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S.Dubnička, O.V.Dumbrais, F.Nichitiu

DETERMINATION OF THE COUPLING CONSTANTS H^3dn, $H^3H^3\pi$ AND $He^3He^3\pi$, FROM THE DATA ON THE DIFFERENTIAL CROSS SECTIONS OF THE nH^3 AND nHe^3 ELASTIC SCATTERING

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S.Dubnička[1], O.V.Dumbrais[2], F.Nichitiu[3]

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[1] On leave of absence from the Inst. of Physics, Slovak Academy of Sciences, Bratislava, Czechoslovakia.

[2] On leave of absence from the Inst. of Nuclear Physics, Moscow State University.

[3] On leave of absence from the Inst. for Atomic Physics, Bucuresti, Romania.

Summary

Determination of the coupling constants H^3dn, $H^3H^3\pi$ and $He^3He^3\pi$ from the existing data on the differential cross sections of the nH^3 and nHe^3 elastic scattering has been carried out. The method is based on the extrapolation of $\frac{d\sigma}{d\Omega}$ to the deuteron and pion poles exploiting the conformal mapping techniques.

Calculations result in the value of the H^3dn coupling constant which is ~20% smaller than the dispersion relation predictions. The comparison of the obtained value with predictions based on nuclear models allows to discriminate between different potentials.

The residue at the pion pole turns out to be too weak and the conclusion is drawn that it is impossible to determine the coupling constants $H^3H^3\pi$ and $He^3He^3\pi$ on the basis of the existing data.

1. Introduction

The practical exploitation of analyticity, one of the fundamental principles of contemporary particle physics, is penetrating persistently the nuclear physics. This is of great importance because the new aspect emerges here. Namely, it is possible to compare the values of the same quantities obtained in completely different ways. So, for example, the quantities called the particle-nucleus coupling constants can be found, on the one hand, on the basis of data on particle-nucleus scattering by means of analyticity, when a nucleus is considered as a structureless object with a definite set of quantum numbers. On the other hand, the same quantities, known as spectroscopic factors, can be found in nuclear physics on the basis of specific nuclear models.

Speaking about the exploitation of analyticity in nuclear physics one should mention first of all the use of the forward dispersion relations of the particle nucleus scattering. An excellent review on this subject is available already for several years [1]. Some later developments can be found in [2-5]. Other familiar consequences of analyticity in energy plane, e.g. sum rules, different bounds on the amplitudes, etc., are hardly to be expected to benefit very much in the near future due to the lack of sufficiently accurate and complete experimental data.

However, there is one immediate possibility of exploitation of hypothesis of analyticity of the elastic scattering amplitude in the $\cos\theta$ plane. It has been known already for a long time [6,7] that one can make use of this hypothesis by extracting the values of coupling constants by extrapolation of the differential cross section from physical region to the poles of scattering amplitude in the $\cos\theta$ plane. The first applications of this idea to nuclear physics are due to /8,9/.

In the present paper we try to determine in such a way the coupling constants H^3dn, $H^3H^3\pi$ and $He^3He^3\pi$ on the basis of the experimental data on the nH^3 and nHe^3 elastic scattering /10,11/.

2. Singularities for the nH^3 Elastic Scattering in the $\cos\theta$ Plane for Fixed Energy

The singularities of the elastic nH^3 scattering amplitude in the $\cos\theta$ plane for fixed T, found by means of the investigation of the analytic properties of the formally written Feynman diagrams, are shown in fig. 1. The positions of the singularities are determined according to the following formulas *

$$x_{\bar{n}H^3} = -\left[1 + \frac{(m_n + m_{H^3})^2}{2k^2} - \frac{(m_n^2 - m_{H^3}^2)^2}{2k^2 s}\right], \quad (1)$$

$$x_{np} = -\left[1 + \frac{(m_n + m_p)^2}{2k^2} - \frac{(m_n^2 - m_{H^3}^2)^2}{2k^2 s}\right], \quad (2)$$

$$x_d = -\left[1 + \frac{m_d^2}{2k^2} - \frac{(m_n^2 - m_{H^3}^2)^2}{2k^2 s}\right], \quad (3)$$

$$x_\pi = 1 + \frac{m_\pi^2}{2k^2}, \quad (4)$$

$$x_{\substack{anom.\\ thr.}} = 1 + \frac{m_\pi^2 F}{2k^2}, \quad (5)$$

$$x_{2\pi} = 1 + \frac{2m_\pi^2}{k^2}, \quad (6)$$

where

$$x = \cos\theta \text{ and } F = \frac{1}{m_\pi^2}\left[4m_p^2 - \frac{(m_{H^3}^2 - m_d^2 - m_p^2)^2}{m_d^2}\right] \quad (7)$$

s is c.m.s. energy squared and k is c.m.s. momentum.

One now could use the expansion /7/

$$(x - x_d)^2 (x - x_\pi)^2 \frac{d\sigma}{d\Omega} = \sum_{n=0}^{N} a_n x^n \quad (8)$$

for determination of the coupling constants H^3dn and $H^3H^3\pi$. After fitting the sum on the right hand side of eq. (8) to the experimental data one could carry out continuation to the deuteron and pion poles of the scattering amplitude and to determine the values of the coupling constants H^3dn and $H^3H^3\pi$ respectively. However, it is well known /12,13/, that this procedure is not the optimal one from the point of view of maximal exploitation of analytic properties under consideration. To achieve the latter we apply the technique of conformal mappings.

3. The Conformal Mapping Method

We map the entire cut x-plane onto an unifocal ellipse in the z-plane /12/*. In figs. 2a,b,c the optimal mappings are shown for determination of the coupling constants H^3dn, $H^3H^3\pi$ and $He^3He^3\pi$, respectively.

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* In the case of the nHe^3 scattering the d-pole is absent and in formulas (1)-(7) m_{H^3} must be replaced by m_{He^3} and $m_n \to m_p$.

* In our case the ellipses coincide practically with the circles.

These mappings are known to accelerate maximally the rate of convergence of the expansions of the type (8) and to make it possible to take into account the analytic nature of the data when estimating the errors of the coupling constants.

4. The Parametrization

To determine the coupling constant H^3dn we use a search of the form

$$(z-z_d)^2 \frac{d\sigma(z)}{d\Omega} = \sum_{n=1}^{M} A_n B_n T_n(z), \tag{9}$$

where $T_n(z)$ are Tschebyscheff polynomials ($T_1=1$, $T_2=z$, $T_{m+1}=2zT_n - T_{n-1}$), $B_n=(R^{2(n-1)}+R^{-2(n-1)}+2\delta_{n-1,0})^{-1/2}$; R is the sum of the semi-axis of the ellipse and A_n are coefficients to be found from a fit.

Analogous expressions are used in the case of determination of the coupling constants $H^3H^3\pi$ and $He^3He^3\pi$. Various definitions of coupling constants and their exact relations to the quantity $\sum_{n=1}^{M} A_n B_n T_n(z_{pole})$ are given in Appendix.

The specific form of the right-hand side of eq. (9) is designed to take into account the analytic nature of the data. For this purpose one defines the quantities

$$t_n = \frac{A_{n+1}^2}{\sum_{k=1}^{n} A_k^2}, \quad (1 \le n \le M-1) \tag{10}$$

and evaluates the integrals

$$H_n = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{n}{2})} \int_0^{t_n} \frac{dx}{x^{1/2}(1+x)^{\frac{n+1}{2}}}, \tag{11}$$

where $\Gamma(y)$ is the Gamma function. After finding the quantities ρ_n from the equation

$$\frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{\rho_n}{2}}} e^{-x^2} dx = H_n \tag{12}$$

one can calculate the Cutkosky's convergence test function Φ defined as [14]

$$\Phi = \sum_{n=1}^{M-1} \rho_n . \tag{13}$$

This function controls the goodness of the convergence of the expansion (9). One now can combine it with the χ^2, which controls the goodness of the fit, and define the new quantity

$$X = \chi^2 + \Phi \tag{14}$$

which is merging the statistic and analytic nature of the data. Now the rule is to stop in the fitting procedure at the M which minimizes X instead of that minimizing χ^2.

5. The Results of the Fit

We have fitted the experimental data on the $\frac{d\sigma(\theta)}{d\Omega}$ of the elastic nH^3 scattering at 1 MeV, 2 MeV, 3.5 MeV, 6 MeV [10] and on the $\frac{d\sigma(\theta)}{d\Omega}$ of the elastic nHe^3 scattering at 1 MeV, 2 MeV [10], 2.67 MeV [11], 3.5 MeV [10], 5 MeV [11], 6 MeV [10], 8.07 MeV, 17.5 MeV [11].

The results are presented in Tables 1,2,3.

From Table 1 one can see that according to the criterium of minimizing the quantity X one should keep three terms in the expansion (9) for the 1 and 2 MeV data and four terms for other energies. This provides, whithin the errors, consistent values of the coupling constant H^3dn for the first three energies. The data at 6 MeV give significantly smaller value for this coupling constant. The reason for this is not clear for us. Anyway, one should mention that the experimental data at 6MeV differ significantly (have larger dip at medium angles and flatter slope at backward angles) both from the data at lower energies and from the corresponding theoretical predictions /15/, at larger angles especially.

Examination of Tables 2 and 3 shows that the values of the coupling constants $H^3H^3\pi$ and $He^3He^3\pi$ exhibit violent jumps when changing the number of parameters in the expansion (9). Therefore we conclude that the corresponding residues at the pion pole are too weak to be determined from the existing data.

6. Conclusions

The confrontation of our averaged value of $r_{H^3dn} \simeq 0.3$ over the first three energies with that obtained from the forward dispersion relations of the nd scattering /2/ ($r_{H^3dn} = 0.382 \pm 0.040$) leads to the conclusion that it is $\simeq 20\%$ smaller.

On the other hand, it is very interesting to compare our value $C^2_{H^3dn} \simeq 2.8$ with those obtained by means of methods of nuclear physics.*

In /16/ the nuclear reactions (p,d) and (d,t) have been compared within the framework of the peripheral model and the value $0.73 \lesssim C^2_{H^3dn} \lesssim 4.66$ has been deduced. The modified phase-shift analysis for the pHe^3 elastic scattering has been carried out in /17/ and the value $1.50 \lesssim C^2_{He^3dp} \lesssim 2.52$ has been found. With a wave function of He^3 calculated using the Faddeev equations and some special potential the value $C^2_{He^3d\bar{p}} = 2.86 \pm 0.03$ was found in the work /18/. Similar calculations /19/ of a wave function of H^3 using the Faddeev equations and

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*We are indebted to Prof. Y.E.Kim for discussion of these methods.

other potentials resulted in the estimation $0.28 \lesssim C^2_{H^3dn} \lesssim 5.32$. The values $C^2_{He^3d\bar{p}} = 3.0$ and $C^2_{H^3dn} = 3.4$ have been derived in /20/ from the data of the two body photodisintegration and electrodisintegration of He^3, from realistic H^3 wave functions utilized in transfer reaction and from three-body elastic scattering data.

Thus if one considers our values as being model-independent estimations, then one can give preference to some potentials. It seems that we have manifested the reasonable accuracy of this particular method of determination of coupling constants. Having in mind that the spectroscopic factors must be given the same status as other nuclear parameters such as the binding energy charge radius and form-factors /16–20/, we think that the vast program of determination of different spectroscopic factors can be carried out by means of this method in those cases when the methods of works /1–5/ or /17–20/ cannot be applied. First of all this refers to the spectroscopic factors $_ZA^N_ZA^{N-1}$ * owing to the availability of large amount of good data on differential cross sections of elastic neutron scattering on different nuclei. The same can be said about the spectroscopic factors $_ZA^N_{Z-1}A^N p$. However, in this case the Coulomb corrections should be taken into account properly. Apart from that, we urge experimenters to measure differential cross sections of elastic scattering of kaons of different nuclei. This would provide the opportunity to determine some more exotic spectroscopic factors involving hypernuclei.

We express our deep gratitude to Dr. V.B.Belyaev for valuable discussions.

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* The special care should be taken for the excited states of the nuclei. They must be considered as the extra poles.

Appendix

The squared residue of the scattering amplitude at the pion pole, defined through the expression

$$\lim_{z\to z_\pi} [z(x)-z(x_\pi)]^2 \frac{d\sigma(z)}{d\Omega} \equiv (Res_\pi)^2 = \sum_{n=1}^{N} A_n B_n T_n(z_\pi) \qquad (I)$$

is related to the coupling constant $g^2_{H^3H^3\pi}$ in the following way*

$$(Res_\pi)^2 = g^2_{H^3H^3\pi} g^2_{NN\pi} \frac{(1-x_\pi)^2 10(\hbar c)^2}{4s} \left(\frac{dz}{dx}\right)^2_{x=x_\pi}, \qquad (II)$$

where $g^2_{NN\pi}$ is the rationalized, renormalized, pseudoscalar πNN coupling constant, $g^2_{NN\pi} \approx 14.6$, and the factor $10(\hbar c)^2$ ($\hbar c \approx 0.1973$ fm GeV) is introduced to match the units when $\frac{d\sigma(\theta)}{d\Omega}$ is measured in mb/sterad. The formula (II) can be derived from the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2 s} \sum_{s_i, s_f=-1/2}^{+1/2} |M(s,t)|^2 \qquad (III)$$

substituting for $M(s,t)$ the pion pole contribution to the invariant nH^3 scattering amplitude

$$M_\pi(s,t) = -m_n m_{H^3} \frac{4\pi g_{H^3H^3\pi} g_{NN\pi}}{t-m_\pi^2} \bar{u}(q_2)\gamma_5 \times \qquad (IV)$$

$$\times u(q_1)\bar{u}(p_2)\gamma_5 u(p_1).$$

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* In the case of the nHe^3 scattering H^3 should be replaced by He^3.

calculated by means of the Feynman rules from the diagram shown in fig. 3a. Then we can write

$$\frac{d\sigma(x)}{d\Omega}\Big|_\pi = \frac{g^2_{H^3H^3\pi} g^2_{NN\pi}}{4s} \frac{(1-x_\pi)^2}{(x-x_\pi)^2} \qquad (V)$$

and the limit

$$\lim_{x\to x_\pi} \left[\frac{z(x)-z(x_\pi)}{x-x_\pi}\right]^2 (x-x_\pi)^2 \frac{d\sigma(x)}{d\Omega}\Big|_\pi \qquad (VI)$$

gives exactly the relation (II) for the $(Res_\pi)^2$.

Instead of $g^2_{H^3H^3\pi}$ we prefer to use an analogue of the pseudovector coupling constant f^2 defined by

$$f^2_{H^3H^3\pi} = g^2_{H^3H^3\pi} \frac{m_\pi^2}{4m^2_{H^3}}. \qquad (VII)$$

The squared residue at the deuteron pole, defined by

$$\lim_{z\to z_d} (z-z_d) \frac{d\sigma(z)}{d\Omega} \equiv (Res_d)^2 = \sum_{n=1}^{N} A_n B_n T_n(z_d) \qquad (VIII)$$

is related to the coupling constant $g^2_{H^3dn}$ in the following way

$$(Res_d)^2 = g^4_{H^3dn} \frac{10(\hbar c)^2}{4sk^4} \{2[(\sqrt{(k^2+m_n^2)(k^2+m^2_{H^3})} + k^2)^2 +$$

$$+[m_n^2 + k^2(1-x_d)][m^2_{H^3} + k^2(1-x_d)] -$$

$$-2m_n m_{H^3}(\sqrt{(k^2+m_n^2)(k^2+m^2_{H^3})} + k^2 x_d) + 2m_n^2 m^2_{H^3}] -$$

$$-2\left(\frac{m_{H^3}-m_n}{m_d}\right)^2\left[2m_n m_{H^3}\left(\sqrt{(k^2+m_n^2)(k^2+m_{H^3}^2)}+k^2\right)+\right.$$

$$+(m_n^2+m_{H^3}^2)k^2(1-x_d)+2m_n^2m_{H^3}^2\Big]+$$

$$+\left(\frac{m_{H^3}-m_n}{m_d}\right)^4\left[\sqrt{(k^2+m_n^2)(k^2+m_{H^3}^2)}+k^2x_d+m_n m_{H^3}\right]^2\Big\}\times$$

$$\times\left(\frac{dz}{dx}\right)^2_{x=x_d} \qquad \text{(IX)}$$

which can be derived in the same way as formula (II) from the differential cross section (III) but substituting for $M(s,t)$ the deuteron pole contribution to the invariant nH^3 scattering amplitude

$$M_d(s,t)=-m_n m_{H^3}\frac{4\pi g^2_{H^3dn}}{u-m_d^2}\left[\bar{u}(p_2)\gamma_\mu u(q_1)\bar{u}(q_2)\gamma_\mu u(p_1)-\right.$$

$$\left.-\left(\frac{m_{H^3}-m_n}{m_d}\right)^2\bar{u}(p_2)u(q_1)\bar{u}(q_2)u(p_1)\right] \qquad \text{(X)}$$

calculated by means of the Feynman rules from the diagram shown in fig. 3b.

On the other hand, in the dispersion relations for the forward nd scattering amplitude it is common [2] to use the residue r_{H^3dn} of spin-averaged amplitude which is related to $g^2_{H^3dn}$ by means of the expression

$$r_{H^3dn}=g^2_{H^3dn}\frac{1}{4m_{H^3}\sqrt{2m_n^2+2m_{H^3}^2-m_d^2}}\left[4m_n m_{H^3}-m_n^2-\right.$$

$$-m_{H^3}^2+m_d^2-\frac{(m_{H^3}-m_n)^2(m_n^2+m_{H^3}^2-m_d^2)}{2m_d^2}-$$

$$\left.-\frac{m_n m_{H^3}(m_{H^3}-m_n)^2}{m_d^2}\right] \qquad \text{(XI)}$$

The last relation is obatined through limit

$$r_{H^3dn}=\lim_{\omega\to\omega_0}(\omega-\omega_0)f_d(\omega,0) \qquad \text{(XII)}$$

where ω is the incident neutron laboratory total energy, ω_0 corresponds to the deuteron pole

$$\omega_0=\frac{m_n^2+m_{H^3}^2-m_d^2}{2m_{H^3}}, \qquad \text{(XIII)}$$

$$f_d(\omega,0)=\frac{1}{8\pi\sqrt{s}}\sum_{s_i,s_f=-1/2}^{+1/2}M_d(s,0), \qquad \text{(XIV)}$$

where

$$M_d(s,0)=M_d(s,t)_{t=0}. \qquad \text{(XV)}$$

For nuclear physicists it is more convenient to use the normalization constant C^2 (the spectroscopic factor) of the asymptotic wave function of H^3 which is related in a simple way [2,18] to the residue r_{H^3dn}

$$C^2_{H^3dn}=\frac{4}{3}R\mu r_{H^3dn} \qquad \text{(XVI)}$$

where $R=(2\mu B)^{-1/2}$, $\mu=\frac{m_n m_d}{m_n+m_d}$ and $B\simeq 6.26$ MeV is the binding energy of the deuteron and neutron in H^3.

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Table 1

The results of the fit and the values of the coupling constant H^3dn *.

Energy (MeV)	M	χ^2	Φ	X	$r_{H^3dn} \pm \Delta r_{H^3dn}$	$C^2_{H^3dn} \pm \Delta C^2_{H^3dn}$
1.0	2	34.72	4.99	39.71	0.123 ± 0.003	1.16 ± 0.03
	3	7.75	13.81	21.56	$0.305^{+0.024}_{-0.026}$	$2.87^{+0.23}_{-0.25}$
	4	6.32	27.43	33.75	$0.733^{+0.220}_{-0.326}$	$6.93^{+2.09}_{-3.07}$
2.0	2	320.40	4.26	324.67	0.102 ± 0.002	0.96 ± 0.03
	3	2.56	15.86	18.42	0.311 ± 0.008	2.92 ± 0.08
	4	2.16	17.11	19.28	$0.364^{+0.072}_{-0.090}$	$3.43^{+0.68}_{-0.86}$
	5	1.68	33.85	35.52	$0.865^{+0.413}_{-0.865}$	$8.16^{+3.91}_{-8.16}$
3.5	2	849.75	1.58	851.35	0.058 ± 0.002	0.55 ± 0.03
	3	4.66	12.59	17.25	0.269 ± 0.004	2.55 ± 0.05
	4	3.88	13.11	16.99	$0.234^{+0.039}_{-0.048}$	$2.22^{+0.38}_{-0.45}$
	5	3.37	26.74	30.10	$0.438^{+0.181}_{-0.419}$	$4.13^{+1.71}_{-3.96}$
6.0	2	1280.08	0.58	1280.66	0.017 ± 0.003	0.15 ± 0.03
	3	28.44	9.46	37.90	0.178 ± 0.003	1.69 ± 0.03
	4	2.09	12.46	14.55	$0.054^{+0.039}_{-0.054}$	$0.50^{+0.38}_{-0.50}$
	5	1.98	13.65	15.63	imaginary	imaginary

* $g^2_{H^3dn} \approx 600\, r_{H^3dn}$

Table 2

The results of the fit and the values of the coupling constant $H^3H^3\pi$.

Energy (MeV)	M	χ^2	Φ	X	$f^2_{H^3H^3\pi} \pm \Delta f^2_{H^3H^3\pi}$
1.0	2	103.95	6.06	110.00	-11 ± 0.4
	3	11.49	16.66	28.15	51 ± 6
	4	6.17	32.20	38.37	-206 ± 112
2.0	2	792.03	5.25	797.28	-8 ± 0.3
	3	54.29	15.73	70.02	50 ± 2
	4	3.57	27.90	31.47	-82 ± 19
	5	1.77	43.67	45.44	192 ± 206
3.5	2	1277.88	3.48	1281.36	-3 ± 0.2
	3	161.29	15.03	176.32	38 ± 1
	4	9.20	26.49	35.70	-55 ± 7
	5	3.64	38.69	42.32	62 ± 50
6.0	2	1207.23	0.63	1207.85	1 ± 0.2
	3	258.48	14.95	273.43	20 ± 1
	4	6.80	24.66	31.46	-23 ± 3
	5	1.90	31.88	33.79	4 ± 13
	6	1.73	43.27	45.00	27 ± 57

Table 3

The results of the fit and the values of the coupling constant $He^3He^3\pi$.

Energy (MeV)	M	χ^2	Φ	X	$f^2_{He^3He^3\pi} \pm \Delta f^2_{He^3He^3\pi}$
1.0	2	65.44	5.18	70.63	-4 ± 0.5
	3	4.60	18.69	23.29	56 ± 8
	4	4.32	28.75	33.07	130 ± 140
2.0	2	1148.90	2.42	1151.32	-0.05 ± 0.2
	3	58.37	19.08	77.45	58 ± 2
	4	8.96	31.50	40.47	-77 ± 19
2.6	2	1500.45	4.26	1504.71	-3 ± 0.2
	3	17.10	19.34	36.44	54 ± 1
	4	8.94	27.61	36.55	2 ± 18
	5	8.94	29.61	38.55	12 ± 17
	6	8.55	63.35	71.90	-953 ± 1561
3.5	2	1398.46	0.20	1398.66	1.5 ± 0.1
	3	134.87	18.79	153.66	33 ± 1
	4	9.10	30.89	39.99	-42 ± 7
	5	7.82	39.81	47.62	12 ± 48
5.0	2	775.33	5.38	780.71	-3 ± 0.2
	3	63.01	15.57	78.59	34 ± 1
	4	17.29	29.75	47.04	-30 ± 10
	5	5.72	36.03	41.75	187 ± 65
	6	4.86	58.12	62.97	-191 ± 412
6.0	2	397.42	2.55	399.97	3 ± 0.2
	3	122.44	16.15	138.59	15 ± 1
	4	10.40	29.64	40.04	-25 ± 4
	5	7.95	38.14	46.09	4 ± 19
	6	7.03	56.34	63.37	70 ± 99
8.0	2	1723.29	0.32	1723.61	1 ± 0.1
	3	114.12	14.51	128.63	18 ± 1
	4	33.26	25.49	58.76	-5 ± 3
	5	22.73	38.81	61.54	32 ± 11
	6	21.87	53.18	75.05	-19 ± 56
17.5	2	114.25	0.07	114.32	0.8 ± 0.05
	3	19.72	10.35	30.07	6 ± 1
	4	7.94	22.72	30.66	-2 ± 2
	5	7.21	29.18	36.38	6 ± 9
	6	4.23	50.34	54.57	-47 ± 32

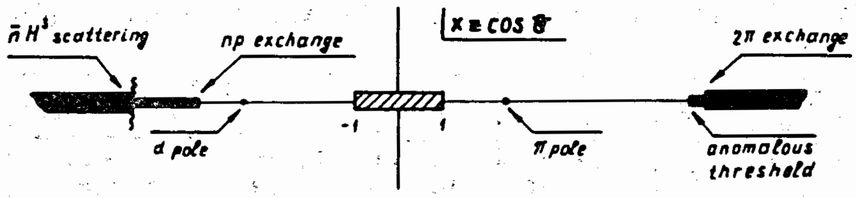


Fig. 1. The analytic structure of amplitude for the nH^3 elastic scattering in the $\cos\theta$ plane. The scale corresponds to $T = 6$ MeV.

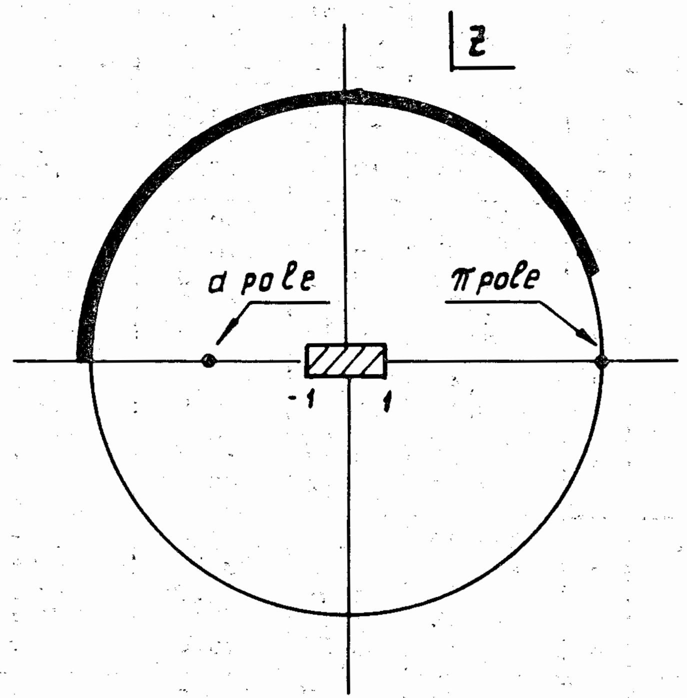
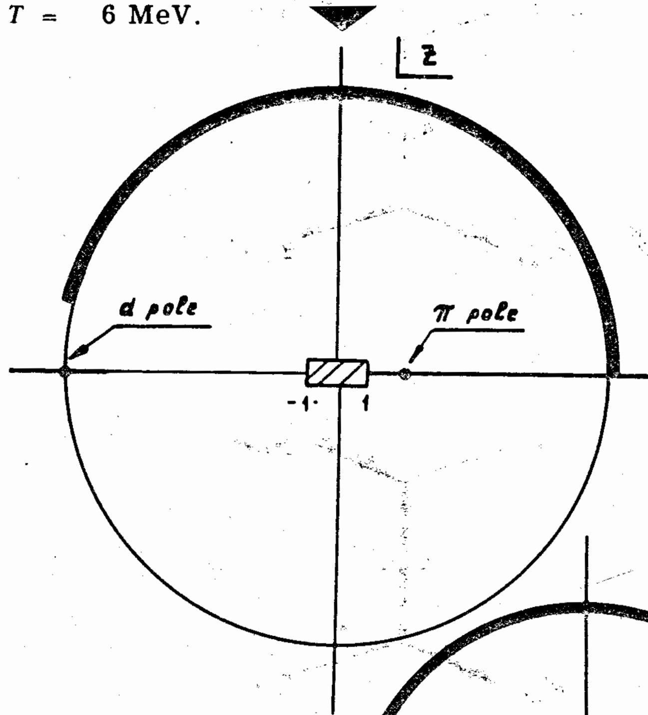


Fig. 2a. The optimal mapping for the determination of the coupling constant H^3dn. The scale corresponds to $T = 6$ MeV.

Fig. 2b. The optimal mapping for the determination of the coupling constant $H^3H^3\pi$. The scale corresponds to $T = 6$ MeV.



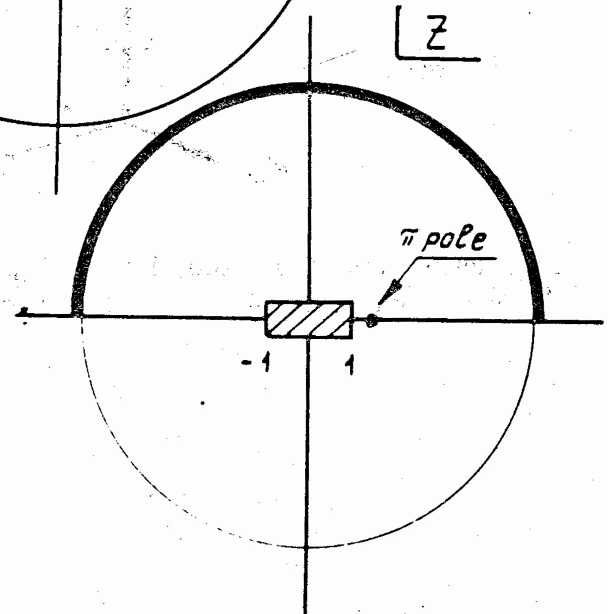


Fig. 2c. The optimal mapping for the determination of the coupling constant $He^3He^3\pi$. The scale corresponds to $T = 17.5$ MeV.

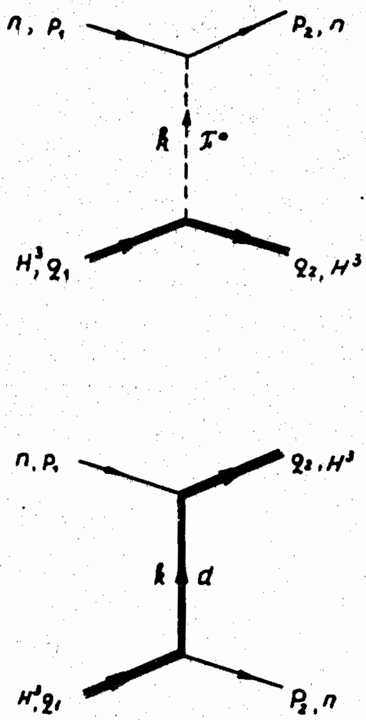


Fig. 3a,b. The pion and deuteron pole contributions to the scattering amplitude of the process $nH^3 \to nH^3$.