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ULTRAVIOLET ASYMPTOTICS IN QUANTUM FIELD THEORY AND SCALE INVARIANCE

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1. Introduction

We consider the general problem of studying ultraviolet asymptotic behaviour of the one-particle and higher Green functions (GFs) in quantum field theory (QFT) and especially the possibility of obtaining power asymptotic expression of scale -invariant type.

We raise the question: What can scale invariance (if indeed observed experimentally) tell us about the ultraviolet structure of QFT (if the latter is applicable to hadron physics)? By "scale invariance" here we mean the asymptotic expressions for GFs of power type (power of big momentum argument multiplied by the function of ratios of large arguments).

To get the answer we use the general properties of QFT condensed in functional equations of renormalization group (RG). For this aim we solve simultaneously the equations for invariant charge (IC) and GFs. Such a functional equation for the photon propagator which coincides with IC in quantum electrodynamics was obtained and analysed by Gell-Mann and Low [1]. The complete set of functional group equations and the corresponding differential Lie equations in quantum electrodynamics were obtained by N.N.Bogolubov and the present author [2], and for pion-nucleon interaction in the paper [3]. The differential group Lie equations were proved to be a very simple and convenient practical tool for asymptotic analysis of spinor electrodynamics in ultraviolet and infrared regions [4], as well as in some other field theories [5,6] (see also review paper [7] and Chapter 8 in [8]).

As was shown by Ovsiannikov [9], they turned out to be effective means for obtaining general solutions of RG functional equations. Ovsiannikov used the Lie equations in the original form of the first-order differential equation in partial derivatives. Unfortunately his paper of 1956 [9] remained little known and the equations in the Ovsiannikov form were rediscovered in fifteen years by a rather complicated technique (Callan [10], Symanzik [11]) and now are called sometimes Callan-Symanzik equations. The asymptotic ultraviolet form of Callan-Symanzik equations completely coincides with the asymptotic form of the Ovsiannikov-Lie equations.

We consider two possible types of IC asymptotic behaviour:

I. Finite limit corresponding to the finite value of Z_3 renormalization constant.

II. Infinite limit $(Z_3^{-1} = \infty)$.

The hypothesis of finite asymptotic values for hadron ICs was proposed about ten years ago [12] (see also [13]) to get Regge-type scattering asymptotics in the framework of QFT. Recently this hypothesis became popular again in connection with scale invariance (see, e.g. [14,15]).

Using the asymptotic differential Lie equations in the Ovsiannikov-Callan-Symanzik form we show that the power behaviour of propagators and higher GFs can be obtained for finite ultraviolet limit of ICs as well as for some cases of its infinite growth.

The main analysis (Sections 2,3,4,5) is performed for the one-charge RG in spinor electrodynamics. Its application to multi-charge theory of strong interactions is discussed in Section 6. Further details of calculations can be found in ref. [16,17].

2. The Ovsiannikov Method

The quantum electrodynamics RG functional equations (in transverse gauge) for the dimensionless electron propagator s and 3-vertex Γ are of the form (see [2,8,13]),

$$s(x,y;a) = s(t,y;a)\, s\left(\frac{x}{t}, \frac{y}{t}; \xi(t,y,a)\right), \tag{1}$$

$$\Gamma(x_1,x_2,x_3,y\;;a) = \Gamma(t,y;a)\,\Gamma\left(\frac{x_1}{t}, \frac{x_2}{t}, \frac{x_3}{t}, \frac{y}{t}; \xi(t,y,a)\right). \tag{2}$$

Here

$$\Gamma(t,y;a) \equiv \Gamma(t,t,t,y;a)$$

is the symmetric vertex,

$$a = \frac{e^2}{4\pi}, \quad x_i = \frac{p_i^2}{\lambda^2}, \quad y = \frac{m^2}{\lambda^2},$$

and the invariant charge ξ, proportional to the transverse photon propagator d

$$\xi(x,y;a) = a\, d(x,y;a)$$

satisfies the functional equation

$$\xi(x,y;a) = \xi\left(\frac{x}{t}, \frac{y}{t}; \xi(t,y;a)\right). \tag{3}$$

The general solution of RG functional equations was given by Ovsiannikov [9]. His method consists in the transition from functional to differential group equations and their solution. The differential equations can be obtained by differentiating Eqs. (1)-(3) with respect to t and then putting $t=1$:

$$\left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} - \phi(y,a)\frac{\partial}{\partial a} \right] \xi(x,y;a) = 0, \tag{4}$$

$$\left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} - \phi(y,a)\frac{\partial}{\partial a} - \psi(y,a) \right] s(x,y;a) = 0, \tag{5}$$

$$\left[\sum_i x_i\frac{\partial}{\partial x_i} + y\frac{\partial}{\partial y} - \phi(y,a)\frac{\partial}{\partial a} - \gamma(y,a) \right] \Gamma(x_1,x_2,x_3,y;a) = 0 \tag{6}$$

with
$$\phi(y,a)=\frac{\partial\xi(t,y;a)}{\partial t}\Big|_{t=1}\;,\;\psi(y,a)=\frac{\partial \ln s(t,y;a)}{\partial t}\Big|_{t=1},$$

$$\gamma(y,a)=\frac{\partial \ln\Gamma(t,y;a)}{\partial t}\Big|_{t=1}. \tag{7}$$

The differential equations of this type are completely equivalent to the Lie equations used in ref. [4,7,8] for the improvement of approximation properties of perturbation expansions. In some sence they are more spectacular as far they give explicitly the response of the GFs to the infinitesimal shift of the normalization point described by the t argument. However they are less informative because they do not contain the normalization conditions

$$\xi(1,y;a)=a\;;\quad s(1,y;a)=\Gamma(1,1,1,y;a)=1$$

built into functional equations.

Eq. (4) was first obtained by Ovsiannikov in 1956. His derivation presented here was rediscovered by Sirlin [18] in 1972. We shall call such a form of the Lie equations Ovsiannikov equations, and their ultraviolet asymptotic form Ovsiannikov-Callan-Symanzik equations. The given derivation of such equations is extremely simple and can be generalized to higher GFs and directly to the multi-charge case, provided the corresponding functional equation of RG is known.

For instance, in the two-charge theory of meson-nucleon interaction

$$\mathcal{L}=g\bar\Psi\gamma^5\tau\Psi\phi+h(\phi\phi)^2 \tag{8}$$

such an equation for 4-pion vertex $\square$ is of the form

$$\Big[\sum_{1\le i\le 6}x_i\frac{\partial}{\partial x_i}+y_1\frac{\partial}{\partial y_1}+y_2\frac{\partial}{\partial y_2}-\phi_1(y_1,y_2;g^2,h)\frac{\partial}{\partial g^2}-$$
$$-\phi_2(y_1,y_2;g^2,h)\frac{\partial}{\partial h}-\delta(y_1,y_2;g^2,h)\Big]\square(x_1,\dots,x_6,y_1,y_2;g^2,h)=0. \tag{9}$$

Here ϕ_1, ϕ_2 and δ are the derivatives of invariant charges and of $\square$ at the normalization point which are defined in a manner analogous to eq. (6), $y_1=\mu^2/\lambda^2$, $y_2=M^2/\lambda^2$.

The Ovsiannikov form of the Lee group equations turns out to be very convenient for a simultaneous analysis of ultraviolet asymptotics of higher GFs and ICs and allows one to relate the property of scale invariance of higher GFs to the coupling constant renormalizations.

3. General Solution in Ultraviolet Region

In the ultraviolet region the mass variable $y=m^2/\lambda^2$ drops out from the group equations. In this case the general solution for eq. (4) can be written in the Gell-Mann-Low [1] form

$$\int_a^{\xi(x;a)}\frac{da}{\phi(a)}=\ln x\;,\quad \phi(a)\equiv\phi(y=0,a). \tag{10}$$

The normalized solution of eq. (5), first obtained in ref. [9] can be represented as follows (for details see ref. [16,17]):

$$s(x,a)=\exp\int_a^{\xi(x,a)}\frac{\psi(a)}{\phi(a)}\,da. \tag{11}$$

The symmetrical solution of eq. (6) has the same form

$$\Gamma(x,a)=\exp\int_a^{\xi(x,a)}\frac{\gamma(a)}{\phi(a)}\,da. \tag{12}$$

If we fix the ratios $x_1/x_3=C_1=const$, $x_2/x_3=C_2=const$ and vary the $x_3=x$ variable then, as can be shown,

$$X(C_1x,C_2x,x;a)=\Gamma(C_1,C_2,1;a)\tilde\Gamma(x,a). \tag{13}$$

Here $\tilde{\Gamma}$ satisfies the same equation as $\Gamma(x,a)$ and can be represented in the form (L12) (with $\gamma(a)$ changed by $\tilde{\gamma}(a)$). The generalization of this result to higher GFs will be

$$F(x_1,...,x_n;a)=f(\frac{x_1}{x_n},...\frac{x_{n-1}}{x_n};a)F(x_n;a). \qquad (14)$$

the last factor being represented in the form (12).

Now consider the multi-charge case. In two-charge meson-nucleon field theory the functional equations for invariant charges form the system [3,7,8]

$$\xi_i(x,y;g^2,h)=\xi_i(\frac{x}{t},\frac{y}{t};\xi_1(t,y;g^2,h),\xi_2(t,y;g^2,h)). \qquad (15)$$

Its general solution obtained in [9], in the ultraviolet region, can be represented [16,17] as

$$\Psi_i(\xi_1(x;g^2,h),\xi_2(x;g^2,h))-\Psi_i(g^2,h)=\ln x, \qquad (16)$$

$$i=1,2,$$

where Ψ_1 and Ψ_2 are arbitrary functions reversible with respect to their arguments. These functions satisfy the differential equations

$$\phi_1(g^2,h)\frac{\partial\Psi_i(g^2,h)}{\partial g^2}+\phi_2(g^2,h)\frac{\partial\Psi_i(g^2,h)}{\partial h}=1$$

$$(17)$$

$$\phi_i(g^2,h)=\frac{\partial\xi_i(x;g^2,h)}{\partial x}\Big|_{x=1}$$

Eqs. (16) are an analog of the Gell-Mann-Low equation for the two-charge case. The generalization of eqs. (15)-(16) for the multi-charge case is evident.

The differential Lie equations corresponding to the system (15) were analysed in the framework of the weak coupling limit of pion-nucleon theory by Ginzburg [5,8].

The result is that even for small physical values of g^2 and h in the high-energy limit (as $x\to\infty$) we enter into the region of strong effective couplings and meet the trouble of "ghost-pole" analogous to one-charge quantum electrodynamics. Because of it, in what follows, we shall not use the perturbation information and analyse two possibilities (in terms of one-charge case)

$$\text{I} \quad \int_a^{a_\infty}\frac{da}{\phi(a)}=\infty, \qquad \xi(\infty,a)=a_\infty<\infty$$

$$\text{II} \quad \int_a^{\infty}\frac{da}{\phi(a)}=\infty, \quad \xi(\infty,a)=\infty$$

4. Finite Charge Renormalization

In the case I of the finite charge renormalization

$$aZ_3^{-1}=a_\infty<\infty$$

the function ϕ has a sufficiently strong zero at $a=a_\infty$.

$$\phi(a)\sim(a_\infty-a)^{1+\epsilon}, \quad \epsilon>0 \qquad (18)$$

Consider now the typical asymptotic of the GF given by eq. (12). Assuming

Ia)

$$I_0\equiv\int_a^{a_\infty}\frac{\gamma(a)}{\phi(a)}da<\infty$$

we get

$$\Gamma(\infty,a)=\exp I_0=const<\infty. \qquad (19)$$

If, however, I_0 diverges but

Ib)

$$I_1\equiv\int_a^{a_\infty}\frac{\gamma(a)-\gamma(a_\infty)}{\phi(a)}da<\infty$$

then using eq. (10) we obtain

$$\Gamma(x,a)=x^{\gamma(a_\infty)}\exp I_1 . \qquad (20)$$

If one subtraction is not sufficient, e.g. $I_1=\infty$, but

$$I_2=\int_a^{a_\infty}\frac{\gamma(a)-\gamma(a_\infty)-(a-a_\infty)\gamma'(a_\infty)}{\phi(a)}da<\infty$$

then

$$\Gamma(x,a)\to x^{\gamma(a_\infty)}f(\ln x)\exp I_2 . \qquad (21)$$

Here $f(\ln x)$ is the function which varies slower than the power of x. For example, in the Redmond-type model for the photon propagator obtained in /19/ where $a_\infty=3\pi$ and $\phi(a)\sim(a_\infty-a)^2$

$$f(\ln x)=(\ln x)^{-3\pi\gamma'(a_\infty)} .$$

We can summarize that in the case I of finite asymptotic for IC (finite coupling constant renormalization) the asymptotic behaviour of symmetric GFs is of a power type. The anomalous dimension $\gamma(a_\infty)$ can be equal to zero that corresponds to finite wave function renormalization. Note that in quantum electrodynamics due to the Ward identity renormalization Z_3 of the coupling constant a coincides with renormalization of the photon Green function. In a more general case (e.g. in $h\phi^4$ theory) the coupling constant renormalization is the product of several Z_i. Here the case of $\gamma_i(a_\infty)\neq 0$ corresponds to compensating of divergences in the product. We call such a case abnormal. In our notation the normal case would correspond to the absence of anomalous dimensions (all $\gamma_i(a_\infty)=0$).

5. Infinite Charge Renormalization

Proceed now to the case II of infinite charge renormalization. Here $\phi(a)/a$ cannot grow faster than $\ln a$, as $a\to\infty$. Consider the cases:

$$\text{IIA)}\ \frac{\phi(a)}{a}\sim\frac{\phi_n}{n}a^{-n},\quad n>0;\quad \xi\sim(\phi_n\ln x)^{\frac{1}{n}}, \qquad (22A)$$

$$\text{IIB)}\ \frac{\phi(a)}{a}\sim\frac{\phi_0}{(m+1)\ln^m a},\quad m\geq 0;\quad \xi\sim x^{\phi_0(\phi_0\ln x)^{-\frac{m}{m+1}}}, \qquad (22B)$$

$$\frac{\phi(a)}{a}\sim\frac{\phi_0}{(m+1)\ln^m a},\quad 0>m\geq -1;\quad \frac{\ln\xi}{\ln x}\to\infty . \qquad (22C)$$

Case A corresponds to (quasi) logarithmic divergence of Z_3^{-1}, case B to power divergences of Z_3^{-1}, case C to the behaviour more singular than the power one. For the ultraviolet asymptotic of the Green functions the behaviour of the ratio $\gamma_i(a)/\phi(a)$ is essential.

In the "normal" case

$$\gamma_i(a)=\nu_i\frac{\phi(a)}{a}\quad \text{as}\quad a\to\infty \qquad (23)$$

and

$$\Gamma_i(x,a)\sim[\xi(x,a)]^{\nu_i}. \qquad (24)$$

There can be also anomalous cases. In the simplest of them

$$\gamma_i(a)\sim n_i\phi(a)+\nu_i\frac{\phi(a)}{a} \qquad (25)$$

and correspondingly

$$\Gamma_i(x,a)\sim[\xi]^{\nu_i}\exp(n_i\xi). \qquad (26)$$

Note that in the product of GFs corresponding to the invariant charge $\xi=\prod_i(\Gamma_i)^{N_i}$ the expotential factors vanish due to $\sum_i n_iN_i=0$.

We can conclude that in the case A of (quasi) logarithmic growth of IC the normal case corresponds to anomalous dimensions different from zero. In the case B of power-type divergence the normal case again corresponds to power behaviour of GFs. In the case C the power behaviour of GFs is impossible.

Note, that the renormalization factors for higher GFs without proper divergences are not independent being the products of Z_i for propagators and lower GFs. As is known, e.g. in quantum electrodynamics for higher vertex with $2f$ fermion and b boson legs we have:

$$K_{2f,b} \to Z_2^f Z_3^{b/2} K_{2f,b} .$$

Because of it the product $s^f d^{b/2} K_{2f,b}$ turns out to be invariant under RG transformation and can depend only on IC ξ . Hence [20,21]

$$K_{2f,b}(x) \sim d^{-b/2}(x) s^f(x) f(\xi(x, a)) . \qquad (27)$$

6. Strong Interactions. Conclusion

As to the validity of the obtained results for the multi-charge case we can say that this case is more complicated in mathematical structure and more rich in different possibilities. However the experience we have got from perturbation theory [5] and two-charge scalar electrodynamics [6] teaches us that the situation can be analogous (in essential features) to the one-charge case.

Thus if in the k-charge case we conjecture that at $x \to \infty$ all ICs ξ_i tend to finite limiting values

$$\lim_{x \to \infty} \xi_i = \xi_i(\infty) \qquad i = 1, \ldots, k ,$$

i.e. the point $\xi_i = \xi_i(\infty)$ in k-dimensional phase space is the stable knot for the system of the corresponding Lie equations, then we get the results of Section 4.

The picture analogous to the one-charge case can arise here for infinite ICs asymptotics. Here the relations between the (infinite) limits become important. For example, the case when the powers of these limits are proportional to one another

$$\xi_i(x) \sim [\xi_1(x)]^{k_i}$$

can be completely analysed and yields the results of Section 5.

Thus, we can conclude that in many-charge theory of strong interactions the power asymptotics for propagators and scale-invariant expressions (like eq. (14)) for higher GFs arise naturally from the ansatz of the finiteness of asymptotic IC values (i.e. finiteness of effective coupling constant renormalizations). In this case, in virtue of (27), the higher GFs asymptotics can be expressed in terms of lower GFs asymptotics, and anomalous dimensions can be ascribed to the corresponding field operators. One can speak about ''field anomalous dimensions''.

As was shown, one can get the power asymptotics also for several cases of infinite charge renormalization constants with ''moderate'' growth of ICs at infinity. Here we must single out the case IIB of power growth for IC (eq. (22B) at $m = 0$). In the normal case (23) all GFs have power asymptotics. However here the exponents are not connected by simple relations leading to field anomalous dimensions. For example, the anomalous dimensions of the pion 4-vertex can not be expressed via the squared anomalous dimension of the pion 2-vertex, being rather an independent quantity, related to the exponent of the 4-pion IC.

It is possible to say that in this case the ICs themselves obey anomalous dimensions. Such an assertion is however a bit conventional since it does not lead to unambiguous determination of anomalous dimensions for higher GFs in terms of the ones for lower GFs. In eq. (27) it is impossible a priori to define $f(\xi)$ explicitly.

With due reference to these comments for the multi-charge case we can answer the question raised in the second paragraph of Section 2:

a. Scaling without anomalous dimensions corresponds to the normal case of finite charge renormalization (I) or of logarithmic divergence (IIA),

b. Scaling with anomalous dimensions different from zero corresponds to an anomalous case of finite (I) and logarithmic (IIA) behaviour and to the normal case of power rize (IIB) of Z_3^{-1} and ξ.

As far as perturbation calculations in existing models of strong interactions do not yield these possibilities we can conclude that such models (or at least the method of their treating) are in principle incorrect for hadron physics.

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