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RELATIVISTIC QUARK MODEL OF MESONS IN INFINITE MOMENTUM FRAME

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In the last years the idea of a dynamical picture of elementary particles, based on the hypothesis of constituent quarks, thought of as true elementary objects, has been a central point in particle physics. Many authors [1-5] discussed relativistic models of mesons as bound states of a quark and an antiquark desctibed by the Bethe-Salpeter equations in four-dimensional formalism. Since the Bethe-Salpeter equation displays a number of undesirable features, a quasipotential approach to the relativistic two-body problem was proposed several years ago [6-8]. The important features of the quasipotential equations are that the wave function depends on one-time argument and allows a probability interpretation.

In the present paper we suggest a three-dimensional equation of the quasipotential type for describing the mesons as bound states of a quark and an antiquark in infinite momentum frame. This equation was proposed and discussed in our previous work [9]. For simplicity, here only the case of spinless quarks and mesons will be considered. The extension to the cases with spins does not seem to involve any essential new difficulty.

We consider a dynamical model wich has the following main features:

1) The constituent quarks are heavy objects and the binding energy is smaller than the quark mass.

2) We suppose that the quarks in the bound states interact via neutral scalar gluon field in such a way that the interaction potential is of the Yukawa type.

3) The gluon mass is small, and we may call our model a long-range potential model.

By using a modified perturbation procedure [10] we obtain asymptotic expansion in inverse powers of the value of the c.m.s. momentum of the two particles for Regge trajectories and wavefunctions. For the proposed model, having in mind the conditions 1, 2 and 3 we get Regge linear trajectories.

The bound state of two scalar particles of masses m_1 and m_2 interacting by the local potential is described in infinite momentum frame by the wave function Ψ obeying the equation

$$\frac{w^2}{\eta_1+\eta_2}\Psi = -\frac{\Delta\Psi}{\mu^*} + \frac{V\Psi}{\mu^*} + \left(\frac{m_1^2}{\eta_1} + \frac{m_2^2}{\eta_2}\right)\Psi , \tag{1}$$

where w is the total energy of the system,

$$\mu^* = \frac{\eta_1 \cdot \eta_2}{\eta_1+\eta_2}, \quad \frac{\eta_1}{\eta_2} = \frac{w^2+m_1^2-m_2^2}{w^2-m_1^2+m_2^2}. \tag{2}$$

In the case of equal masses we get

$$\Delta\Psi + \left(\frac{w^2}{4} - m^2 - V\right)\Psi = 0 . \tag{3}$$

This equation was obtained by performing the Foldy-Wouthuysen transformation on the wave function obeying a relativistically covariant equation for two particles with spin $1/2$ in quantum field theory [11] and was applied to the meson quark model with harmonic oscillator potential. In all the quark models of mesons with harmonic interaction the infinitely harmonic oscillator potential acts drastically if the quarks are separated by large distances. Therefore in the proposed model the interaction between the quarks will be described by Yukawa type potential

$$V(r) = -g^2 \frac{e^{-\mu r}}{r}, \tag{4}$$

where μ is the mass of exchange gluon. If in the corresponding radial equation

$$f_\ell''(r) + \left[\frac{w^2}{4} - m^2 + g^2\frac{e^{-\mu r}}{r} - \frac{\ell(\ell+1)}{r^2}\right] f_\ell(r) = 0 \tag{5}$$

following [10] we perform the substitutions:

$$f_\ell(r) = e^{ikr} r^{\ell+1} \phi(r) \tag{6}$$

$$z = -2ikr, \quad \kappa = ik = -\sqrt{m^2 - \frac{w^2}{4}} \tag{7}$$

we get

$$z\frac{d^2\phi}{dz^2} + (2\ell + 2 - z)\frac{d\phi}{dz} - \left[\ell + 1 + \frac{z}{(2\kappa)^2} V\left(-\frac{z}{2\kappa}\right)\right]\phi = 0. \tag{8}$$

Next we expand the potential in ascending powers of r :

$$V(r) = \sum_{i=1}^{\infty} M_{i+1} (-r)^i = \sum_{i=-1}^{\infty} M_{i+1} \left(\frac{z}{2\kappa}\right)^i, \tag{9}$$

where

$$M_i = g^2 \frac{\mu^i}{i!}. \tag{10}$$

Applying the perturbation approach procedure for regular interactions suggested by Muler for asymptotic solutions of equation (5) for large c.m.s. momentum we get

$$f_\ell(r) = e^{ikr} r^{\ell+1} \left[\Phi(a) + \frac{1}{(2ik)^3}\{P_2(a,1)\Phi(a+1) + P_2(a,-1)\Phi(a-1)\} + \right.$$

$$+ \frac{1}{(2ik)^3}\{P_3(a,2)\Phi(a+2) + P_3(a,1)\Phi(a+1) +$$

$$\left. + P_3(a,-2)\Phi(a,-2) + P_3(a,-1)\Phi(a-1)\} + \dots\right]. \tag{11}$$

In the last equation $\Phi(a)=\Phi(a,b;-2ikr)$ is the confluent hypergeometric function for $a=-n$; $n=0,1,2\ldots$; $b=2\ell+1$ and P are the well-known coefficients for which a recurrent formula is explicitly given in ref./10/. For instance, $P_2(a,1)=M_1\,a \quad P_2(a,-1)=-M_1(a-b)$.

The asymptotic expansion for Regge pole position for large k is

$$\ell_n=-n-1-\frac{1}{2}\frac{M}{\sqrt{m^2-\frac{w^2}{4}}}+\frac{1}{4\left(\sqrt{m^2-\frac{w^2}{4}}\right)^3}\{n(n+1)M_2+M_1M_0\}+$$

$$+\frac{(2n+1)M_2M_0}{8\left(\sqrt{m^2-\frac{w^2}{4}}\right)^8}+\ldots . \tag{12}$$

Neglecting in (12) the terms maintaining the largest powers of $\frac{1}{k^3}$ and terms with μ^2 and expanding $\sqrt{1-\frac{w^2}{m^2}}$ in ascending powers of $\frac{w}{m}$, we obatin Regge linear trajectories for meson states

$$\ell_n=-n-1-\frac{1}{2}\frac{g^2}{m}+\frac{g^4\mu}{4m^3}+\left(\frac{3g^4\mu}{32m^5}-\frac{g^2}{16m^3}\right)w^2 . \tag{13}$$

From the masses in the table /12/ for $S=0$

$J^{PG}=$	0^{--}	1^{++}	2^{--}
	$\pi(140)$	$B(1235)$	$A_3(1640)$
$w^2=$	$0{,}0195\ \mathrm{Gev}^2$	$1{,}52\ \mathrm{Gev}^2$	$2{,}69\ \mathrm{Gev}^2$

we obtain the numerical values of the parameters of the Regge trajectories

$$\ell_n=-n-1+a+\beta w^2 \tag{14}$$

$$a=0{,}87,\quad \beta=0{,}76\ \mathrm{Gev}^{-2}.$$

If we identify (13) with (14) we have

$$a=\frac{g^4\mu}{4m^3}-\frac{g^2}{2m}, \tag{15}$$

$$\beta=\frac{3g^4\mu}{32m^5}-\frac{g^2}{16m^3}. \tag{16}$$

This model permits also to consider the Regge trajectories corresponding to $n=1$, i.e. the first radial excited state. For example, the first excited state of a π meson can be assumed $F_1(1540)$. From eqs. (15) and (16) the parameters g^2 and μ will be expressed by quark masses by the relations:

$$g^2=m(8m^2\beta-3a), \tag{17}$$

$$\mu=\frac{2m(8m^2\beta-a)}{(8m^2\beta-3a)^2}. \tag{18}$$

Neglecting in the last equation the value of the parameter a which is smaller than m^2 we get a simple relation between the quark mass and the mass of exchange gluon

$$\mu\sim\frac{1}{4m\beta}. \tag{19}$$

Since from experimental information the quark mass is $m>5\,\mathrm{Gev}$ in accordance with (17) and (18)

$$g^2>744\ \mathrm{Gev},\quad \mu<0{,}067\ \mathrm{Gev}. \tag{20}$$

In the presented paper it is not our intention to draw possible conclusion from eq. (1) and to compare them critically with experimental data. But we wanted to discuss a few phenomenological comparisons in order to prove the correctness of our physical assumptions.

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