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ON LOCAL PROPERTIES OF DUAL FIELD THEORY

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ON LOCAL PROPERTIES OF DUAL FIELD THEORY

Usually dual models [1] are studied in momentum space which is sufficient for the consideration of most of the interesting physical properties (asymptotic and j -plane behaviour, construction of physical states, elimination of ghost states and so on). Only recently, in connection with the construction of dual amplitudes [2], some interest has been turned to space-time properties of dual models [3,4].

In the following we suppose that currents or fields are determined from off-shell amplitudes on the basis of the classical S -matrix reduction formalism. In the foregoing paper [3] this method has been applied to naturally extrapolated tree amplitudes (Veneziano model). Here we will investigate a dual field theory [5] which has an explicit given off-shell extrapolation like ordinary Feynman field theories. This theory appears as a set of generalized Feynman amplitudes in momentum space which has to be understood as a perturbation series. Restricting our investigations to the simplest amplitudes, we study the self-energy parts of the ground state particle, especially their discontinuities.

As usual we take the asymptotic behaviour of spectral functions as a criterion for local properties [6]. Our calculations done in the second order of perturbation theory announce a non-local behaviour of dual field theory. This is in accordance with the conclusions of previous investigations [3] reached by different methods.

The dual field theory in its standard form [5] is given as a Feynman like theory constructed from vertices and two types of propagators, twisted and untwisted, which are operators in a Hilbert space which contains the states of an infinite set of harmonic oscillators. This corresponds to the fact that there is an infinite number of stable particles with increasing spins.

As the simplest case we consider the self-energy parts of the lowest mass state.

The diagrams and the corresponding expressions are:
a) the self-energy part without twists

$$\Sigma(s) = 4\pi^2 g^2 \int_0^1 dx \int_0^1 dy \; \frac{\omega^{-1-a}}{\log^2 \omega} \; f^{-4}(\omega) \, [(1-x)(1-y)]^{a-1} \times$$

$$\times \{ [\psi(x)]^s - [\tilde{\psi}(x)]^s \}$$

$$\psi(x) = -2\pi i \, e^{\frac{\log^2 x}{2 \log \omega}} \; \frac{\Theta_1\left(\frac{\log x}{2\pi i} \,\middle|\, \frac{\log \omega}{2\pi i}\right)}{\Theta_1'\left(0 \,\middle|\, \frac{\log \omega}{2\pi i}\right)}$$

$$= \exp\left\{ - \frac{\log x \log y}{2 \log \omega} + \sum_1^\infty \frac{2\omega^n - x^n - y^n}{n(1-\omega^n)} \right\}$$

$$\tilde{\psi}(x) = -\frac{\log \omega}{\pi} \sin\left(\pi \frac{\log x}{\log \omega}\right) \qquad (1)$$

$$f(\omega) = \prod_1^\infty (1-\omega^n)$$

$$\omega = xy$$

b) The self-energy part with one twist

$$\Sigma_T(s)=4\pi^2 g^2 \int\limits_{\substack{0\le v\le 1\\ 0\le u^2 v\le 1}} du \int dv\, \frac{(1+\omega)^2}{\log^2\omega}\,\omega^{-1-a} f^{-4}(-\omega)\times$$

$$\times[(1-u^2v)(1-v)]^{a-1}\,\{[\psi_N(v)]^s - [\tilde{\psi}_N(v)]^s\}$$

$$\psi_N(v)=-2\pi i\, e^{\frac{\log^2 v}{2\log\omega}}\,\frac{\Theta_1\left(\frac{\log v}{2\pi i}\,\middle|\,\frac{\log\omega}{2\pi i}+\frac{1}{2}\right)}{\Theta_1'\left(0\,\middle|\,\frac{\log\omega}{2\pi i}+\frac{1}{2}\right)} \qquad (2)$$

$$\tilde{\psi}_N(v)=-\frac{2\log\omega}{\pi}\sin\left(\pi\frac{\log v}{\log\omega}\right)$$

$$\omega = u v$$

c) The double twisted self-energy part

$$\Sigma_{TT}(s)=4\pi^2 g^2\int_0^1 dx\int_0^1 dy\,\frac{(1-\omega)^{1+a}}{\log^2\omega}\,\omega^{-1-a} f^{-4}(\omega)[\psi_T(x)]^s$$

$$\psi_T = 2\pi\, e^{\frac{\log^2 x}{2\log\omega}}\,\frac{\Theta_1\left(\frac{\log x}{2\pi i}+\frac{1}{2}\,\middle|\,\frac{\log\omega}{2\pi i}\right)}{\Theta_1'\left(0\,\middle|\,\frac{\log\omega}{2\pi i}\right)}$$

$$=\exp\left\{-\frac{\log x\log y}{2\log\omega}+\sum_1^\infty\frac{2\omega^n-(-x)^n-(-y)^n}{n(1-\omega^n)}\right\}$$

Θ_1 : Jacobi's Θ function (3)

Θ_1 : Jacobi's Θ function.
The parameter a is the intercept of the real input trajectory taken negative to avoid tachyon states. The counter terms $\tilde{\psi}(x)$ and $\tilde{\psi}_N(x)$ guarantee finite expressions for $\Sigma(s)$ and $\Sigma_T(s)$ respectively (for s in the left half-plane). This represents one possible renormalization procedure /5/. As usual this ambiguity does not influence the discontinuity. The expression for $\Sigma_{TT}(s)$ exists for $Re\, s < -4/3$ and needs no counter term. The latter restriction corresponds to a cut at $s=-4/s$ which is not a consequence of two-particle unitarity. Note that all expressions exist in the left half s-plane only and have to be continued analytically to determine the discontinuities along the positive axis.

The analytical continuation can be done by introducing the integration variables

$$\xi = -\log\omega \qquad 0 \le \xi < \infty$$
$$\theta = \frac{\log x}{\log\omega} \qquad 0 \le \theta \le 1 \tag{5}$$

and choosing a suitable integration path in the ξ plane.

Because we are interested in the discontinuities only we are allowed to simplify the original expressions. Let us begin with Σ_{TT} which has the structure (see Appendix)

$$\Sigma_{TT}(s) = \int_0^1 d\theta \int_0^\infty d\xi\, g(\theta,\xi)\, e^{sF(\xi,\theta)} \quad . \tag{6}$$

Contributions to $disc\,\Sigma_{TT}$ arise from the regions where $F(\xi,\theta) \to \infty$ that is in our case $\xi = 0$ and $\xi = \infty$. Thus we have to study the discontinuity for integrals of the type

$$\Sigma(s,A) = \int_0^1 d\theta \int_A^\infty d\xi\, g(\xi,\theta)\, e^{sF(\xi,\theta)}$$

$$= \int_0^1 d\theta \int_0^\infty dy \; \frac{g(\xi,\theta)}{F_{,\xi}(\xi,\theta)} \; e^{sy} \; e^{sF(A,\theta)} \tag{7}$$

$$y = F(\xi,\theta) - F(A,\theta).$$

A similar consideration of the region $\xi \approx 0$ leads to an identical result for the asymptotics.
Rotating the integration path about the angle ϕ (or $\phi-$) and performing the limit $\phi \to \frac{\pi}{2}$ we get

$$disc\, \Sigma(s,A) = \int_0^1 d\theta\, e^{sF(A,\theta)} \int_{-i\infty}^{+i\infty} dy \, \frac{g(\xi,\theta)}{F_{,\xi}(\xi,\theta)} \, e^{sy} \tag{8}$$

$$y = F(\xi,\theta) - F(A,\theta).$$

Note that $g/F_{,\xi}$ is an analytic function and is bounded in the right y half plane. By choosing A sufficient large we avoid poles of $1/F_{,\xi}$. The limit $\phi \to \frac{\pi}{2}$ exists for all finite s. For the evaluation of the integral (8) we translate the integration path to the left across the nearest singularities of $g/F_{/\xi}$ (stationary points), which determine the leading terms for $s \to +\infty$.

Let us consider the contribution of a stationary point $\xi_0 = \xi_0(\theta)$ corresponding to an expansion

$$F(\xi,\theta) = F(\xi_0,\theta) + b(\theta)(\xi-\xi_0)^2 + \dots \tag{9}$$

(This case with $b(\theta) > 0$ is realised for Σ_{TT}) we get

$$\frac{g(\xi,\theta)}{F_{,\xi}(\xi,\theta)} \approx \frac{g(\xi_0(\theta),\theta)}{2\sqrt{b(\theta)}\,\sqrt{y - F_{,}(\xi_0(\theta),\theta) + F(A,\theta)}}\,. \tag{10}$$

Therefore the contribution from the ξ integration is

$$\int_{-i\infty}^{+i\infty} dy \frac{g(\xi,\theta)}{F_{,\xi}(\xi,\theta)} e^{sy} \approx i\sqrt{\pi}\, \frac{g(\xi_0(\theta),\theta)}{\sqrt{b(\theta)}} \frac{e^{[F(\xi_0(\theta),\theta)-F(A,\theta)]s}}{\sqrt{s}}. \tag{11}$$

Insertion of this leading term into expression (8) gives

$$disc\,\Sigma(s,A) \approx \frac{i\sqrt{\pi}}{\sqrt{s}} \int_0^1 d\theta \frac{g(\xi_0(\theta),\theta)}{\sqrt{b(\theta)}} e^{F(\xi_0(\theta),\theta)s}. \tag{12}$$

One can show that the function $F(\xi_0(\theta),\theta)$ takes its absolute maximum at $\theta = 1/2$, so that the leading contribution arises from the neighbourhood of $\theta=1/2$. Using

$$F(\xi_0(\theta),\theta) = F(\xi_0(1/2),1/2) - c(\theta - 1/2)^2 + \dots \tag{13}$$

we get

$$disc\quad \Sigma(s,A) \approx \frac{i\pi}{2} \frac{g(\xi_0(1/2),1/2)}{\sqrt{b(1/2)c}} \frac{e^{F(\xi_0(1/2),1/2)s}}{s}. \tag{14}$$

Applying this method to Σ_{TT} we have to take into account that the function $F_{,\xi}(\xi,\theta)$ is periodic with respect to ξ for $\theta = 1/2$ (compare eqs. (1c), (2c), (3c)):

$$F_{,\xi}(\xi + 4\pi m i,\ \theta = 1/2) = F_{/\xi}(\xi,\ \theta = 1/2) \tag{15}$$

(m integer).
That means that the stationary points occur in infinite sets $\xi_0 + 4\pi m i$. So we obtain finally

$$disc\ \Sigma_{TT} \approx i\pi \frac{4\pi^2 g^2}{\sqrt{b(1/2)c}} (1-e^{-\xi_0})^{a+1} \prod_1^\infty (1-e^{-n\xi_0})^{-4} \times \tag{16}$$

$$\times \frac{e^{sF(\xi_0,1/2)}}{} \left\{ \frac{1}{\xi_0} + 2\sum_{m=1}^\infty \frac{\xi_0 \cos(4a+\frac{s}{2})\pi m + 4\pi m \sin(4a+\frac{s}{2})\pi m}{\xi_0^2 + (4\pi m)^2} \right\}$$

In the appendix it is shown that there is at least one stationary point ξ_0 with $F(\xi_0, 1/2) > 0$. (A factor 2 arises because of the contributions from $\xi \simeq 0$).

Let us now turn back to the other parts Σ and Σ_T . At first it should be remarked that the integration in the expression for Σ_T has the same range by introducing the variables $x = \sqrt{v}$, $y = u\sqrt{v}$. Therefore both functions may be treated in the same manner. Instead of the variables x, y it is more appropriate to use the variables

$$\theta = \frac{\log x}{\log \omega} \tag{17}$$

$$\eta = -\theta(1-\theta) \log \omega .$$

The typical expression is

$$\Sigma(s) = \int_0^1 d\theta \int_0^\infty d\eta \; g(\eta, \theta) \{ e^{sF(\eta,\theta)} - e^{s\tilde{F}(\eta,\theta)} \}. \tag{18}$$

The presence of the counter term is necessary for convergence at $\eta = 0$. Note that the part

$$\int_0^1 d\theta \int_0^A d\eta \; g \{ e^{sF} - e^{s\tilde{F}} \} \tag{19}$$

exists for all values of s and therefore does not contribute to the discontinuity (Appendix). The same is true for the counter term in the remaining part, so that we have to discuss

$$disc\, \Sigma(s) = disc \int_0^1 d\theta \int_A^\infty d\eta \; g(\eta, \theta) e^{sF(\eta,\theta)} . \tag{20}$$

which has the same structure as the already discussed expressions. By this way we get

$$disc\, \Sigma(s) \approx 4 i\pi \frac{4\pi^2 g^2}{\sqrt{b\,(1/2)c}} \left(1 - e^{-\frac{\xi_0}{2}}\right)^{2(a-1)} e^{a \operatorname{Re} \xi_0} \times$$

$$\times \prod_{1}^{\infty} (1-e^{-n\xi_0})^{-4} \frac{e^{s\,Re\,F(\xi_0,1/2)}}{s} \times \tag{21}$$

$$\times \sum_{\ell=1/2,3/2} \frac{Re\,\xi_0 \cos(4\pi a+\frac{\pi s}{2})\ell + 4\pi\ell \sin(4a+\frac{s}{2})\pi\ell}{(Re\,\xi_0)^2+(4\pi\ \ \ )^2}$$

with

$$\xi_0 = -2\log(0.107) \pm 2\pi i$$

$$F(\xi_0, 1/2) = 0.25 \pm \frac{i\pi}{4} \tag{22}$$

This result has been derived in the same manner as that for Σ_{TT}.

3.

The self energy parts in the second order have an exponential growing discontinuity eventually multiplied by an oscillating function. The failure of positive definiteness of the discontunuity may be traced back to the well known ghost difficulty in the dual theory. Remark that the double twisted self-energy part contains an exponentially growing discontinuity. This is in contrast to the behaviour of the double twisted box amplitude (dual "Pomeron graph") which is polynomially bounded /7/ . Furthermore we should compare our result with the investigations of the second order corrections to the Regge trajectory orginating from a dual box diagram /8/. Despite of some structural similarities between the self-energy part $\Sigma(s)$ and the trajectory corrections the latter has been shown to be polynomially bounded in the whole s plane. Apparently the on-shell amplitudes investigated up to now behave better than the off-shell amplitudes.

The discontinuities considered above are related to the Lehmann spectral function of the ground state propagator by

$$\sigma(s) = \frac{1}{2i}\,\frac{1}{(s-m^2)^2}\,\{\,disc\,\Sigma(s) + 2\,disc\,\Sigma_T(s) + disc\,\Sigma_{TT}(s)\,\} \qquad (23)$$

(in 2 order).

Consequently we get an exponentially growing $\sigma(s)$ which announces non-local properties of dual field theories.

We gratefully acknowledge discussions with H.J.Kaiser.

Appendix

I. Further expressions for the self-energy parts[5].

a) Self-energy part without twist

$$\Sigma(s) = 4\pi^2 g^2 \int_0^\infty d\xi \int_0^1 d\theta\, \frac{e^{a\xi}}{\xi}\,[(1-e^{-\theta\xi})(1-e^{-(1-\theta)\xi})]^{a-1} \times$$

$$\times f^{-4}(\omega)\{[\psi(x)]^s - [\tilde{\psi}(x)]^s\}$$

$$\psi(x) = \xi\,\frac{\Theta_1(\theta\,|\,\frac{2\pi i}{\xi})}{\Theta_1'(0\,|\,\frac{2\pi i}{\xi})} = \sin\pi\theta \prod_1^\infty \frac{1-2q^{2n}\cos 2\pi\theta + q^{4n}}{(1-q^{2n})^2}\cdot\frac{\xi}{\pi}$$

$$\tilde{\psi}(x) = \frac{\xi}{\pi}\sin\pi\theta$$

$$q = e^{-\frac{2\pi^2}{\xi}}, \quad \omega = e^{-\xi}.$$

b) Self-energy part with one twist

$$\Sigma_T(s) = 8\pi^2 g^2 \int_0^\infty d\xi \int_0^1 d\theta \frac{e^{a\xi}}{\xi} (1+e^{-\xi})^2 f^{-4}(-\omega) \times$$

$$\times [(1-e^{-2\theta\xi})(1-e^{-2(1-\theta)\xi})]^{a-1} \{[\psi_N(x^2)]^s - [\tilde{\psi}_N(x^2)]^s\}$$

$$\psi_N(x^2) = 2\xi \frac{\Theta_1(\theta \mid -\frac{1}{2} + \frac{\pi i}{2\xi})}{\Theta_1'(0 \mid -\frac{1}{2} + \frac{\pi i}{2\xi})}$$

$$= \frac{2\xi}{\pi} \sin\pi\theta \prod_1^\infty \frac{1 - 2q_N^{2n} \cos 2\pi\theta + q_N^{4n}}{(1-q_N^{2n})^2}.$$

$$\tilde{\psi}_N(x^2) = \frac{2\xi}{\pi} \sin\pi\theta$$

$$q_N = -i e^{-\frac{\pi^2}{2\xi}}$$

c) Double twisted self-energy part

$$\Sigma_{TT}(s) = 4\pi^2 g^2 \int_0^\infty d\xi \int_0^1 d\theta \frac{e^{a\xi}}{\xi} (1-e^{-\xi})^{a+1} f^{-4}(\omega)[\psi_T(x)]^s$$

$$\psi_T(x) = \xi \frac{\Theta_4(\theta \mid \frac{2\pi i}{\xi})}{\Theta_1'(0 \mid \frac{2\pi i}{\xi})}$$

$$= \frac{1}{2} \xi q^{-1/4} \prod_1^\infty \frac{1 - 2q^{2n-1} \cos 2\pi\theta + q^{4n-2}}{(1-q^{2n})^2}$$

$$f^{-4}(\omega) = \frac{1}{4\pi^2} \omega^{1/6} \log^2\omega \, q^{-1/3} f^{-4}(q).$$

II. Asymptotic behaviour of the functions F.

$$F(\xi,\theta) \equiv \log\psi \simeq$$

$$\approx \begin{cases} \dfrac{\theta(1-\theta)\xi}{2} + \log\left[1 - e^{-\theta\xi} - e^{-(1-\theta)\xi}\right], & Re\,\xi \to \infty, \\ \log\xi, & \xi \to 0 \end{cases}$$

$$F_T(\xi,\theta) \equiv \log\psi_N \approx \begin{cases} (1-2\theta)\theta\xi, \; Re\,\xi \to \infty, & 0<\theta<1/2 \\ (3\theta - 2\theta^2 - 1)\xi, \; Re\,\xi \to \infty, & 1/2<\theta<1 \\ \log\xi, & \xi \to 0 \end{cases}$$

$$F_{TT}(\xi,\theta) \equiv \log\psi_T \approx$$

$$\approx \begin{cases} \dfrac{\theta(1-\theta)\xi}{2} + \log\left[1 + e^{-\theta\xi} + e^{-(1-\theta)\xi}\right], & Re\,\xi \to \infty \\ \dfrac{\pi^2}{2\xi}, & \xi \to 0 \end{cases}$$

III. Existence of stationary points and their θ dependence.

The existence of at least one stationary point for Σ_{TT} follows from an inspection of formula (3c). It is easily seen that F_{TT} is infinite in the points $x=y=0$ and $x=y=1$ and has a minimum along each curve $\theta = const.$

A straight-forward calculations shows that $F_{TT}(\xi_0(\theta),\theta)$ takes its maximum value at $\theta = 1/2$. This follows from

$$F_{,\theta} = \frac{\partial \log \psi_T}{\partial \theta} = 4\pi \sin 2\pi\theta \sum_{n=0}^{\infty} \frac{q^{2n+1}}{1-2q^{2n+1}\cos 2\pi\theta + q^{4n+2}} = 0$$

$$F_{,\theta\theta} = \frac{\partial^2 \log \psi_T}{\partial \theta^2} = [\, 8\pi^2 \cos 2\pi\theta + 4\pi \sin 2\pi\theta \frac{\theta}{\partial \theta}] \times$$

$$\times \sum \frac{q^{2n+1}}{1-2q^{2n+1}\cos 2\pi\theta + q^{4n+2}} < 0 .$$

Turning to $\Sigma(s)$ we may repeat the foregoing considerations for F which again show the importance of the line $\theta = 1/2$. It is therefore worthwile to start the search for stationary points with respect to ξ taking $\theta = 1/2$ from the beginning. In the case of $F(\xi, 1/2)$ the existence of two zeros of $F_{,\xi}$ with $Jm\,\xi = \pm 2\pi i$ can directly be shown by studying the limits of for $Re\,\xi \to \{ {0 \atop \infty}$ along $Jm\,\xi = \pm 2\pi i$.

References

1. V.Alessandrini, D.Amati, M.LeBellac, D.Olive. Physics Reports, 1, 269 (1971).
2. I.T.Drummond. Nucl. Phys., B35, 269 (1971).
   A.Neveu, J.Scherk. Nucl. Phys., B41, 365 (1972).
   E.Cremmer, J.Scherk. CERN 1623, Geneve 1973 and further papers.
3. D.Robaschik, E.Wieczorek. JINR, E2-6328, Dubna, 1972.
   R.J.Rivers. Imperial College, ICTP/71/2 and ICTP/71/6, London, 1971.

4. P.Olesen. Niels Bohr Institute. NBI-HE-73-9, Copenhagen, 1973.
5. D.J.Gross, A.Neveu, J.Scherk, J.H.Schwarz. Phys. Rev., D2, 697 (1970).
A.Neveu, J.Scherk. Phys.Rev., D1, 2355 (1970).
6. Н.Н.Мейман. ЖЭТФ, 47, 1966 /1964/.
A.V.Jaffe. Phys.Rev., 158, 1454 (1967).
G.V.Efimov. Commun. Math. Phys., 7, 138 (1968).
7. V.Alessandrini, D.Amati, B.Morel. Nuovo Cim., 7, 797 (1972).
8. H.Dorn, H.J.Kaiser. PHE 72-15, Berlin, 1972.

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