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STRAIGHT LINE PATH METHOD AND THE EIKONAL PROBLEM

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Introduction

As the theoretical analysis of the experimental data shows, the eikonal representation of the scattering amplitude with smooth effective quasipotential correctly reproduces the main features of the high energy particle scattering at small angles.

In this connection the study of the local quasipotential qualities and the foundation of eikonal representation in the quantum field theory models are of great interest /1,2/.

Studying the eikonal problem the Dubna group has formulated the "straight line path approximation" /1,3/. The essence of this method is in the assumption that the large momentum transfers are suppressed in each act of high energy particle interaction. So large momenta, carried by the particles in the collision process, have a conservation tendency ("inertia" of large momenta).

The type of the particles transferring large momenta may change during the interaction process according to the empirical regularities observed in the inclusive reactions. Thus, for example, in the collision of fast nucleons it is necessary to take into account the possibility of radiation of hard mesons which take away the greatest part of the initial nucleon momenta.

Generally to obtain the eikonal formula by summation of the perturbation theory series one takes the initial particles as the leading ones transferring large momenta. The results thus obtained are in essence equivalent to those of the " $k_i k_j = 0$ approximation" /4,5/.

However the existence of virtual processes with the alteration of the "leading" particle type must lead, generally speaking, to the violation of the orthodox eikonal representation.

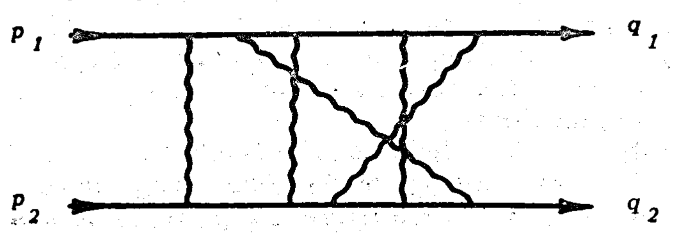
The possibility that such extra-terms in the asymptotics of some diagrams can appear was first noted in the paper /6/.

In our paper we study a structure of the "noneikonal" contribution to the two nucleon scattering amplitude described by a ladder-type diagram sum without taking into account radiative corrections and vacuum polarization effects in the scalar model.

It will be shown in particular that there exist in the sum of all ladder-type graphs of the eighth order the terms which violate the orthodox eikonal formula but disappear in the limit $\frac{\mu}{m} \to 0$ where μ and m are meson and nucleon masses, respectively. These terms are associated with the contribution to the effective quasipotential corresponding to the nucleon-antinucleon pair exchange.

1. High Energy Asymptotics of Feynman Graphs and Modification of the Particle Propagators

Now we choose to study the scattering amplitude of two scalar nucleons in the model $\mathcal{L}_{int} = g : \psi^+ \psi \quad \phi :$ neglecting the radiative corrections and closed nucleon loops. This amplitude is represented as the sum of the following diagrams



$= F$

where p_1 and p_2 are the in-particle momenta and q_1, q_2 are the out-particle ones. If the number of

integration momenta is ℓ and the number of internal lines is I (for the diagrams of Fig. 1 type $I = 3\ell + 1$)

$$F = \int dk_1 \ldots dk_\ell \prod_{i=1}^{I} \frac{1}{r_i^2 - m_i^2 + i\epsilon}, \tag{1.1}$$

where r_i are linear combinations of integration momenta k_j.

Using the Feynman parametrization we have

$$F = (I-1)! \int_0^1 d\alpha_1 \ldots d\alpha_I \, \delta(1 - \sum_{i=1}^{I} \alpha_i) \int \frac{dk_1 \ldots dk_\ell}{[\Psi(k, \alpha, s, t)]^I}, \tag{1.2}$$

where

$$\Psi = \sum_{i=1}^{I} \alpha_i (r_i^2 - m_i^2 + i\epsilon) = \sum_{i,j=1}^{\ell} a_{ij} k_i k_j + 2 \sum b_i k_i + c. \tag{1.3}$$

After this procedure it is possible to obtain a representation for F in the Chisholm form /7/, integrating over k_i in eq. (1.2)

$$F = (i\pi)^\ell (I - 2\ell - 1)! \int_0^1 d\alpha_1 \ldots d\alpha_I \, \delta(1 - \sum_{i=1}^{I} \alpha_i) \frac{[C(\alpha)]^{I-2\ell-2}}{[\mathfrak{D}(\alpha, s, t)]^{I-2\ell}}. \tag{1.4}$$

In the formula (1.4)

$$C = \det \| a_{ij} \|, \quad \mathfrak{D} = \det \left\| \begin{array}{c|c} a_{ij} & \begin{array}{c} b_1 \\ \vdots \\ b_\ell \end{array} \\ \hline b_1 \ldots b_\ell & c \end{array} \right\| \tag{1.5}$$

and the Chisholm determinant $\mathfrak{D}$ can be represented in the following form

$$\mathfrak{D}(\alpha, s, t) = f(\alpha) s + g(\alpha) t + h(\alpha). \tag{1.6}$$

Now we shall give a brief account of the results obtained in /8/ which we'll use to study the asymptotic behaviour of the expression (1.4) *.

Definition

A t-path is a set of lines forming a continuous arc, such that

a) If we short-circuit all these lines, the entire graph is split into two parts having no common lines and only one common vertex (to which these lines have been reduced). The p_1 and q_1 external lines of the graph are attached to one of the two parts and p_2 and q_2 ones to the other.

b) None of its subsets has property (a). A $\bar{t}$-path is a t-path of minimum length (i.e. number of lines).

Rule

If the graph F is such that there exist M $\bar{t}$-paths of length ρ its asymptotics is

$$F \simeq (i\pi^2)^{\ell} \frac{(I-2\ell-1-\rho)!\,\rho!}{(M-1)!} \frac{(\ln s)^{M-1}}{s^{\rho}} \int \frac{[C_0(a)]^{I-2\ell-2}}{(g_0 t + h_0)^{I-2\ell-\rho}\, \tilde{f}_0^{\rho}} \cdot \tag{1.7}$$

$$\prod_{j=1}^{M} \delta\Big(\sum_{\nu=1}^{\rho} a_\nu^{(j)} - 1\Big)\, \delta\Big(\sum_{\nu \not\subset p} a_\nu - 1\Big)\, \{da\}.$$

In the formula (1.7)

$$g_0 t + h_0 = \mathfrak{D}(a,s,t)\big|_{a_\nu^{(j)}=0}, \tag{1.8}$$

$$C_0(a) = C(a)\big|_{a_\nu^{(j)}=0}$$

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* Results similar to those of ref. /8/ have been obtained also in papers /9/.

$a_\nu^{(j)}$ are parameters of those lines which belong to the j-th path; a_ν $(\nu \not\subset p)$ are the remaining parameters and the quantity $\tilde{f}_0$ is obtained from f (see (1.6)) as follows: Let us perform the replacement

$$a_\nu^{(j)} \to \lambda_j\, a_\nu^{(j)}, \tag{1.9a}$$

then

$$f \to \lambda_1 \lambda_2 \ldots \lambda_M \tilde{f}(\lambda) \qquad \text{and} \qquad \tilde{f}_0 = \tilde{f}\,\big|_{\lambda_j = 0} \tag{1.9b}$$

Now after having written out the formulae we need, we can give further account.

In case of the momentum transfers in the graphs 1 being a zero, i.e. $p_1 = q_1$ and $p_2 = q_2$, we shall call a set of lines, whose propagators depend on the momentum p_1, a p-path.

Thus in the graphs F there are two p-paths each forming a continuous arc. Note that each p-path is a t-path according to the Definition. However the configurations of the p-paths depend on the concrete arrangement of the integration momenta while the t-paths are the topological characteristics of the given graph. In the studied graphs the integration momenta can be chosen so that the p-paths will coincide with any pair of t-paths not forming a closed loop.

Statement 1

Let the given graph be such that the contribution to the leading asymptotics is due to the pair of $\bar{t}$-path having no common line. Then the asymptotics of this graph will not be changed if the integration momenta are placed so that the p-paths coincide with $\bar{t}$-paths and the following modification of the propagators depending on external momenta p is performed

$$\frac{1}{(\Sigma k_i)^2 + 2p\,\Sigma k_i - \tilde{m}^2 + i\epsilon} \to \frac{1}{2p\,\Sigma k_i + i\epsilon}, \tag{1.10}$$

i.e. we neglect masses and products of integration momenta.

Proof

The propagator modification (1.10) results in the following alterations of determinants C and $\mathfrak{D}$. In the determinant C parameters corresponding to the $\bar{t}$-paths become equal to zero, i.e. C turns into C_0. In the determinant $\mathfrak{D}$ the quantity c in which the same parameters become equal to zero also is changed. As a result the quantity $f(\alpha)$ (see (1.6)) conserves its main properties which determine the asymptotic dependence on s. Determinants C_0, $g_0 t + h_0$ and $\bar{f}_0$, calculated according to eqs. (1.8) and (1.9) are also not changed. Thus we make sure that whenever the propagator modification (1.10) is performed the expression (1.7) is the correct asymptotic form of our Feynman integral.

Statement 2

Let the given graph be such that the contribution to the leading asymptotics is due to a pair of $\bar{t}$-paths having a common line. Let also the integration momenta be placed so that p-paths coincide with $\bar{t}$-paths. Then the asymptotics of the graph is equal to the factor $(\pm \frac{1}{s})$ multiplied by the asymptotics of the reduced graph obtained when we short-circuit the common line. We choose the plus sign when the external momenta in this line have the same direction, if not we choose the minus sign. Dealing with the reduced graph we can use Statement 1.

Proof

Let a parameter β be associated with the common line to which there corresponds the propagator

$$\frac{1}{(\Sigma k_i)^2 + 2(p_1 \pm p_2)(\Sigma k_i) - M^2 \pm s + i\epsilon}. \qquad (1.11)$$

It is sufficient to show that the propagator (1.11) can be replaced by $(\pm \frac{1}{s})$. Really, quantities C_0 and $g_0 t + h_0$ aren't changed because of the arguments used in proof of the Statement 1. The quantity f has a structure

$$f = \beta C + \left\| \begin{array}{c|c} C & (\alpha^{(1)}+\beta) \\ \hline (\alpha^{(2)}+\beta) & 0 \end{array} \right\| , \qquad (1.12)$$

where $\alpha^{(1)}$ and $\alpha^{(2)}$ are sets of parameters corresponding to the two $\bar{t}$-path. It is evident now that instead of f we can use the quantity

$$f^{(1)} = \beta C(\beta = 0) + \left\| \begin{array}{c|c} C(\beta=0) & (\alpha^{(1)}) \\ \hline (\alpha^{(2)}) & 0 \end{array} \right\| \qquad (1.13)$$

which proves our statement.

2. Eikonal and Noneikonal Contributions to the Scattering Amplitude

As is known /10/, two scalar nucleon scattering amplitude can be represented in the form

$$f(p_1, p_2; q_1, q_2) = = \frac{i g^2}{(2\pi)^4} \int d^4x\, \mathfrak{D}(x)\, e^{-ix(p_1 - q_1)} \int_0^1 d\lambda\, S_\lambda + (q_1 \to q_2) \qquad (2.1)$$

if we neglect the radiative corrections and vacuum polarization. In the formula (2.1)

$$S_\lambda = \int [\delta\nu_1]_{-\infty}^{\infty} [\delta\nu_2]_{-\infty}^{\infty} \exp\{ig^2 \lambda \int_{-\infty}^{\infty} d\xi\, d\tau\, \mathfrak{D}\, [-x+$$

$$+2\xi a_1(\xi)-2\tau a_2(\tau)-2\int_{-\xi}^{0}\nu_1(\eta)d\eta+2\int_{-\tau}^{0}\nu_2(\eta)d\eta], \qquad (2.2)$$

$$a_{1,2}(\xi)=p_{1,2}\theta(\xi)+q_{1,2}\theta(-\xi)$$

In the expressions (2.1) and (2.2) the integration momenta of each diagram are placed so that the p -paths coincide with nucleon lines.

Putting the variables ν_1 and ν_2 equal to zero, i.e. neglecting the terms of $k_i k_j$ -type in nucleon propagators, we obtain according to the statement 1 a sum of contributions in each diagram of those t -paths which coincide with nucleon lines. Note that for the present twisted graphs corresponding to the term $(q_1 \to q_2)$ in eq. (2.1) are not under discussion.

As a result we have the well known eikonal representation for the scattering amplitude

$$f=\frac{is}{(2\pi)^4}\int d^2\vec{x}_\perp\, e^{-i\vec{x}_\perp\vec{\Delta}_\perp}\left(e^{-\frac{ig^2}{4\pi s}K_0(\mu|\vec{x}_\perp|)}-1\right) \qquad (2.3)$$

when $s=(p_1+p_2)^2\to\infty$ and $t=(p_1-q_1)^2$ is fixed. According to this we'll call the contributions of the t -paths, coinciding with nucleon lines, the eikonal ones.

In the paper [6] it was pointed to the fact that in diagrams of higher orders (namely, beginning from the 8-th) in powers of coupling constant g it is necessary to take into account other t -paths whose contributions may be comparable with those of the eikonal t -paths. We begin our study of the noneikonal contributions with a diagram shown in Fig. 2

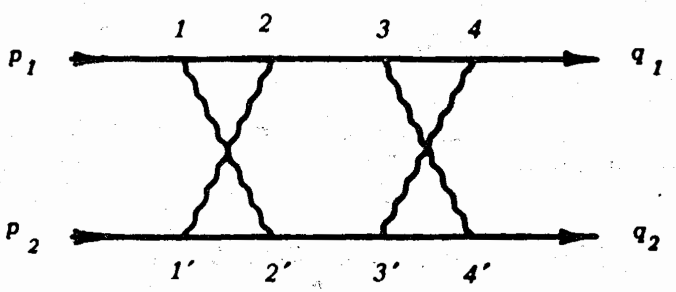


Fig. 2

In this graph which we'll call "xx -diagram" there exist four $\bar{t}$ -paths of the same length three: (1234), (1'2'3'4'), (1'234') and (12'3'4). A formal account of all the $\bar{t}$ -paths lead us to the asymptotics $\frac{\ln^3 s}{s^3}$. However that corresponds to the conversion into zero of all line parameters and this is impossible due to the factor $\delta(1-\Sigma\alpha_i)$. Use of any three paths should lead to the asymptotics $\frac{\ln^2 s}{s^3}$, but in that case the coefficient uncluding determinant C_0 also becomes equal to zero so long as these three paths form a closed loop.

Then it is necessary to calculate a sum of contributions from the following pairs of $\bar{t}$ -paths:

$$(1234;\,1'2'3'4'),\ (1234;1'234'),\ (12'3'4;1'2'3'4'), \qquad (2.4)$$
$$(12'3'4;1'234').$$

Pairs $(1234;\,12'3'4)$ and $(1'2'3'4';\ 1'234')$ have no influence upon the asymptotics since these $\bar{t}$ -paths formed a closed loop. All pairs of $\bar{t}$ -paths (2.4) lead to the same asymptotic dependence on s namely $\frac{\ln s}{s^3}$ thus we'll be interested in coefficients.

The contribution to the xx -diagram from the pair $(1234;1'2'3'4')$ is included in formula (2.3) and will be indicated

$$\frac{\ln s}{s^3}\, f_{eik}^{(xx)}(t). \qquad (2.5)$$

Now we'll get the contribution from the $\bar{t}$ -paths $(12'3'4)$ and $(1'234')$. Let us choose the integration momenta so that these paths coincide with the p-paths.

Then according to the Statement 1 we can modify propagators of lines forming the $\bar{t}$- paths.

After that perform the substitution of integration momenta

$$k_i \to \frac{m}{\mu}k_i \qquad (2.6)$$

which results in the replacements of nucleon lines by meson ones

$$\mathfrak{D}_m(k\frac{m}{\mu}) = \frac{1}{k^2\frac{m^2}{\mu^2} - m^2 + i\epsilon} = \frac{\mu^2}{m^2}\mathfrak{D}_\mu(k) , \qquad (2.7)$$

$$\mathfrak{D}_m(p_1-q_2-k) \to \frac{\mu^2}{m^2}\mathfrak{D}_\mu[(p_1-q_1)\frac{\mu}{m} - k] , \text{ i.e. } t \to t\frac{\mu^2}{m^2} .$$

The propagators corresponding to the $\bar{t}$ -paths will be multiplied by $\frac{\mu}{m}$. Thanks to this fact we may consider all the lines of $\bar{t}$-paths as modified nucleon lines. As a result we obtain a diagram of the same type (Fig. 2) but the p -paths being directed along nucleon lines

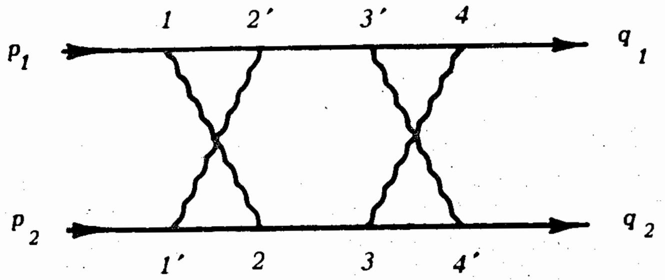


Fig. 3

So the described contribution is of the form

$$\frac{\ell n\, s}{s^3} f^{(1)}_{noneik}(t),$$

$$f^{(1)}_{noneik}(t) = \frac{\mu^2}{m^2} f^{(xx)}_{eik}(t\frac{\mu^2}{m^2}). \qquad (2.8)$$

If the particle masses satisfy the condition

$$\frac{\mu^2}{m^2} \ll 1 , \quad \frac{t}{m^2} \ll 1 \qquad (2.9)$$

the contribution of noneikonal $\bar{t}$ -paths will be less than that of eikonal ones.

Now we have nothing to consider but the contribution to the asymptotics of xx -diagram from the pair of $\bar{t}$ -paths $(1'2'3'4')$ and $(12'3'4)$. The other remaining pair (1234) and $(1'234')$ (see (2.4)) evidently lead to the same contribution. The $\bar{t}$ -paths $(1'2'3'4')$ and $(12'3'4)$ being short-circuited we obtain the reduced graph

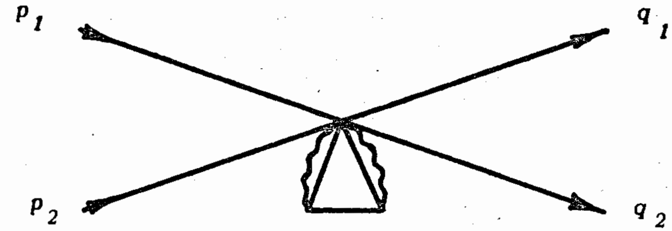


Fig. 4

Then it follows that the contribution of these $\bar{t}$ -paths does not depend on momentum transfers, i.e. can be represented in the form

$$\frac{\ell n\, s}{s^3} \frac{1}{\mu^2}\phi(\frac{\mu^2}{m^2}). \qquad (2.10)$$

Let us find the form of function $\phi(\frac{\mu^2}{m^2})$ if the condition (2.9) is satisfied. For this purpose choose the integration momenta in xx -diagram so that the p- paths coincide with the $\bar{t}$ -paths $(1'2'3'4')$ and $(12'3'4)$. Then using the Statement 2 we obtain that the desired contribution will be equal to the reduced graph (Fig. 5) asymptotics multiplied by $\frac{1}{s}$.

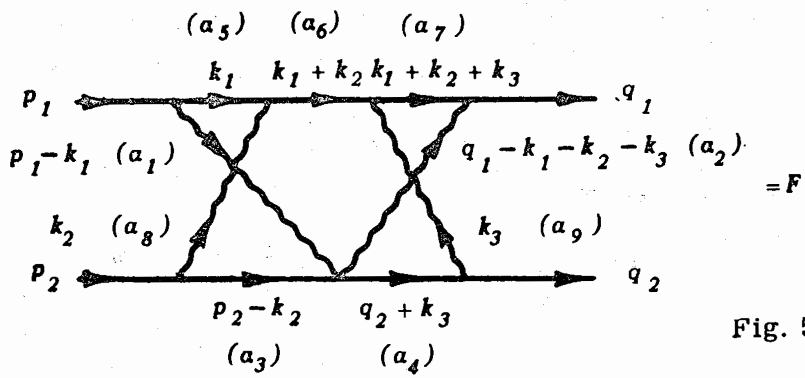


Fig. 5

When $s \to \infty$ the asymptotics of F' according to the formula (1.7) will be of the form

$$F' \simeq \frac{\ln s}{s^2}\, const \int da_1 \dots da_9\, \delta(1-a_1-a_2)\,\delta(1-a_3-a_4)\,\cdot$$

$$\cdot\, \delta(1-a_5-a_6-a_7-a_8-a_9)\,\frac{C_0}{(g_0 t + h_0)\tilde{f}_0^2}, \qquad (2.11)$$

where

$$g_0 = 0,$$
$$h_0 = -\mu^2\left[\frac{m^2}{\mu^2}(a_5+a_6+a_7)+(a_8+a_9)\right]C_0 \qquad (2.12)$$

From eqs. (2.11) and (2.12) we get the expression for the function ϕ defined by the relation (2.10).

$$\phi\left(\frac{\mu^2}{m^2}\right) = const \int \{da\} \times$$
$$\times\, \Pi\,\delta(1-\Sigma\delta a)\,\frac{\delta(1-a_5-a_6-a_7-a_8-a_9)}{\tilde{f}_0^2\left[\frac{m^2}{\mu^2}(a_5+a_6+a_7)+(a_8+a_9)\right]}. \qquad (2.13)$$

At large $\frac{m^2}{\mu^2}$ the main contribution comes from a domain $a_5+a_6+a_7=0$ and we can use again the Tiktopoulos method /8/, performing the substitution $a_{5,6,7} \to \lambda a_{5,6,7}$. As a result

$$da_5\, da_6\, da_7 \to \lambda^2\,\delta(1-a_5-a_6-a_7)\, da_5\, da_6\, da_7\, d\lambda,$$
$$\delta(1-a_5-a_6-a_7-a_8-a_9) \to \delta(1-a_8-a_9) \qquad (2.14)$$
$$\tilde{f} \to \lambda\, \tilde{\tilde{f}}_0,$$

from which follows

$$\phi\left(\frac{\mu^2}{m^2}\right) = const \int_0^1 \frac{d\lambda}{\lambda\frac{m^2}{\mu^2}+1}, \quad \text{i.e. } \phi\left(\frac{\mu^2}{m^2}\right) = const\, \frac{\mu^2}{m^2}\ln\frac{\mu^2}{m^2}. \qquad (2.15)$$

under condition (2.9). Note that $const$ in eq. (2.15) now includes in itself all the integrals over a_i. Taking into account the equality $f_{eik}(t=0) = \frac{const}{\mu^2}$ and eqs. (2.5), (2.8), (2.10), (2.15) we obtain the asymptotics of the xx-diagram

$$f^{(xx)}(t) \simeq \frac{\ln s}{s^3}\left\{ f^{(xx)}_{eik}(t) + f^{(xx)}_{noneik}(t)\right\}, \qquad (2.16)$$

where

$$f^{(xx)}_{noneik}(t) = \frac{\mu^2}{m^2}\, f^{(xx)}_{eik}\left(t\frac{\mu^2}{m^2}\right) + const\, f^{(xx)}_{eik}(t=0)\,\frac{\mu^2}{m^2}\ln\frac{\mu^2}{m^2}$$

when $s \to \infty$, t is fixed and $\frac{\mu^2}{m^2} \ll 1$.

3. Asymptotics of the Nucleon-Nucleon Scattering Amplitude. Eighth Order

In the previous paragraph we have considered one of the eighth order diagrams. Now turn to the remaining diagrams except for the twisted graphs described by the term $(q_1 \to q_2)$ in formula (2.1). In these diagrams there are three types of noneikonal $\bar{t}$-paths which can give a contribution to the leading asymptotics.

In the first type we include noneikonal $\bar{t}$-paths which have no common line. Except the xx-diagram there is only one graph with such $\bar{t}$-paths (see Fig. 6) and two cross-symmetric diagrams.

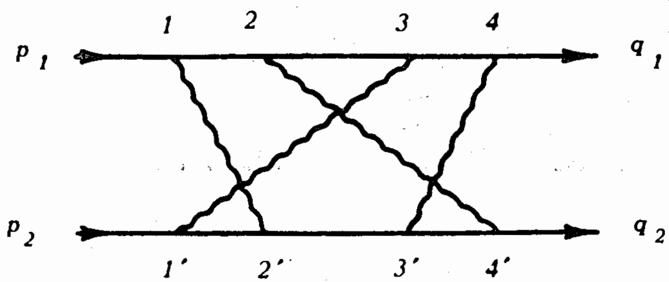


Fig. 6

The contribution to the asymptotics of the diagram 6 can be written down in the same form as (2.8)

$$f^{(2)}_{noneik}(t)=\frac{\ln s}{s^3}\,\frac{\mu^2}{m^2}\,\text{cross}\,f^{(2)}_{eik}\left(t\frac{\mu^2}{m^2}\right). \qquad (3.1)$$

If add the eikonal contribution of diagrams xx and 6 to those of cross-symmetric ones, then $\ln s$ cancell and we obtain the total eikonal contribution

$$\frac{1}{s^3}\,f_{eik}(t). \qquad (3.2)$$

Then according to eqs. (2.8) and (3.1) the noneikonal $\bar{t}$-path contribution to the same sum has the form

$$f_{noneik}(t)=\frac{1}{s^3}\,\frac{\mu^2}{m^2}\,f_{eik}\left(t\frac{\mu^2}{m^2}\right). \qquad (3.3)$$

In the eighth order there aren't any other noneikonal contributions depending on the momentum transfers.

We attribute the noneikonal $\bar{t}$-paths, having a common nucleon line, to the second type. Its contribution does not depend on the momentum transfers and have been considered above for the xx-diagram (see eqs. (2.10)-(2.16)). However the similar contributions are cancelled in the sum of all diagrams with such $\bar{t}$-paths.

Consider, for example, the diagram

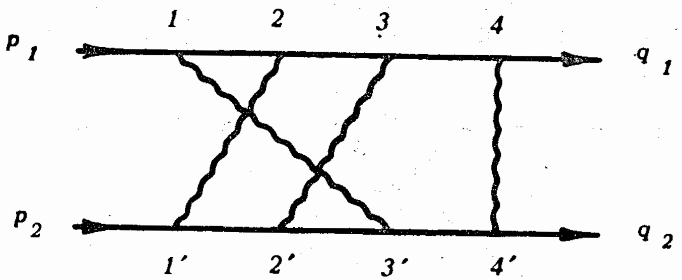
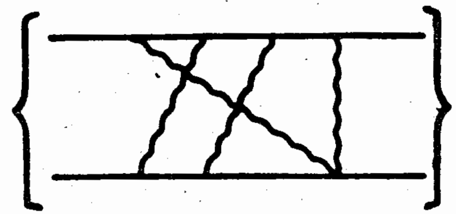
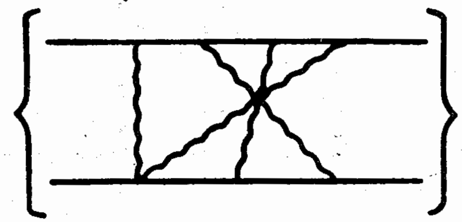


Fig. 7

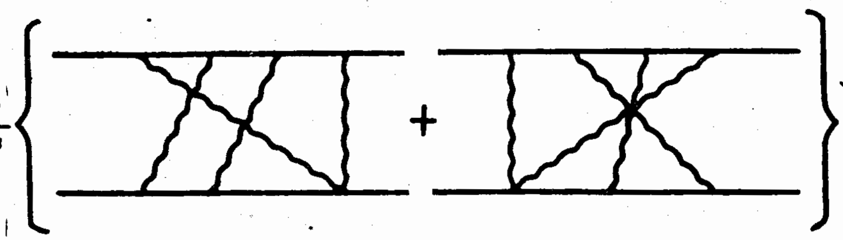
whose paths $(1'2'3'4')$ and $(13'4'4)$ belong to the second type. Its contributions may be taken into account with the help of Statement 2. Namely, this diagram asymptotics may be graphically represented in the form

$\frac{1}{s}$  (3.4)

The asymptotics of the graph which appears as a result of mirror reflection of 1 and 2 vertices relatively to the vertex 4 may be represented as follows

$\frac{1}{s}$  (3.5)

Now let us consider cross-symmetric graphs. According to the Statement 2 we have to replace the common lines by the factors $(-\frac{1}{s})$. Then obtain

$-\frac{1}{s}$ 

(3.6)

The first term in eq. (3.6) corresponds to the non-eikonal contribution to the diagram which is cross-symmetric to the graph shown in Fig. 7.

Summing the expressions (3.4), (3.5) and (3.6) we convince ourselves of the cancellation of the noneikonal second type $\bar{t}$-paths contributions.

Evidently the same arguments are faithful for the other similar diagrams. To the third type we attribute those of $\bar{t}$-paths which have common meson line. Its contribution to the leading asymptotics also does not depend on the momentum transfers. In the eighth order there are the same diagrams with the third type $\bar{t}$-paths. As an example we consider only one of these graphs (see Fig. 8), keeping in mind the validity of all the results for other similar diagrams.

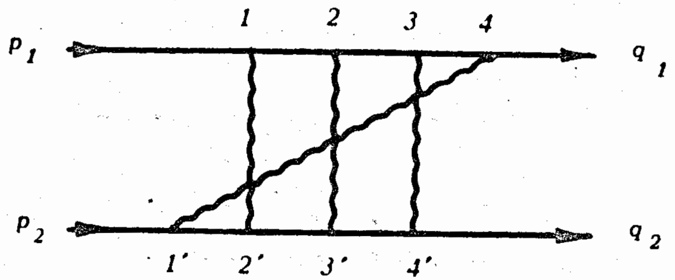


Fig. 8

In this diagram the $\bar{t}$-paths $(1'434')$ and $(12'1'4)$ are noneikonal and belong to the third type. Its contribution may be written down in the form (2.10).

$$\frac{\ln s}{s^3}\,\frac{1}{\mu^2}\,\Phi\left(\frac{\mu^2}{m^2}\right). \tag{3.7}$$

We'll search the behaviour of function Φ under condition (2.9). Let us choose the integration momenta so that the p-paths coincide with $\bar{t}$-paths $(1'434')$ and $(12'1'4)$ (see Fig. 8). According to the Statement 2 the desired contribution will be equal to the reduced graph (Fig. 9) asymptotics multiplied by $1/s$.

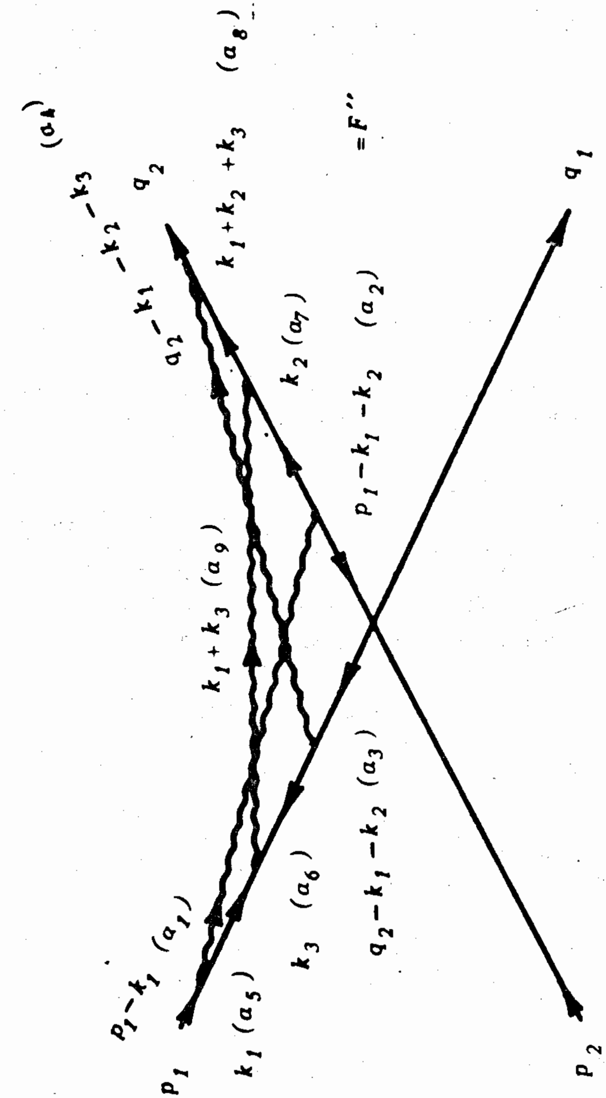


Fig. 9

Using eq. (1.7) when $s \to \infty$ we get the asymptotics

$$F'' \simeq \frac{\ln s}{s^2}\, const \int da_1 \dots da_9\, \delta(1-a_1-a_2)\,\delta(1-a_3-a_4)\, \delta(1-a_5-a_6-a_7-a_8-a_9)\frac{C_0}{(g_0 t + h_0)\tilde{f}_0^{\,2}}, \tag{3.8}$$

where

$$g_0 = 0, \quad h_0 = -\mu^2 C_0 \left[\frac{m^2}{\mu^2}(a_5+a_6+a_7+a_8)+a_9\right]. \tag{3.9}$$

From eqs. (3.8) and (3.9) we obtain the expression for the function $\Phi\left(\frac{\mu^2}{m^2}\right)$, defined by relation (3.7)

$$\Phi\left(\frac{\mu^2}{m^2}\right) = const \int \{d\alpha\} \times \prod \delta(1-\Sigma a)\, \frac{\delta(1-a_5-a_6-a_7-a_8-a_9)}{\tilde{f}_0^{\,2}\left[\frac{m^2}{\mu^2}(a_5+a_6+a_7+a_8)+a_9\right]}. \tag{3.10}$$

At large $\frac{m^2}{\mu^2}$ the main contribution is due to the region $a_5+a_6+a_7+a_8=0$. By the substitution

$$a_{5,6,7,8} \to \lambda\, a_{5,6,7,8}$$

we get

$$da_5 \dots da_8 \to \lambda^3 \delta(1-a_5-a_6-a_7-a_8)\, da_5 \dots da_8\, d\lambda$$
$$\delta(1-a_5-a_6-a_7-a_8-a_9) \to \delta(1-a_9) \tag{3.11}$$
$$\tilde{f}_0 \to \lambda\, \tilde{\tilde{f}}_0 .$$

It follows then under condition (2.3) that

$$\Phi\left(\frac{\mu^2}{m^2}\right) = const \int_0^1 d\lambda\, \frac{\lambda}{\lambda \frac{m^2}{\mu^2}+1}, \quad i.e. \quad \Phi\left(\frac{\mu^2}{m^2}\right) = const\, \frac{\mu^2}{m^2}. \tag{3.12}$$

The results of the second and third sections can be expressed by a single formula in which the cancellation of $\ln s$ in the cross-symmetric sum of diagrams is taken into account. Really, at large s an asymptotic behaviour of the nucleon-nucleon scattering amplitude in the eighth order of the perturbation theory has the form

$$f^{(8)} \simeq \frac{g^8}{s^3}\left\{ \frac{1}{8\cdot 4!\,(2\pi)^8}\int d^2\vec{x}_\perp\, e^{-i\vec{x}_\perp \vec{\Delta}_\perp} K_0^4(\mu|\vec{x}_\perp|) + f^{(8)}_{noneik}\right\}, \tag{3.13}$$

where

$$f^{(8)}_{noneik}(t) = \frac{\mu^2}{m^2}\, f_{eik}\left(t\frac{\mu^2}{m^2}\right) + \frac{const}{\mu^2}\,\Phi\left(\frac{\mu^2}{m^2}\right).$$

The $f_{eik}(t)$ in eq. (3.13) denotes the t-dependent factor in the main asymptotic term of the sum of the diagram shown in Figs. 2 and 6 together with its cross-symmetric partners, when only the contributions of the eikonal paths are taken into account. The function $\Phi\left(\frac{\mu^2}{m^2}\right)$ goes as $\frac{\mu^2}{m^2}$ at $\frac{\mu^2}{m^2} \ll 1$. The first term in curly brackets belongs to the sum of the eikonal contributions from all the graphs of the eighth order (compare with eq. (2.3)).

Under condition of a smallness of the ratio $\frac{\mu^2}{m^2}$ one can neglect the dependence on momentum transfers, if $\frac{t}{m^2} \ll 1$

$$f_{eik}\left(t\frac{\mu^2}{m^2}\right) \simeq f_{eik}(0) = \frac{const}{\mu^2} \tag{3.14}$$

that gives the result:

$$f^{(8)}\Big|_{\substack{s\to\infty \\ t-fixed \\ \mu^2/m^2 \ll 1}} \simeq \frac{g^8}{s^3}\left\{ \frac{1}{8\cdot 4!\,(2\pi)^8} \times \int d^2\vec{x}_\perp\, e^{-i\vec{x}_\perp \vec{\Delta}_\perp} K_0^4(\mu|\vec{x}_\perp|) + \frac{const}{\mu^2}\,\frac{\mu^2}{m^2}\right\}. \tag{3.15}$$

In conclusion of this section it should be stressed that only the contributions of various $\bar{t}$ -paths corresponding to zeroes of the function $f(a)$ in eqs. (1.4)-(.1.7)were taken into account.

4. The Asymptotics of the Nucleon-Nucleon Scattering Amplitude. Higher Orders

In Sec. 3 we have considered the high-energy behaviour of a scattering amplitude in the eighth order in powers of g. We have shown that in this order there exist graphs which give the noneikonal contributions to the asymptotics of the amplitude of the same order in s as the eikonal one.

However, as it was shown in paper /6 /, in higher orders in powers of g there exist graphs in which the noneikonal asymptotic term dominates the eikonal one. The typical example of these graphs with noneikonal paths of the first type (see Sec. 3) is illustrated in figure 10.

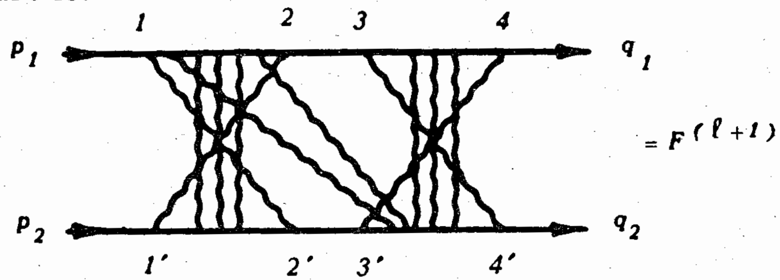


Fig. 10

This diagram just as xx -diagram has two $\bar{t}$ -paths with length equal three: $(12'3'4)$ and $(1'234')$. To study its asymptotic behaviour we use the same method as in Sec. 2, i.e. directing the p -paths along the $\bar{t}$ -paths and replacing the momenta as in (2.6). The asymptotics of the graph with $\ell+1$ meson lines (of the $2\ell+2$ order in powers of g) shown in Fig. 10 coincides with the

asymptotics of the graph shown in Fig. 11 up to a factor $(\frac{\mu^2}{m^2})^{\ell-2}$

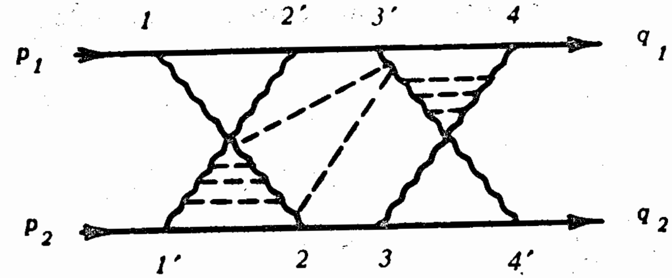


Fig. 11

Moreover, the substitution $t \to t\frac{\mu^2}{m^2}$ in the graph in Fig. 11 should be done (compare with eq. (2.7)). The dot-lines of this reduced graph correspond to the virtual particles with the mass $\frac{\mu^2}{m}$. These lines are due to the meson lines (see Fig. 10), which do not belong to the $\bar{t}$ -paths

$$\mathfrak{D}_\mu(k) \to \mathfrak{D}_\mu(k\frac{m}{\mu}) = \frac{1}{k^2\frac{m^2}{\mu^2} - \mu^2 + i\epsilon} = \frac{\mu^2}{m^2}\mathfrak{D}_{\frac{\mu^2}{m}}(k), \quad (4.1)$$

Under the condition (2.3) one may put $t=0$ in the asymptotics of this diagram.

Thus using eq. (1.7) for the main asymptotic term of the graph of the $2\ell+2$ order considered above we get the following expression:

$$F^{(\ell+1)} \doteq \frac{\ln s}{s^3}\frac{const}{\mu^{2(\ell-2)}}\int\{da\}\{d\beta\}\{d\gamma\}\Pi\delta(1-\Sigma\gamma_i)\cdot \quad (4.2)$$

$$\cdot\delta(1-\Sigma a_i - \Sigma\beta_i)\frac{C_0}{\tilde{f}_0^3[\frac{m^2}{\mu^2}\Sigma a_i + \Sigma\beta_i]^{\ell-2}}, \quad \ell \geq 3$$

In formula (4.2) the parameters α_i correspond to the wave meson lines, β_i to the dot-lines and γ_i to the nucleon lines. Apparently the region $\Sigma \alpha_i = 0$ does not give the essential contribution to the integral (4.2) at $\frac{m^2}{\mu^2} \gg 1$ *.

Hence

$$F^{(\ell+1)} \simeq \frac{\ell n s}{s^3} \frac{const}{(m^2)^{\ell-2}}, \quad \ell \geq 3. \tag{4.3}$$

In the considered case of the order $2\ell+2$ in powers of g there exist the graphs with noneikonal $\bar{t}$ -paths of the third type, which have the form:

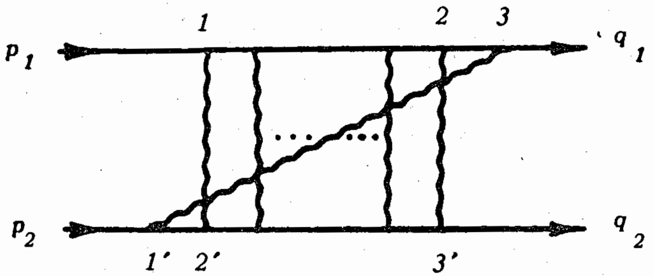


Fig. 12

In the diagram in Fig. 12 there exist two $\bar{t}$ -paths of the length three: $(12'1'3)$ and $(1'323')$, which lead to the asymptotics $\frac{\ell n\, s}{s^3}$. The method used above in Sec. 3 for the eighth order graphs gives here the formula similar to the eq. (4.3).

The noneikonal $\bar{t}$ -paths of the second type, whose contributions have cancelled in the sum of the eighth order graphs, give here nonleading asymptotic terms.

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* This can be shown by calculating the power of λ, appearing in the nominator when the substitution is performed (compare with eqs. (3.11), (3.12)).

All the graphs of the given $2\ell+2$ order belong either to the type described in this section and lead to the asymptotics of the form (4.3), or have the $\bar{t}$ -paths of the length more than three and consequently do not dominate in asymptotic region $s \to \infty$.

Taking into account the cancellation of $\ell n s$ when graphs with its cross-symmetric partners are being summed, we get the following asymptotic expression for the amplitude $f^{(2\ell+2)}$ in $2\ell+2$ order in power of g :

$$f^{(2\ell+2)}\Big|_{\substack{s\to\infty \\ t-fixed \\ \frac{\mu^2}{m^2} \ll 1}} \simeq \frac{1}{s^3} \frac{const}{(m^2)^{\ell-2}}, \quad \ell \geq 3 \tag{4.4}$$

Note, that the eikonal formula (2.3) when $t=0$ in the same order of g gives the following result:

$$f_{eik}^{(2\ell+2)}(t=0) = \frac{const}{s^{\ell} \mu^2}. \tag{4.5}$$

Thus if one neglects twisted graphs one gets for the ratio of the noneikonal and the eikonal contributions to the amplitude of the given order the result:

$$\frac{f_{noneik}^{(2\ell+2)}}{f_{eik}^{(2\ell+2)}}\Bigg|_{\substack{s\to\infty \\ t-fixed \\ \frac{\mu^2}{m^2} \ll 1}} \simeq const \frac{\mu^2}{m^2} \left(\frac{s}{m^2}\right)^{\ell-3}, \quad \ell \geq 3. \tag{4.6}$$

From eq. (4.6) it follows that in the region

$$s \to \infty, \quad \frac{\mu^2}{m^2} \ll 1, \quad s \sim m^2, \; t = 0 \tag{4.7}$$

the eikonal part of scattering amplitude dominates over the non-eikonal one, and the eq. (2.3) gives the main asymptotic terms in each order in powers of g^2. On the contrary, in the region (4.7) when $s \gg m^2$ the noneikonal contributions dominate as it follows from eq. (4.6) over the eikonal one.

Conclusion

So the investigation of the ladder type graphs in the scalar model demonstrates that the eikonal formula corresponds to the account of the $\bar{t}$-paths, coinciding with nucleon lines. The "leading" particles, carrying large momenta, are nucleons in that case and do not change their type in virtual processes.

The noneikonal contributions to the amplitude are due to the processes with alteration of the leading particle type, i.e. with the large momenta transfer from nucleons to mesons and vice versa. Then the important question arises about the significance of twisted graphs in which the final momenta q_1 and q_2 are exchanged (compare Fig. 1 and eq. (2.1)).

The possibility of large momentum carried by meson brings to the fact, that the corresponding contribution may dominate over the eikonal one in the same order of coupling constant. For example, in the fourth order the twisted graph has the asymptotics $\frac{\ln s}{s}$.

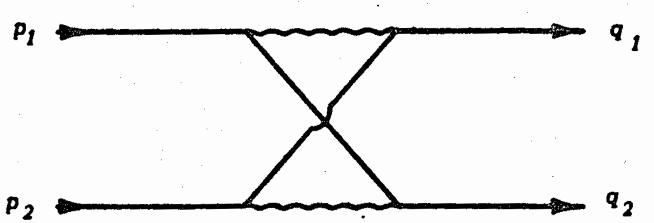

Fig. 13

Note that while orthodox eikonal formula corresponds to the scattering on Yukawa quasipotential (i.e. one meson exchange), the account of the graph shown in Fig. 13 leads to the quasipotential correction of non-Yukawa type. The derived correction corresponds to the nucleon-antinucleon pair exchange and has an effective radius $\sim \frac{\hbar}{2m}$. At small distances this correction has a singularity $\frac{\ln r}{r}$.

This example illustrates the importance of investigating the system of corrections to the effective quasipotential at high energies and gives the argument in favour of quasipotential interpretation of eikonal representation in quantum field theory.

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