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PRESENT STATUS OF QUANTUM FIELD THEORY

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Introductory talk at the International Seminar on Nonlocal Quantum Field Theory, Alushta, April, 1973.

1. Introduction

Our Seminar is called a Seminar on Nonlocal Quantum Field Theory. However the range of problems we are dealing with is much wider. As a matter of fact, the Seminar is devoted to a review of the status of modern quantum field theory as a whole.

The traditional name of the conference is called to emphasize the fact that we are ready to discuss also problems which go beyond the framework of the canonical ones.

My talk does not aim at giving the outline of the problems to be discussed here, the agenda of the conference is clear from the programme. It will be a brief survey of the present -day situation in field theory without having claims to discussion of different conceptions.

I think I am not mistaken to say that for the past three years interest in the field theory, as a basis for understanding the world of elementary partibles, has greatly increased.

Moreover, the times when one suggested to bury completely the field theory and replace it by the conception of the analytical properties of the amplitude seem to be quite distant.

In reality, it turned out that this conception has no bases other than those which are presented by the field theory, in particular, the principle of local microcausality:

$$[\varphi(x), \varphi(y)] = 0 \qquad (x-y)^2 < 0. \qquad (1)$$

Here $\varphi(x)$ and $\varphi(y)$ are any two quantum fields at two space-time points x and y, $(x-y)^2 < 0$ means that the interval $x-y$ is a space-time one.

Next it was found that almost all the results of the phenomenological approach can be reproduced or even improved by means of the field theory.

II. Review of Some Results of the Field Theory

Current Algebra

Recently the attention of physicists has been drawn to the current algebra by means of which a series of useful relations has been derived. As an example, we may recall the sum rule for high-energy neutrino reactions which is based on the local commutation relations of currents:

$$\int d\omega \left[W^{\bar{\nu}p}(\omega, q^2) - W^{\nu p}(\omega, q) \right] \simeq 2, \qquad (2)$$

where $W^{\nu p}(\omega, q^2)$ is the structure function which describes inelastic lepton-nucleon interaction $\nu + N = \ell + X$;

$\omega = E_\nu - E_\ell$ is the energy transferred to the lepton; q^2 is the squared momentum transfer. The current density $J(x)$ is obviously a notion related to the field theory. In a certain sense, it supplements the notion of field $\varphi(x)$. In fact,

$$J(x) = \frac{\delta S}{\delta \varphi(x)} S^{-1}, \qquad (3)$$

where S is the scattering matrix.

Chiral Symmetry

In theoretical calculations much attention is paid to the investigation of the nonlinear representation of the chiral group. These investigations have led to the study of essentially nonlinear lagrangians of the type

$$\mathcal{L} = \frac{1}{2} g_{ab}(\pi) \frac{\partial \pi_a}{\partial x_\mu} \frac{\partial \pi_b}{\partial x_\mu}, \qquad (4)$$

where $\vec{\pi}$ is the meson field vector, and $g_{ab}(\pi)$ is the metric tensor in the same isotopic space. This tensor is of the form

$$g_{ab}(\vec{\pi}) = \delta_{ab}\, a(\vec{\pi})^2 + \pi_a \pi_b\, b(\vec{\pi}^2) \qquad (5)$$

the shape of the functions a and b depends on the parametrization chosen.

Until recently this lagrangian was thought of as a purely classic lagrangian from which, in the spirit of the principle of correspondence, it was possible to obtain relations between different processes of multiple pion production.

Now it is known that the probability amplitudes following from this lagrangian can be calculated by the superpropagator method first developed at our Laboratory, at Dubna.

Superpropagator

The superpropagator $Д(x-y)$ is of the form:

$$Д(x-y) = \sum_n C_n \langle \varphi^n(x)\varphi^n(y)\rangle = \sum_n C_n [\Delta^c(x-y)]^n, \quad (6)$$

where $\langle \ldots \rangle$ is the vacuum average of the T -product; $\Delta_c(x)$ is the free field causal function. The calculation of the superpropagator is performed by substituting the complex variable function $C(z)$ for the coefficients C_n and representing the sum (6) in the form of the Sommerfeld-Watson integral. This method has yielded some hopeful results for $\pi\pi$ interaction.

One has also undertaken some fascinating applications of this method for taking into account gravitation in quantum electrodynamics. Taking into consideration multiple production of gravitons, one has succeeded in deriving the final expression for the field mass

$$\frac{\delta m}{m} = \frac{3\alpha}{4\pi} \ln\left(\frac{4\pi}{\varkappa m}\right), \quad (7)$$

where $\alpha = e^2/\hbar c$, $\varkappa$ is the gravitational constant.

As far as we know, other methods do not allow us for the time being to take into account in a consistent manner quantum gravitation phenomena, and the most interesting investigations of the role of gravitation in the elementary particle world are restricted to the study of classical models.

Yang-Mills Fields

Another trend in the field theory is due to the development of the remarkable idea about local gauge invariant fields. We imply here the fields which transform by the formula

$$\psi'(x) = \exp\{ig\tau_a(x)\alpha^a(x)\}\psi(x), \quad (8)$$

$$B_\mu^{\prime\alpha}(x) = B_\mu^\alpha(x) + g\varepsilon^{\alpha bc} B_\mu^b \alpha^c(x) + \partial_\mu \alpha^\alpha(x). \quad (8')$$

The vector fields B_μ^α are called compensating fields. Their particular, perhaps predominant, role in the interaction is given the name of vector predominance.

Recently this idea has offered serious hopes for combining electromagnetic and weak interactions in one scheme.

Vacuum Splitting

In this scheme the spontaneous vacuum splitting is of great importance. In its simplest form, it is contained in any field theory with nonlinear lagrangian:

$$\mathcal{L} = \frac{1}{2}(\Box\varphi)^2 - U(\varphi), \tag{9}$$

where the "potential energy" $U(\varphi)$ has several minima for $\varphi = \varphi_i$ $(i = 1, 2 \ldots)$.

This scheme is very elegant in a classical case and becomes however very complicated in its quantized form.

In any case, these investigations remain an excellent illustration of possibilities of the field theory.

Eikonal Approximation

The eikonal (geometric-optical) approximation which was first developed many years ago at Dubna has proved to be very fruitful for the study of the scattering of strongly interacting particles.

In the past years attempts have been made to give grounds for this method starting from the field theory.

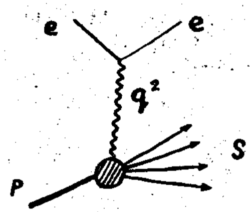
The use of the functional integration procedure was found to be most successful. One has succeeded in obtaining asymptotic solutions for some optical models which give the optical picture of particle scattering. With certain physically clear approximations it was found possible to get an eikonal approximation for the scattering amplitude

$$A(s,q) = \frac{s}{4\pi i}\int d^2 b\, e^{iqb}\left[e^{i\chi(b,s)} - 1\right], \tag{10}$$

where b is the impact parameter, q - is the momentum transfer, $\chi(b,s)$ is an eikonal which depends on the mode of particle interaction. The result of these calculations leads to the Gauss-potential with logarithmically increasing range.

Scale Invariance

is one of the most interesting results for the past years. Two facts have stimulated its development. First of all, this is the automodel behaviour of deeply inelastic $e-p$ scattering



the amplitude of which, at large q^2 and s, depends only on their ratio, so that

$$\sigma \sim \frac{1}{s} f(s/q^2). \tag{11}$$

Then it was found that the total cross section and the cross section for inclusive processes is in satisfactory agreement with "longitudinal" automodelity:

$$\sigma_{tot} \sim const \tag{12}$$

$$\frac{d\sigma_{incl}}{d\vec{q}} = \frac{1}{q} \begin{cases} \times f\left(\frac{q_\parallel}{E}, q_\perp\right) & \text{in the region of fragmentation} \\ \times f(q_\perp) & \text{in the region of pionization} \end{cases}$$

On the basis of the field theory it was shown that the automodel behaviour of deeply inelastic processes does not contradict the local field theory. This important result has been obtained by studying the analytic properties of the Compton effect amplitude for large q which corresponds in the current commutator to the region near the light cone. Scale invariance can surely be expected when all the scalar products of external momenta are larger than all the hadron masses:

$$p_i p_j \gg m^2 \quad \text{including} \quad i=j. \tag{13}$$

In the language of space-time this condition would mean the smallness of all the distances between pairs of interacting particles. However this is not a physical domain. Using the methods of summation of the Feynmann diagrams developed at our Laboratory one has succeeded in studying the amplitude behaviour on the mass shell and, at the same time, obtaining a rather complete picture of high-energy processes.

The important consequence of scale invariance is a restriction imposed on the choice of interaction lagrangians. In particular, for a system of baryons and pseudoscalar mesons scale invariance conditions lead to a classical case

$$\mathcal{L}_{int} = g\bar{\Psi}\gamma_5\Psi\varphi + h\varphi^4 \tag{14}$$

under the condition of finite renormalization.

<u>Nonlocal Field Theory</u>

During the period considered we learnt to estimate the difficulty of constructing the scattering matrix based on the nonlocal field $\phi(x)$, i.e. the field for which condition (1) is violated. In this connection, a success has recently been achieved in proving the possibility of constructing a nonlocal matrix S_α obeying unitarity

$$S_\alpha^+ S_\alpha = 1 \tag{15}$$

and macroscopic causality

$$\frac{\delta^2 S_a}{\delta\,\phi(x)\,\delta\phi(y)}\,S_a^{-1} = \mathcal{K}(x-y). \qquad (16)$$

In this case the operator $\mathcal{K}(x-y)$ vanishes when $(x-y)^2 < 0$ as applied to a definite class of trial functions. It was found to be important to give a strict definition of the class of trial functions allowed by the nonlocal field theory. Therefore there appears some possibility of calculating the scattering matrix in the case of nonrenormalizable interactions.

Curved Momentum Space

The development of the field theory methods in the curved space of relative momenta belongs to the same field of investigations. The law of summation of relative momenta is changed

$$q_1 + q_2 \longrightarrow q_1 \oplus q_2 \qquad (17)$$

in accordance with the assumption that the space $\mathcal{R}_4(q)$ has a constant curvature $\hbar^2/a^2$. This curvature is responsible for the existence of the upper limit of the elementary particle masses, a peculiar "maximon" with $M = \hbar/a$. Most recently a great success has been achieved in obtaining an axiomatic formulation of this interesting theoretical scheme.

Quantization in the Riemannian Space

The field quantization in the Riemannian space $\mathcal{R}_4(x)$ of constant curvature (in the de-Sitter space) would seem to be supplementary to this study. As far as I know, this is the first example of quantization in the space with a metric which differs from the Minkowski metrio and is remarkable by the faot that the conformal invarianoe of the lagrangian (at $m \to 0$) is found to be necessary for a corpuscular interpretation of the state veotors.

Quantization of an Essentially Nonlinear Field

In this conneotion we may recall the work on quantization of an essentially nonlinear field which is approximately decomposed into a classic(strong) $\psi_{cl}(x)$ and a quantum (weak) field $\hat{\psi}(x)$

$$\psi(x) = \psi_{cl}(x) + \hat{\psi}(x). \qquad (18)$$

In this oase the problem reduces to taking the Feynmann integral with a quadratic form, which has variable coefficients, depending on $\psi_{cl}(x)$ and consequently, is also an example of quantization in the curved Riemannian space.

Quarks

I do not dwell on the theory of quarks since one has failed to formulate it as a consistent field theory. The main results of the quark theory are for the time being more successful when they are obtained from special models, e.g. from the parton model or by means of the theory of similarity and the dimensional analysis.

In the case of inclusive electromagnetic or weak interactions it is enough to use the usual dimensional analysis, while, when studying strong inclusive processes, it is necessary to employ a generalized dimensional analysis with two independent length scales, i.e. in the longitudinal and transverse directions.

III. Critical Aspects

Once the quantum field theory is regarded as a basis for understanding microworld, we should be aware of the fact that as to our ideas we return to the conceptions which appeared as long ago as in the thirties, and we encounter very old problems at the new level.

Noncompleteness of S-Matrix Description

The renormalization method was conceived by their creators as a purely formal receipt for avoiding very small distances

a or, as we say now, the region of large momentum transfers $q \sim \hbar/a$.

The incompleteness of the S-matrix description does not consist in its applicability only to a definite class of interactions, but rather in that, it does not in principle allow one to give a description of the course of events in time.

In particular, to describe the behaviour of K mesons we should specify the initial state at a time moment $t_1 = 0$ (rather than $t_1 = -\infty$) and then for all the time moments $t_2 > t_1$ (but not only for $t_2 = +\infty$). In general, the necessary accuracy of time description is $\Delta t \ll \hbar/\Gamma$, where Γ is the decay constant of an unstable particle.

However, for the operator $U(t_2, t_1)$ transforming the state given at a moment t_1 into the state at a moment t_2 , one has not yet found methods of removing divergences, and this old problem can, by no means, be thought of as either solved or declined. The operator $U(t_2 t_1)$ itself may, of course, turn out to be a not quite strictly defined notion.

In nonlocal theory the precise time moments t_2 , t_1 may be only approximately defined ones.

Thus, the first and main shortcoming of modern description of phenomena in the elementary particle world is the incompleteness of the S-matrix approach. The second shortcoming is

Abundance of Fields

At present each sort of stable or unstable particles should be assigned its proper field. Nevertheless, it is clear that some sequences of "particles" are nothing less than excitations of certain original particles. In any case, it appears that there is no doubt that such an interpretation can be applied to particles lying on the same Regge trajectory:

$$j = \alpha(m^2) \tag{19}$$

j is the particle spin, m is the particle mass. Following the Regge theory, when exchanging an elementary particle or a complex particle, the behaviour of the cross section is different. In the former case there must not be observed a narrowing of the diffractional cone.

In this connection, a somewhat unexpected result has been obtained in Serpukhov when studying backward $\pi^+ p$ scattering. The obtained cross section as a function of energy s (24 and 40 GeV) contradicts the supposition that the proton lies on the Regge trajectory.

There is no doubt that studies of such a kind will help to classify particles and understand their hierarchy.

An attempt made by Heisenberg to consider the whole manifold of particles on the basis of a unified field ("Uhrfeld") has not been successful. However, this failure removes in no way the problem of variety of variables that should be used in describing the structure and dynamics of elementary particles.

The most characteristic phenomenon in the elementary particle world is the cascade. Every particle A or an ensemble of particles B given at $t_1 = 0$ turns asymptotically, as $t_2 \to +\infty$, into a cascade consisting of stable particles. The stable particles are described by five fields: ψ_N, ψ_e, ψ_{ν_e}, ψ_{ν_μ} and A_γ. Due to reversibility we can return this cascade in its initial state A (or B). It follows that the variables which are used to describe an ensemble of free stable particles are equipotent with those used to describe any particle state.

Therefore to describe any states of particles five fields are to be sufficient which, of course, should not necessarily coincide with the fields of stable particles.

It is interesting that the property called strangeness S is not related to stable particles. Therefore it should be viewed as a characteristic of a certain internal symmetry of the wave functional

$$\Omega = \Omega(\varphi_1 \ldots \varphi_5) \tag{20}$$

which describes baryons, mesons or leptons in terms of primitive fields, $\varphi_1, \varphi_2, \dots \varphi_5$.

The choice of these fields remains the most important problem of contemporary particle theory.

VI. On the Limits of Local Theory

Whatever the choice of fundamental fields may be, the supposition on accurate localization of fields (or currents) leads to the fact that the wave functionals are represented as asymptotio series <u>which are not determined unambiguously</u>. This gives rise to the question as to what are the limits of applicability of local field theory.

<u>The Point Event $\mathcal{P}(x)$</u>.

The looal theory is based on the assumption about a possible infinitely precise looalization of <u>point events</u> in space-time $\mathcal{R}_4(x)$.

The analysis shows that the local theory allows implioitly the existenoe of elementary partioles of arbitrarily large mass $M\ (M \to \infty)$ as representatives of point events. In this oase the allowed indeterminaoy

$$\Delta x \simeq \frac{\hbar}{Mc} \to 0 \qquad (21)$$

and the local theory beoomes <u>self-oonsistent</u>.

However it is unknown whether particles of mass $M \to \infty$ exist or not.

With infinitely increasing mass gravitational effects also inorease, and in the domain defined by the equality of the Compton wave length of a partiole to its gravitational radius

$$\hbar/Mc \sim \frac{2kM}{c^3} \qquad (22)$$

(here $k = 6{,}67 \cdot 10^{-8} \frac{cm^3}{g \cdot sec^2}$ is the Newton oonstant), we arrive at a maximum heavy particle - <u>gravitational maximon</u>

$$M = M_g \simeq 1{,}5 \cdot 10^{-5} g.$$

The problem of the role of gravitation in the elementary particle world is still debatable. However, there is no doubt that if gravitation plays there any role whatsoever this role will be appreciable only in the region

$$\Lambda_g = \hbar/M_g c = 0{,}23 \cdot 10^{-32}\ cm\ .$$

There naturally arises the question: is there a possibility for the existenoe of an earlier limit for the elementary particle mass? This problem reduoes to the problem of the asymptotic behaviour of the ratio

$$\gamma = \Gamma/M\ , \qquad (23)$$

where Γ is the decay width. It is clear that if $\Gamma \simeq M$ then the particle does not exist as a definite space-time object.

If with increasing energy the <u>weak interaction</u> tends to its unitary limit, it is quite possible that the ratio γ approaches unity. In this case we are dealing with a <u>weak maximon</u> $M = M_F$. Then the limit of the local theory is defined by the Fermi length

$$\Lambda_F = \frac{\hbar}{M_F c} = 0.6\mathrm{x}10^{-16}\ \mathrm{cm}$$

i.e. far earlier than dictated by gravitation.

A mathematical formulation of the theory in which the notion of exactly defined coordinate of a point event does not exist is an attractive problem, related to the theory of <u>stochastic spaces.</u>

In these spaces the arithmetization of events is approximate since the coordinates of the event $\mathcal{P}(x)$ are <u>stochastic quantities</u>

$$\hat{x}_\mu = x_\mu + \hat{\xi}_\mu \tag{24}$$

and $\langle \xi_\mu \rangle = 0$, $\langle \xi_\mu^2 \rangle \neq 0$ +)

+) In particular, if a gravitational field is quantizable then the Riemannian space $R_4(x)$ becomes inevitably stochastic.

V. Conclusion

The troubles which are due to the use of the local field conception have undoubtedly serious grounds. The fact itself of the existence of these troubles is beyond any doubts. On the other hand, we should bear in mind that the experimental facts are nowhere in contradiction with the basic condition of the local theory (I). In particular also there is not observed any length scale "a" that would point to a deviation from locality. Moreover the behaviour of deeply inelastic processes emphasized <u>automodelity</u> of phenomena at large momentum transfers.

The only phenomenon that can favour nonlocality is the increase of the weak interaction cross section with increasing energy E as it follows from the study of inelastic processes of the type

$$\nu + N \rightarrow \ell + X ,$$

where ν is a neutrino, ℓ – a lepton, N a nucleon, and X are any particles obeying baryon number conservation. From the theoretical viewpoint the total cross section for this process contains the length and has the form

$$\sigma_{tot} = \alpha \cdot \Lambda_F^4 \left(\frac{mc}{\hbar}\right)^2 \left(\frac{s}{m^2}\right) , \tag{25}$$

where α is the numerical coefficient, Λ_F =0.23x10^{-32}cm, m is the nucleon mass, s is the invariant energy. Recent measurements show that in complete agreement with (25) we have

$$\sigma_{tot} = (0,7 \pm 0,14)\, E_\nu \cdot 10^{-38}\ \mathrm{cm}^2,$$

for E_ν of the order of a few GeV. It is quite possible that the cross section for these processes may be comparable with the cross section of electromagnetic processes as was predicted many years ago at Dubna.

However, whatever the experimental facts may be, the imperfection of the mathematical apparatus of the local theory is so obvious that not only the investigations on the refinement of this formalism seem to be more than well founded, but also any research efforts devoted to the very bases of the local theory deserve every support.

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