СООБЩЕНИЯ
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ЯДЕРНЫХ
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E2 - 7170

A.B. Govorkov

UNIVERSAL DISTRIBUTION FOR MULTIPLICITY AND A HYPOTHESIS OF INDUCED RADIATION OF PIONS IN HIGH ENERGY PROTON-PROTON COLLISIONS

1973

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I. Introduction

The multiplicity distributions for the charged particles produced in high energy proton-proton collisions exhibit two wonderful peculiarities. One of them consists in the approximate constancy of the ratio of the average multiplicity of charged particles, $\langle n_{ch} \rangle$, to the dispersion of charged particle number, $D_{ch} = [\langle n_{ch}^2 \rangle - \langle n_{ch} \rangle^2]^{1/2}$. The experimental points for this ratio taken from ref. [1] are presented in Fig.I. For the first time the stability of this ratio apparently was mentioned by Chyzewski and Rybicki in their investigation of the multiplicity distributions at relatively low energies [2].

The second peculiarity is the asymptotic so-called "KNO-scaling" that has been discovered by Koba, Nielsen and Olesen [3,4]: if one plots partial cross sections multiplyed by average charged particle multiplicity at given energy, $\langle n_{ch} \rangle P(n_{ch}) = \langle n_{ch} \rangle \sigma_n / \sum_{n=2,4,\dots} \sigma_n$, versus $n_{ch} / \langle n_{ch} \rangle$, then at sufficiently high energies the experimental points form a universal dependence.

The experimental data points at lab.proton incident momenta: I9,50,69,I02,205 and 303 Gev/c are presented in Fig.2 taken from ref. /5/ . Indeed, except the relatively low incident momentum I9 Gev/c, these data justify the KNO-scaling. Koba, Nielsen and Olesen derived their scaling starting from Feynman scaling for all the multiparticle inclusive cross sections /4/.

Each of these peculiarities is alien to the Poisson distribution x) which appears in the independent particle production.

However we know from the history that there is another distribution for the multiparticle productions. It appears when we draw our attention to an ample multitude of branch or chain processes. The main feature of these processes is the dependence of a particle number in the following generation on a particle number in the preceding generation.

Two peculiarities are typical for these processes:

I) The dispersion of particle number depends on the average multiplicity by the law

x) For the Poisson distribution the ratio $\langle n \rangle / D$ changes as $\langle n \rangle^{1/2}$. Consequently it increases with initial energy. Beside the Poisson distribution possesses a scaling properties but it goes to the asymptotic distribution with the infinitesimal width /4/.

$$D^2 = \langle n \rangle + \zeta \langle n \rangle^2 \qquad (I)$$

where ζ is some parameter;

2) There is a limiting distribution with finite width .

$$\langle n \rangle P(n) \rightarrow \psi\left(\frac{n}{\langle n \rangle}, \zeta\right) \qquad (2)$$

when $\frac{1}{\langle n \rangle} \ll \min(1; \zeta)$.

We have just a distribution with the desirable properties if parameter ζ is a function slightly dependent on a change of external conditions. Usually the negative binomial Polya distribution is very convenient as a distribution for the branch processes. It was used for a description of the distribution of particle numbers in the electron-photon showers /6/ as well as for the statistics of the amplitudes of the pulses from the pulsed fast reactor /7/.

We propose to employ this distribution as the empirical distribution for the multiplicity of secondary particles in proton-proton collisions. In these collisions mainly pions are produced and below we shall mean their production.

In section 2 we compare this distribution with experimental data, then in section 3 we discuss possible reasons for its origin in the case of interest and give its phenomenological derivation.

2. The Polya distribution as the empirical distribution of multiplicity

The Polya distribution for the probability of n-particle state is x)

$$P(n) = P(0)\left(\frac{\langle n\rangle}{1+\zeta\langle n\rangle}\right)^n \frac{(1+\zeta)(1+2\zeta)\dots[1+(n-1)\zeta]}{n!} \quad (3)$$

where

$$P(0) = (1+\zeta\langle n\rangle)^{-1/\zeta} . \quad (4)$$

Its factorial moments are given by

$$\langle n(n-1)\dots(n-s+1)\rangle =$$
$$= (1+\zeta)(1+2\zeta)\dots[1+\zeta(s-1)]\langle n\rangle^s . \quad (5)$$

For the square of $D/\langle n\rangle$ - ratio we have

$$\frac{D^2}{\langle n\rangle^2} = \frac{1}{\langle n\rangle} + \zeta \quad (6)$$

x) We shall derive this distribution on the basis of generating function in the following section.

The first term in right-hand side of (6) corresponds to independent fluctuations of the particle number around its mathematical expectation, while the second term corresponds to genetic correlations between particles. The Polya distribution turns into the Poisson distribution when $\zeta = 0$.

In limit $\frac{1}{\langle n\rangle} \ll \min(1; \zeta)$ there is a limiting distribution which is the χ^2 - distribution with respect to $\chi^2 = 2n/\zeta\langle n\rangle$:

$$\langle n\rangle P(n) = \left(\frac{x}{\zeta}\right)^{\frac{1}{\zeta}-1} \exp\left(-\frac{x}{\zeta}\right) \frac{1}{\zeta\,\Gamma(\zeta^{-1})} , \quad (7)$$

where $x = n/\langle n\rangle$ and $\Gamma(\zeta^{-1})$ is Euler gamma-function. If $\zeta \ll 1$ then in its turn the limiting distribution (7) goes to the normal Gauss distribution.

When we compare the theoretical results with the experimental data we ought to take into account the following:

It is naturally to suppose that our empirical distribution is applyed to the total particle number while the charged particle multiplicity is determined experimentally. At the present time there are some indications about the correlation between the charged and neutral particles produced in pp - collisions at high energies /8 - IO/. With these data, we take for the total produced particle number a simple formula

$$n = 1.5\, n_{ch} , \quad (8)$$

where n_{ch} is a number of charged particles.

Further, as a consequence of the conservation law of baryons two baryons must be present in the final state. Thus they must be excluded from the particle number for which we apply our distribution:

$$n' = n - 2 = 1.5\, n_{ch} - 2 \quad . \tag{9}$$

Instead of (6) we have

$$\frac{D_{ch}^2}{\langle n_{ch}\rangle^2} = \frac{1.5\langle n_{ch}\rangle - 2}{(1.5\langle n_{ch}\rangle)^2} + \left(\frac{1.5\langle n_{ch}\rangle - 2}{1.5\langle n_{ch}\rangle}\right)^2 \zeta \tag{IO}$$

Proceeding from the data /5/ for $D_{ch}^2/\langle n_{ch}\rangle^2$ and $\langle n_{ch}\rangle$, we obtain the values of parameter ζ presented in Table I.

Table I. The values of parameter ζ

I9 Gev/c	50 Gev/c	69 Gev/c	I02 Gev/c	205 Gev/c	303 Gev/c
0.I77 ±0.007	0.253 ±0.027	0.257 ±0.0I4	0.266 ±0.022	0.273 ±0.026	0.25I ±0.02I

We see that except the region of relatively low energies the values of parameter ζ are weakly changing. With this exception the average value weighted with squares of the inverse dispersions is x)

$$\zeta = 0.2588 \pm 0.0034\,. \tag{II}$$

In Fig.I we present the ratio $\langle n_{ch}\rangle / D_{ch}$ calculated by means of eq.(IO) with the constant value of parameter ζ from (II). We observe a good agreement between calculated and experimental data at high energies, while at relatively low energies we cannot consider the parameter ζ as being constant.

Now we calculate the multiplicity distribution in KNO-scale according to eq.(3) and eq.(9) with the constant value of parameter ζ from (II). In order to compare directly our calculations with the experimental data we get the calculations for n_{ch} =2,4,6,... (n' =I,4,7,...) then normalize their sum to unity and multiply them by $\langle n_{ch}\rangle$. We carry out the calculations also for the case of incident momentum I9 Gev/c and with the parameter ζ = 0.I77 according to data from Table I. The agreement between theory and experiment is demonstrated in Table II. In this table we indicate also the values of χ^2 when we exclude some of points which evidently drop out of the common regularity. Usually they are the last points.

x) One should not attach very importance to the concrete value of parameter ζ because it is dependent on the way of the connection between charged and neutral particles.

Table II. The agreement between theoretical distribution and experimental data

19 Gev/c	50 Gev/c	69 Gev/c	102 Gev/c	205 Gev/c	303 Gev/c
ζ	ζ	ζ	ζ	ζ	ζ
0.177	0.2588	0.2588	0.2588	0.2588	0.2588
χ^2_4	χ^2_5	χ^2_6	χ^2_6	χ^2_8	χ^2_{10}
29.3	13.7	47.8	7.4	15.9	22.5
6.9[a)]	9.1[b)]	22.9[c)]			10.2[d)]

a) n_{ch} =2,6 are excluded
b) n_{ch} =16 is excluded
c) n_{ch} =18 is excluded
d) n_{ch} =22 is excluded

In Fig.2 we present the results of calculations for the cases of 19,50 and 303 Gev/c. Although the distributions are discrete we joint the calculated points by the smooth curves by hand. We see that the curves for 50 and 303 Gev/c lie tightly one to other. These curves almost coincide with the limiting curve calculated by means of (7). Thus we indeed arrive at a universal distribution and it is a consequence of the nature of the Polya distribution.

We see that there are some regular deviations of the experimental points from the calculated curves in the region of the distribution maximum and these deviations become smaller in the region of high multiplicities.

On the whole we get the satisfactory agreement between our empirical distribution and the experimental data. This distribution depends on two parameters. One of them is the average charged multiplicity $\langle n_{ch} \rangle$ and depends on the proton incident momentum while the other parameter ζ connected with $\langle n_{ch} \rangle / D_{ch}$-ratio is found to be constant in the region of high incident momentum as well as this ratio itself. At this point the proposed empirical distribution possesses, in our opinion, an advantage as compared with other empirical distributions.

3. Hypothesis of the induced radiation of particles in proton-proton collisions at high energies

We have noted in Introduction that the Polya distribution appears in the branch processes. Now, if it appears in our case what is a mechanism which could lead to this distribution?

At the present time the author does not know any microscopic theories of multiparticle production in hadron collision which could lead to the Polya type distributions. Perhaps the mechanism of the multiparticle production is more complicated than it is assumed in the contemporary theories. Qualitatively we could try to consider it starting from the following arguments;

In a collision between two protons in lab.system the projectile is compressed by the Lorentz contraction into a thin disk. This thin disk can radiate a coherent,e.g.,"pion wave" /II/ .x) However, it is possible that this pion-wave is not radiated by the system immediately but it runs through the target proton. Our hypothesis consists in the assertion that the target proton is a "pion - active - substance" which is capable of absorptions and spontaneous and induced productions of the pion-waves. The terms "spontaneous and induced productions" mean that the radiation of pion-waves by the stuff is independent or dependent on a presence of other pions in the system,respectively. The same reasonings are applicable, of course, to the projectile proton in the projectile (antilaboratory) system.

Moreover, we consider this complicated process as a consequence of the elementary acts of the absorptions and productions of particles and we shall describe them by means of

x) Perhaps, it radiates a coherent "vecton-wave",e.g., " ω -meson-wave", which then decays into pions.

a kinetic equation. x) For the description of the behaviour of pions (or any other particles) in the system we propose the following conceptions:

The mean life time of pion with respect to its annihilation including its going out of the system - τ_a ;

The mean life time of pion with respect to its absorbtion with the following emission of secondary pions (multiplication) - τ_m ;

In the latter case the conditional probability of irradiation of a number ν of secondary pions - p_ν ($\sum_{\nu=2,3,\ldots} p_\nu = 1$);

The total mean life time of pion in the system

$$\frac{1}{\tau} = \frac{1}{\tau_a} + \frac{1}{\tau_m} . \tag{I2}$$

The strength of source of pions is characterized by the average number of spontaneously irradiated pions per time unity - Q .

Now, for the probability of a presence of pion number n in the system at an instant t , $P(n,t)$, we have the equation

$$\frac{dP(n,t)}{dt} = -\left(Q + \frac{n}{\tau}\right)P(n,t) + \frac{n+1}{\tau_a}P(n+1,t) + \sum_{\nu=2,3,\ldots} p_\nu \frac{n-\nu+1}{\tau_m} P(n-\nu+1,t) + Q\,P(n-1,t) . \tag{I3}$$

x) We should emphasize that the same approach was developed by V.M.Maltsev and N.K.Dushutin /I2/.

Multiplying the eq.(I3) by n and summing the result over all n we arrive at the equation for the average pion number

$$\frac{d\bar{n}}{dt} = \alpha\,\bar{n}(t) + Q\,, \tag{I4}$$

where

$$\bar{n}(t) = \sum_{n=0}^{\infty} n\,P(n,t) \tag{I5}$$

and "the rate of the pion multiplication" is

$$\alpha = \frac{\bar{\nu}-1}{\tau_m} - \frac{1}{\tau_a} \quad \text{,where} \quad \bar{\nu} = \sum_{\nu} \nu\,p_{\nu} \,. \tag{I6}$$

It is convenient to introduce the generating function

$$H(z,t) = \sum_{n=0}^{\infty} z^n\,P(n,t)\,. \tag{I7}$$

Multiplying eq.(I3) by z^n and summing the result over all n we find the equation for the generating function

$$\frac{\partial H(z,t)}{\partial t} = \frac{G(z)-z}{\tau}\,\frac{\partial H(z,t)}{\partial z} + (z-1)\,Q\,H(z,t), \tag{I8}$$

where $G(z)$ is a generating function for a single act of disappearance of pion in the system

$$G(z) = \frac{\tau}{\tau_a} + \frac{\tau}{\tau_m}\sum_{\nu} p_{\nu}\,z^{\nu}\,. \tag{I9}$$

The boundary condition for $H(z,t)$ has a form

$$H(1,t) = 1. \tag{20}$$

An initial condition is not important because it relaxes in the presence of a sufficiently strong source.

The manifest solution of eq.(I8) can be obtained only in the case of a multiplication of pions with redoubling (p_2 =I) /I3/. However for a small value of α from (I6) we can decompose $G(z)$ around z = I /7/

$$\frac{G(z)-z}{\tau} = \alpha(z-1) + \zeta\,Q\,(z-1)^2 + \ldots \tag{2I}$$

where

$$\zeta = \frac{\overline{\nu(\nu-1)}}{2Q\tau_m}\,, \quad \overline{\nu(\nu-1)} = \sum_{\nu} \nu(\nu-1)\,p_{\nu}\,. \tag{22}$$

If we restrict the series (2I) only to two written terms then the solution of eq.(I8) is reduced to the above mentioned case /I3/. It is

$$H(z,t) = \left[\,1-(z-1)\,\zeta\,\bar{n}(t)\,\right]^{-1/\zeta}. \tag{23}$$

The distribution itself is defined by

$$P(n,t) = \frac{1}{n!}\left[\frac{\partial^n H(z,t)}{\partial z^n}\right]_{z=0} . \quad (24)$$

We can change the multiplicity at an instant t , $\bar{n}(t)$, by the final multiplicity $\langle n \rangle$ and arrive at the distribution (3). The expression (5) for its factorial moments is obtained by the formula

$$\langle n(n-1)\dots(n-s+1)\rangle = \left[\frac{\partial^s H(z,t)}{\partial z^s}\right]_{z=1} . \quad (25)$$

It is interesting that although our initial phenomenological model includes a variety of parameters the final formula for the multiplicity distribution is a function of the average multiplicity with the only parameter ζ , defined by eq.(22). It is likely that the product of the strength of a spontaneous source Q with the multiplication mean life time τ_m is constant: The system which is capable to irradiate pions is capable also to absorb them with subsequent multiplication. Thus it is likely that the parameter ζ is constant during the whole process of multiplication of pions as well as with a change of the incident energy.

4. Conclusion

We propose to consider the process of the multiparticle production in high energy hadron collisions as a particular case of an ample variety of the branch processes. By this analogy we suggest to use the Polya distribution as the multiplicity distribution. It depends on two parameters. One of them is the average multiplicity of charged particles and depends on the incident energy while the other one is parameter ζ , connected with the ratio of the average multiplicity to the dispersion of particle number and is constant at high incident energies. A comparison of the empirical distribution with the experimental data shows a satisfactory agreement.

If indeed there is some mechanism of the induced production of particles then we can expect its most visible revealing in the cases with high multiplicities and its intensification with the increasing incident energy. As we could see such a tendency was really observed. On the same foundation we can predict that at very high energies (for example, at colliding beam energies) the multiplicity distribution will coincide with a limiting distribution and if parameter ζ will not differ from its present value then the distribution will coincide with that has been found already. We saw there are forcible arguments to consider parameter ζ as a constant. Perhaps the same reasons lie in the basis of the early onset of the KNO-scaling. Then the latter is indebted rather to the nature of the induced multiparticle production process than to the Feynman asymtotics.

The author is very much obliged to Professor A.M.Baldin for initiating this work. He would like to thank Dr.S.B. Gerasimov and Dr.V.A.Meshcheryakov for helpfull discussions.

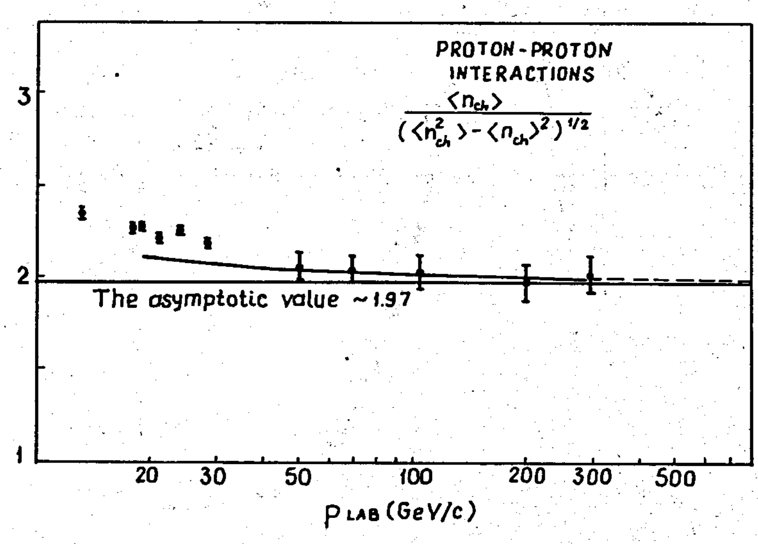

Fig.I. The ratio $\langle n_{ch}\rangle / D_{ch}$ vs lab. incident momenta of incoming proton. The solid curve indicates the results of calculation by means of eq.(IO) with the constant value of parameter ζ = 0.2588.

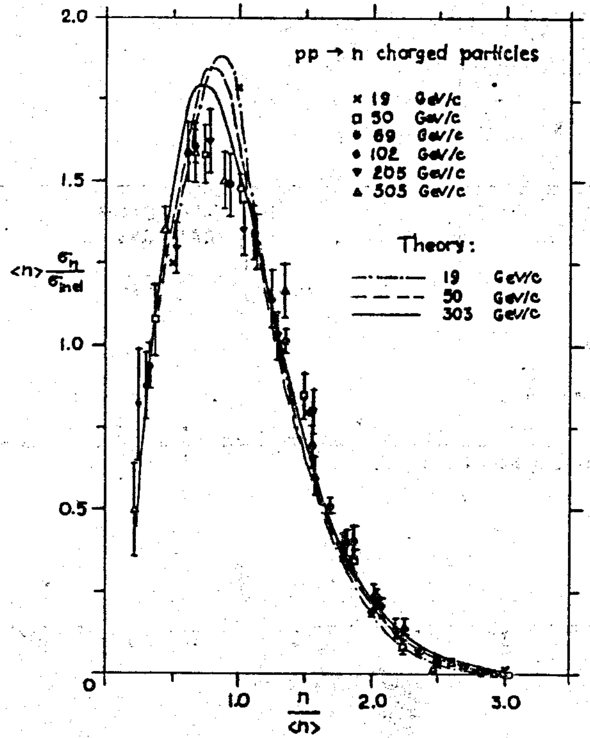

Fig.2. KNO-plot: $\langle n \rangle \sigma_n / \sigma_{inel}$ vs $n/\langle n \rangle$ for the reaction $pp \to n$ charged particles at incident momenta indicated in picture. The theoretical curves represent the Polya distributions. Though the distributions are discrete we joint by hand the calculated points by the smooth curves.

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Received by Publishing Department
on May 21, 1973.