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ANALYSIS OF THE $\pi\pi$ SCATTERING δ_0^0 UNCERTAINTY BY THE DISPERSION RELATION METHOD

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ANALYSIS OF THE $\pi\pi$ SCATTERING δ_0^0 UNCERTAINTY BY THE DISPERSION RELATION METHOD

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The properties of $\pi\pi$ scattering are known rather badly. Information about the $\pi\pi$ interaction is mainly extracted from the reactions of the following type:

$$\pi + B \to \pi + \pi + B', \qquad (1)$$

where B, B' are the baryons.

The partial wave analysis of these reactions does not allow one to determine the phase shifts and the inelasticity factors sufficiently precisely. The main ambiguity is connected with the δ_0^0 phase shift.

The most complete partial wave analysis is performed in papers [1-4]. The following different possibilities for the δ_0^0 behaviour were got:

1. The "up" solution with resonance at $W \sim 750$ MeV [1] (Here W is the c.m. energy).
2. The "down" solution without resonance up to $W \sim 1000$ MeV [1].
3. The δ_0^0 phase shift without resonance up to $W \sim 1400$ MeV [2].
4. The δ_0^0 phase shift with resonance at $W \sim 950$ MeV [3].
5. The δ_0^0 phase shift with resonance at $W \sim 1200$ MeV [4].

The phase shifts (2,3,5) are not far from one another up to $W \sim 1000$ MeV.

Let us note that different theoretical models for $\pi\pi$ scattering can lead to δ_0^0 resonance solutions [5-8] as well as to solutions without resonance [9-10]. The related experimental data and the theoretical models are reviewed in refs. [11,12].

It is interesting to analyse this situation on the basis of the dispersion relations (DR). However it is difficult to use the usual once-subtracted DR because of lack of high energy $\pi\pi$ scattering data and the DR slow convergence.

Note that in paper [13] the once-subtracted DR for the partial amplitudes with definite isospin in s-channel was used to show that up to $W \sim 1000$ MeV the solution (2) is more preferable than (1).

In this paper instead of it we use the quickly convergent DR for the amplitude with isospin 2 in t-channel to analyse the ambiguity of the δ_0^o phase shift:

$$Re\, T^{(2)}(s) = \frac{1}{\pi} \int_{4\mu^2}^{\infty} \frac{(2s' - 4\mu^2)\, Jm\, T^{(2)}(s')\, ds'}{(s' - s)(s' + s - 4\mu^2)} . \qquad (2)$$

Here $s = W^2$, the amplitude $T^{(2)}$ is connected with the amplitudes $S^{(I)}$ with definite isospin I in s-channel by crossing relation:

$$T^{(2)}(s) = \frac{1}{3} S^{(0)}(s) - \frac{1}{2} S^{(1)}(s) + \frac{1}{6} S^{(2)}(s) . \qquad (3)$$

The s-channel amplitudes can be expressed in terms of the real phase shifts $\delta_\ell^{(I)}$ and the inelasticity factors $\eta_\ell^{(I)}$:

$$S^{(I)}(s) = \sum_{\ell=0}^{\infty} (2\ell + 1)\, f_\ell^{(I)}(s) = \sum_{\ell=0}^{\infty} (2\ell + 1) \frac{\eta_\ell^{(I)}(s)\, e^{2i\delta_\ell^{(I)}(s)} - 1}{2i\sqrt{\frac{s - 4\mu^2}{s}}} . \qquad (4)$$

It may be shown that the high-energy contribution to eq. (2) has the form:

$$v(s) = \frac{1}{\pi} \int_{A^2}^{\infty} \frac{(2s' - 4\mu^2)\, Jm\, T^{(2)}(s')\, ds'}{(s'-s)\,(s'+s-4\mu^2)} \sim \tag{5}$$

$$\sim \alpha + \beta \ln\left(1 - \frac{s}{A^2}\right) \quad \text{at} \quad s < A^2.$$

Here A lies outside the region of the resonances. Thus, $v(s)$ is a slowly varying function when $W < A$.

Performing the subtraction at the crossing-symmetrical point $s = u = 2\mu^2$ and passing to the new variable $z = ((s - 2\mu^2)/2\mu^2)^2$ we can rewrite DR (2) in the form:

$$Re f_0^{\circ}(z) = C + 3\frac{z}{\pi} \int_1^{z(A)} \frac{Jm\, T^{(2)}(z')dz'}{z'(z'-z)} - 5\, Re f_2^{\circ}(z) + \tag{6}$$

$$+ \frac{3}{2} Re\, S^{(1)}(z) - \frac{1}{2} Re\, S^{(2)}(z).$$

The slowly changing contribution $v(s)$ is included into the subtraction constant.

For numerical calculations we parametrized the following phase shifts and inelasticity factors:

$I = 0$	s	and d - waves	(inelastic)
$I = 1$		p - wave	(inelastic)
$I = 2$	s	and d waves	(elastic)

We take into account the g -meson $(I = 1,\ \ell = 3)$ in the Breit-Wigner form.

We substitute such parametrized expressions for the cases (1,2,4,5) in the r.h.s. of eq. (6) and make an integration up to $W \sim 1700$ MeV. The C -parameter is chosen from the condition of the minimum of χ^2.

The $Re f_0^\circ$ values obtained from the l.h.s. DR are compared with the experimental data. The results of this comparision are plotted in figures 1-4. The corresponding

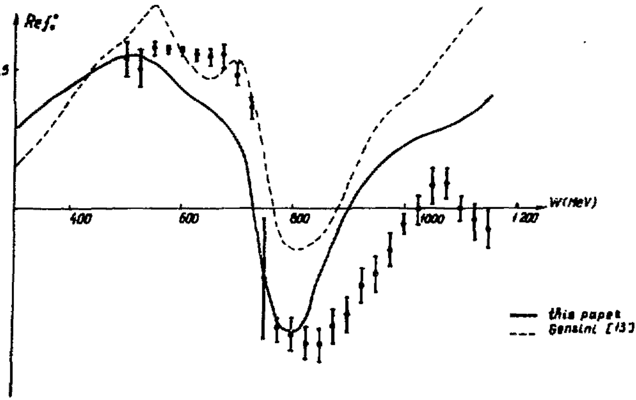

Fig. 1. $Re f_0^\circ$ - for the case (1).

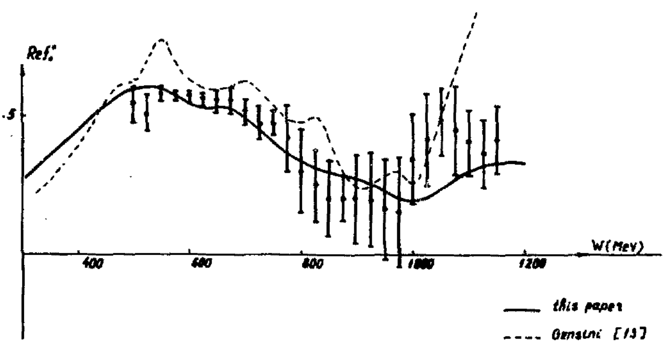

Fig. 2. $Re f_0^\circ$ - for the case (2).

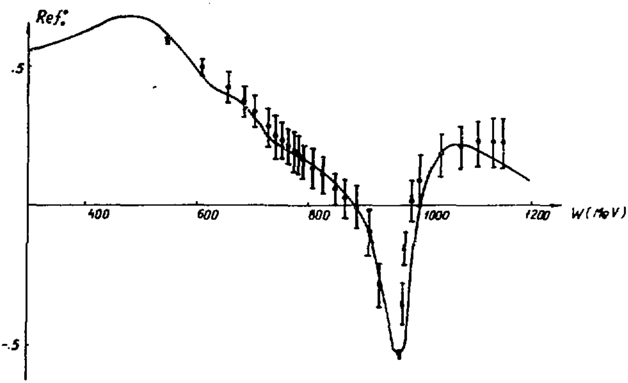

Fig. 3. $Re\, f_0^0$ - for the ca e (4).

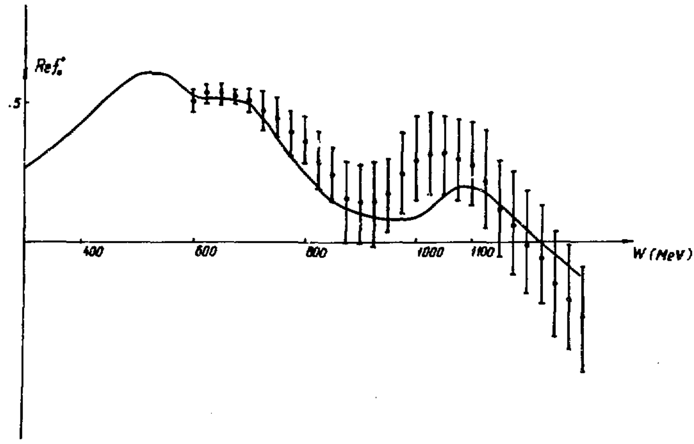

Fig. 4. $Re f_0^0$ - for the case (5).

C and χ^2/N values (where N is the number of the experimental points) are given in the table.

It is easy to see that unsatisfactory description is obtained only in the case (1). The other possibilities for δ_0^0 are practically equivalent. This is due to large errors in the experimental data used which results in wide "allowed corridors" for the dispersion curves.

The results of ref. [13] are shown in figs. 1,2 for the comparison with our results.

The cases (2,3,5) and the corresponding parametrizations for δ_0^0 are plotted in fig. 5. These parametrizations were used in the r.h.s. of DR (6).

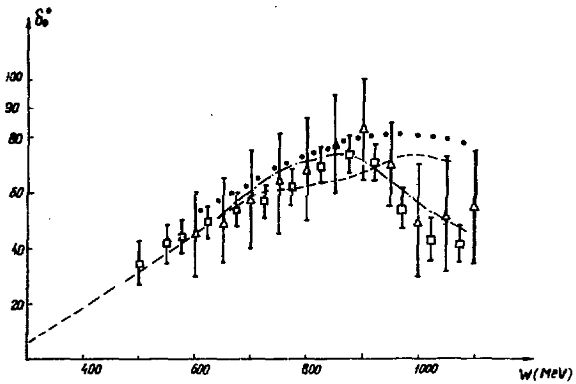

Fig. 5. The experimental data; (2) - □ ; (3) - • ; (5) - Δ ; and the parametrizations: (2) - —·—; (5) - - - - - for the δ_0^0 phase shift.

Notice that the calculated curves for $Re f_0^0$ at $W \gtrsim 500$ MeV are in sensitive to the variation of $Jm f_0^0$ in the threshold region at $W < 500$ MeV. In virtue of it, we can not determine the a_0^0 scattering length.

Thus we can make the following conclusions.

1) The solutions without resonance are essentially favoured up to $W \sim 900$ MeV.

2) The behaviour of the δ_0^0 phase shift at higher energies can not be analysed by the DR for $T^{(2)}$ amplitude. In order to succeed in analysing this region the precision of the experimental data in the inelastic region ($W \gtrsim 1$ GeV) should be essentially improved.

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Table

Variant	C	χ^2/N
I	0,252	24,08
2	0,196	0,88
4	0,486	1,16
5	0,221	0,59