ensure the selfconsistency of the sum rule derivation procedure from the different (μ,ν) components of the tensor $A^{\mu\nu}_{ab}(p,q)$ in the left-hand side of Eq. (6). The necessity of retaining such terms less singular than the leading singularity was previously already discussed in connection with sum rule derivation from light-cone commutators /6,11/. The diagonal matrix elements of the anticommutator (10) which enter Eq. (6) are defined through ''bilocal form-factors'':

$$\langle p,s|v_c^\mu(x|0)|p,s\rangle = p^\mu V_c^1(x^2, xp) + x^\mu V_c^2(x^2, xp)$$

$$\langle p,s|a_c^\mu|p,s\rangle = s^\mu A_c^1(x^2, xp) + p^\mu (xs) A_c^2(x^2, xp) + \tag{12}$$

$$+ x^\mu (xs) A_c^3(x^2, xp).$$

Analogous expressions may be written for $\bar{v}_c^\mu(x|0)$ and $\bar{a}_c^\mu(x|0)$. Note that the exact operator part of the null-plane vector current commutator in the quark free field theory may be obtained by means of the substitution $D^{(1)}(x) \to -iD(x)$ in Eq. (10), where $D(x)$ is the conventional Pauli-Jordan commutator function. The operator structure of the commutator coincides with the corresponding expressions of DJT /6/.

Let us point out one difference of principle occuring in the calculation of the right-hand side of Eq. (6) of the current anticommutator matrix element.

Because of the noncausal structure of the $D^{(1)}(x)$ functions, no δ -function of the transverse coordinates $x_\perp = (x_1, x_2)$ enters the r.h.s. of Eq. (6) and in principle there might appear a sensible q^2 -dependence if the bilocal form-factors are not sufficiently smooth-behaved near the light-cone $x^2 \to 0$. The explicit form of the integrals listed in table I is a consequence of this additional assumption of smooth behaviour near the light-cone:

$$V_c^i(x^2, xp) \simeq V_c^i(0, xp), \quad i = 1, 2$$

$$A_c^i(x^2, xp) \simeq A_c^i(0, xp), \quad i = 1,2,3. \tag{13}$$

On the right-hand sides of our sum rules stand the light-cone Fourier-transforms of the bilocal form-factors (13):

$$\tilde{V}_c^i(\kappa) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} da \exp(-i\kappa a) V_c^i(0, a), \quad i = 1,2. \tag{14}$$

and analogous expressions for $A_c^i(0,a)$, $\bar{V}_c^i(0,a)$ and $\bar{A}_c^i(0,a)$.

4. Discussion

Table I provides the complete set of sum rules proceeding from the basic relations (10), (12)-(14) determining the vector current anticommutator near the light-cone.

Note that the um rules derived from the anticommutator of two "bad" current operator components $(\mu,\nu = -,1,2)$ required the additional condition $M^2/q^2 \to 0$, M -nucleon mass. If the structure functions $W_i^{ab}(\nu,q^2)$ obey the "conventional" Regge asymptotia for $\nu\to\infty$ the integrals (I.7) - (I.11) diverge at the upper limit and we shall not further discuss them. Let us first briefly compare our results with the parton model ones. The parton model version of the sum rule (I.1) has a very simple and obvious interpretation:

$$\frac{1}{2\pi}\int_0^\infty d\nu\, W_2(\nu,q^2) = \sum_N P(N)\sum_{i=1}^N Q_i^2 , \qquad (15)$$

where $P(N)$ is the probability for a configuration of N partons carrying charges Q_i in the nucleon.

The spin-dependent sum rules can be expressed through parton distribution functions with fixed values of the spin components which were introduced, for example, in Ref. /12/.

If partons are identified with quarks, Eq. (15) leads to the well-known relations:

$$\frac{1}{2\pi}\int_0^\infty d\nu\,[W_2^{ep}(\nu,q^2)-W_2^{en}(\nu,q^2)]=\frac{1}{3}. \qquad (16)$$

The basic ingredients in the derivation of the parton sum rules (15) and (16) are the assumptions of point-like

Table I

Sum rules for virtual forward Compton scattering amplitude from light-cone current anticommutators.

1. $\int_0^\infty d\nu\, W_2^{(ab)}(\nu,q^2)$	$= 2 d_{abc}\int_0^\infty da\, \frac{\tilde{V}_c^1(0,a)}{a}$	$= \int_0^1 \frac{d\omega}{\omega} F_2^{(ab)}(\omega)$
2. $\int_0^\infty d\nu\, W_3^{(ab)}(\nu,q^2)$	$= \pi d_{abc}\int_0^\infty d\kappa\, \frac{\tilde{A}_c^1(\kappa)}{\kappa}$	$= \int_0^1 \frac{d\omega}{\omega} F_3^{(ab)}(\omega)$
3. $\int_0^\infty d\nu\,\nu\, W_4^{(ab)}(\nu,q^2)$	$= i\pi d_{abc}\int_0^\infty d\kappa\, \frac{\tilde{\tilde{A}}_c^2(\kappa)}{\kappa^2}$	$= \int_0^1 \frac{d\omega}{\omega} F_4^{(ab)}(\omega)$
4. $\int_0^\infty d\nu\, W_4^{[ab]}(\nu,q^2)$	$= 0$	$= \int_0^1 d\omega\, F_4^{[ab]}(\omega)$
5. $\int_0^\infty d\nu\, \frac{\nu}{-q^2}\, W_2^{[ab]}(\nu,q^2)$	$= \pi f_{abc}\int_0^\infty d\kappa\, \frac{\tilde{V}_c^1(\kappa)}{\kappa}$	$= \int_0^1 \frac{d\omega}{2\omega^2} F_2^{[ab]}(\omega)$
6. $\int_0^\infty d\nu\, W_L^{(ab)}(\nu,q^2)$	$= 0$	$= \int_0^1 \frac{d\omega}{\omega^3} F_L^{(ab)}(\omega)$
7. $\int_0^\infty d\nu\, \frac{\nu^2}{q^4}\, W_2^{(ab)}(\nu,q^2)$	$= \frac{i\pi}{2} d_{abc}\int_0^\infty d\kappa\, \frac{\tilde{V}_c^1(\kappa)}{\kappa^2}$	$= \int_0^1 \frac{d\omega}{4\omega^3} F_2^{(ab)}(\omega)$
8. $\int_0^\infty d\nu\, \frac{\nu}{-q^2}\, W_3^{[ab]}(\nu,q^2)$	$= \frac{i\pi}{2} f_{abc}\int_0^\infty d\kappa\, \frac{\tilde{\tilde{A}}_c^1(\kappa)}{\kappa^2}$	$= \int_0^1 \frac{d\omega}{2\omega^2} F_3^{[ab]}(\omega)$
9. $\int_0^\infty d\nu\, \frac{\nu^2}{-q^2}\, W_4^{[ab]}(\nu,q^2)$	$= -\pi f_{abc}\int_0^\infty d\kappa\, \frac{\tilde{\tilde{A}}_c^2(\kappa)}{\kappa^3}$	$= \int_0^1 \frac{d\omega}{2\omega^2} F_4^{[ab]}(\omega)$
10. $\int_0^\infty d\nu\,\nu\, W_L^{[ab]}(\nu,q^2)$	$= 0$	$= \int_0^1 \frac{d\omega}{\omega^4} F_L^{[ab]}(\omega)$
11. $\int_0^\infty d\nu\,\nu^2\, W_L^{(ab)}(\nu,q^2)$	$= 0$	$= \int_0^1 \frac{d\omega}{\omega^5} F_L^{(ab)}(\omega)$

partons and impulse approximation, i.e. that the deep inelastic electron-nucleon interaction cross section is supposed to be simply the sum of the quasi-elastic scattering cross section off each parton individually. These assumptions automatically lead to x^2 -independent bilocal form-factors (13) which are expressed through the parton distribution functions in the composite model. The parton model assumption of incoherence for the virtual photon-parton interaction amplitudes is equivalent to our assumption that the four-operator product contribution in Eq. (1) is negligible. Diagrammatically this means that one takes into account only diagrams of the type (a) (see fig. 1) and neglects the contribution arising from the "coherent" ones (of type (b)).

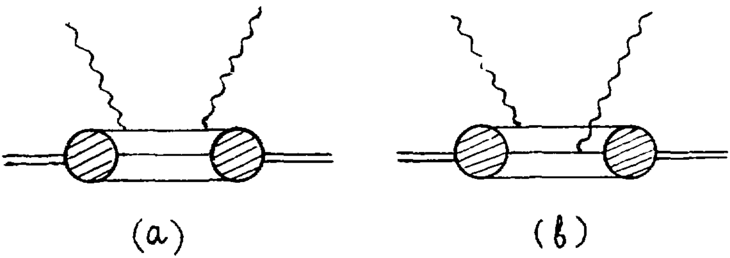

Fig. 1. Feynman graphs describing the "incoherent"(a) and "coherent" (b) virtual photon-parton interactions.

One expects the characteristic parton transverse momenta inside the nucleon are of the same order of magnitude as the experimentally observed average transverse momentum of produced particles in multihadron production processes [13]: $\langle p_\perp \rangle \simeq 300 - 400$ MeV and therefore the parton dynamical correlation may be neglected for $(-q^2)^{1/2} \gg \langle p_\perp \rangle$.

Thus our more formal approach receives an obvious interpretation in parton model language.

Moreover, by introducing the more general concept of bilocal operators we preserve the possibility to include proper corrections to the sum rules of the type (16) if the bilocal form-factors (13) would appear to be insufficiently smooth near the light-cone.

Eq. (11) shows that in the free quark model the bilocal operators coincide with the local vector current operators in the limit $x \to y$. Let us require this condition to hold in the general case of interacting fields and let us try to extract from it, on the basis of our sum rules, supplementary information about the structure function behaviour. Consider the relation (I.2) which we rewrite:

$$\int_0^\infty d\nu\, W_3^{(ab)}(\nu, q^2) = \int_0^\infty \frac{d\omega}{\omega} F_3^{(ab)}(\omega) = \tag{17}$$

$$= \pi d_{abc} \int_0^\infty \frac{d\kappa}{\kappa} \tilde{A}_c^1(\kappa) = d_{abc} \int_0^\infty \frac{d\kappa}{\kappa} \int_0^\infty da\, A_c^1(0,a) \cos \kappa a,$$

where we made use of the symmetry property of the form-factor A_c^1 with respect to the substitution $x \to -x$. At the lower limit of integration over a , A_c^1 coincides with the axial-vector current diagonal matrix element, i.e. it is a finite number. This suggests that the integral (17) over ν logarithmically diverges i.e. that $\nu W_3^{(ab)}(\nu, q^2) \to const,$ when $\nu \to \infty$.

We want to make a last comment on the derivation of photoproduction sum rules from electroproduction ones by means of the transition $q^2 \to 0$. In the limit $q^2 \to 0$ one may no longer neglect the terms nonsingular on the light-cone as we did in Eq. (10).

Within the framework of composite models we interpret these terms as the manifestation of a dynamical parton correlation i.e. the contribution of diagrams (b) in Fig. 1. Without additional assumptions about the particle structure nothing can be said about the contribution of such terms and therefore the sum rules for photoabsorption cross sections essentially express them through other experimentally measurable quantities. Let us write, for example, the photon-nucleon interaction sum rule obtained by taking the derivative with respect to q^2 in Eq. (I.1) at $q^2=0$:

$$\frac{1}{4\pi^2 a}\int_0^\infty \frac{d\nu}{\nu}\,\sigma_{\gamma N}(\nu)-\left(\frac{1}{3}<r_1^2>_N-\frac{\kappa_N^2}{4M^2}\right)=\lim_{q^2\to 0}\frac{\partial}{\partial q^2}C(q^2), \quad (18)$$

where $<r_1^2>_N$ is the mean squared charge radius of the nucleon and κ_N the anomalous magnetic moment. The non-zero right-hand side of Eq. (18) is connected with the possible existence of a q^2 -dependence of the bilocal form-factor integral entering the right-hand side of the sume rule (I.1), which may proceed from the non causal behaviour of the singular function $D^{(1)}(x)$ and also from the contribution of the terms non singular on the light-cone which describe the parton correlations. A more detailed discussion and a calculation of the right-hand side of Eq. (18) considered as a parameter characterizing the parton space correlation inside the nucleon was given in a recent paper /14/.

Additional dynamical assumptions about the parton nature and the symmetry of the ground state of the composite system would permit one to fix the right-hand side of Eq. (18) or to express it through other observables. We hope to publish in a forthcoming paper the results of a verification of photoproduction sum rules within the framework of a perturbative theory.

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