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ELECTROMAGNETIC MASS DIFFERENCES OF NUCLEONS AND SUM RULES IN THE AUTOMODEL REGION

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Электромагнитная разность масс протона и нейтрона и правило сумм для автомодельных асимптотик

Рассматривается электромагнитная поправка к разности масс протона и нейтрона в связи с глубоко-неупругим рассеянием. Используя представления Йоста-Лемана-Дайсона для амплитуды виртуального комптон-эффекта, найдены правила сумм для автомодельных асимптотик амплитуды, обеспечивающие конечность электромагнитной разности масс протона и нейтрона.

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Electromagnetic Mass Differences of Nucleons and Sum Rules in the Automodel Region

The electromagnetic mass corrections for nucleons are considered in connection with deep inelastic scattering. Using a Dyson-Jost-Lehmann representation for the virtual Compton scattering amplitude conditions for the finiteness of δm are given in terms of sum rules for the structure functions in the automodel region.

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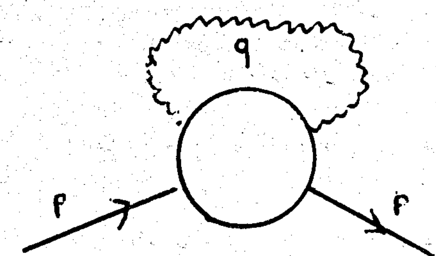
It is one of the classical problems of Quantum Field Theory to understand theoretically the $p-n$ mass difference. According to standard ideas about the electromagnetic origin of this mass difference it is related, in lowest order at least, to the virtual Compton scattering. This assumption has led to the so-called Cottingham formula for electromagnetic mass corrections [1]:

$$\delta m^{p,n} = \frac{\pi \alpha}{i} \int \frac{d^4 q}{q^2 + i0} \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) T^{p,n}_{\mu\nu}(q,p) \qquad (1)$$



Here $T^{p,n}_{\mu\nu}$ denotes the virtual Compton scattering amplitude in forward direction averaged over the nucleon spins (for p or n respectively)

$$T_{\mu\nu}(q,p) = \frac{i}{4\pi} \sum_\sigma \int dx\, e^{iqx} \langle p,\sigma | T(J_\mu(x), J_\nu(0)) | p,\sigma \rangle$$

$$\langle p',\sigma' | p,\sigma \rangle = 2p_0 (2\pi)^3 \delta(\vec{p} - \vec{p}\,') \delta_{\sigma\sigma'} . \qquad (2)$$

Most of the previous investigations have used dispersion relations for $T_{\mu\nu}$ at fixed q^2 [2,3]. In this connection assumptions about the asymptotic behaviour of $T_{\mu\nu}$ for large q^2 play an important role, in particular assumptions about the number of subtractions in dispersion relations for arbitrary, fixed q^2.

In this note we consider the question of finite electromagnetic mass corrections for nucleons using a Dyson-Jost-Lehmann representation for the virtual Compton scattering amplitude.

The DJL representation has been used for the investigation of the automodel behaviour of invariant causal formfactors connected with the imaginary part of the virtual Compton scattering amplitude /4,5/

$$W_{\mu\nu}(q,p)=\frac{1}{8\pi}\sum_{\sigma}\int dx\, e^{iqx}\langle p,\sigma|[J_\mu(x),J_\nu(0)]|p,\sigma\rangle=$$
$$=(-g_{\mu\nu}q^2+q_\mu q_\nu)V_1+(q^2p_\mu p_\nu-(qp)(p_\mu q_\nu+p_\nu q_\mu)+(pq)^2 g_{\mu\nu})V_2 \quad (3)$$

where

$$v_i(q^2,qp)=\int\epsilon(q_0)\delta(q_0^2-(\vec{q}-\vec{u})^2-\lambda^2)\psi_i(\vec{u},\lambda^2)d\vec{u}\,d\lambda^2. \quad (4)$$

It has been shown /6/ that the condition of integrability of the first primitive of the weight function $\psi(\vec{u},\lambda^2)$ in formula (4) allows the following representation of the Compton scattering amplitude

$$T_{\mu\nu}(q,p)=\frac{1}{\pi}(-g_{\mu\nu}q^2+q_\mu q_\nu)\int\frac{d\vec{u}\,d\lambda^2}{q_0^2-(\vec{q}-\vec{u})^2-\lambda^2+i0}\psi_1(\vec{u},\lambda^2)+$$
$$+\frac{1}{\pi}(q^2p_\mu p_\nu-(pq)(p_\mu q_\nu+p_\nu q_\mu)+(pq)^2g_{\mu\nu})\times$$
$$\times\int\frac{d\vec{u}\,d\lambda^2}{q_0^2-(\vec{q}-\vec{u})^2-\lambda^2+i0}\psi_2(\vec{u},\lambda^2). \quad (5)$$

Inserting the representation (5) into the expression for electromagnetic mass corrections and performing two partial integrations we obtain

$$\delta m=\delta m^p-\delta m^n=\frac{2a}{i}\int d\vec{u}\,d\lambda^2\Big\{\psi_{1(2)}(u,\lambda^2)\int\frac{-3q^2d^4q}{q^2[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2]^3}+$$
$$+\psi_{2(2)}(\vec{u},\lambda^2)\int\frac{q^2+2q_0^2}{q^2[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2]^3}d^4q\Big\}, \quad (6)$$

where

$$\psi_{i(1)}(\vec{u},\lambda^2)=\int_{0-}^{\lambda^2}ds\,\psi_i(\vec{u},s), \quad (7)$$

$$\psi_{i(2)}(\vec{u},\lambda^2)=\int_{0-}^{\lambda^2}ds\,\psi_{i(1)}(\vec{u},s). \quad (8)$$

Now the q-integrals converge and show the following behaviour for large values of λ^2

$$\int d^4q\frac{-3q^2}{q^2[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2]^3}\to\frac{3\pi^2}{2\lambda^2}i$$
$$\int d^4q\frac{q^2+2q_0^2}{q^2[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2]^3}\to\frac{-3\pi^2 i}{4\lambda^2}+O(\frac{1}{\lambda^4}). \quad (9)$$

Let us discuss the convergence of the λ^2 integrations in formula (6). At first we consider the behaviour of the 2-primitives of $\psi_i(u,\lambda^2)$ for large values of λ^2.

In accordance with the experimental data we assume the following automodel behaviour of the formfactors v_1 and v_2:

$$v_1(\nu,q^2)\approx\frac{1}{\nu^2}h_1(\xi) \qquad \nu=2pq\to\infty \quad (10)$$
$$v_2(\nu,q^2)\approx\frac{1}{\nu^2}h_2(\xi), \qquad \xi=-\frac{q^2}{\nu}\ \text{fix}.$$

This asymptotic behaviour can be reproduced with the help of weight functions satisfying

$$\lim_{\lambda^2\to\infty}\psi_{i(2)}(\vec{u},\lambda^2)=\int_{0-}^{\infty}d\lambda^2\psi_{i(1)}(\vec{u},\lambda^2)=\psi_{i0}(\vec{u})\neq 0 \quad (11)$$

furthermore, as shown in /4,5/

$$h_i(\xi)=2\pi\xi\,\psi_{i0}(|\xi|). \quad i=1,2. \quad (12)$$

Note that the limit (11) leads to generalized functions $\psi_{i0}(\vec{u})$..

From the foregoing formula we see that the intagral determining the electromagnetic mass correction diverges logarithmically in general

$$\delta m = 6\pi^2 a \{ c \int_{\Lambda^2}^{\infty} \frac{d\lambda^2}{\lambda^2} + \int_0^{\Lambda^2} d\lambda^2 \ \dots \}, \tag{13}$$

where

$$c = \int_0^1 d\xi \, \xi \, (h_1^{p-n}(\xi) - \frac{1}{2} h_2^{p-n}(\xi)). \tag{14}$$

For finite electromagnetic mass corrections expression (14) has to vanish necessarily. (Note that the integarl (14) is finite bacause $h_i(\xi)$ appears as a generalized function due to equations (11) and (12).)

Usually the experimental data are written in terms of the structure functions w_1 and w_2

$$w_1 = q^2 v_1 - (pq)^2 v_2 , \quad w_2 = q^2 v_2 . \tag{15}$$

Let us write the leading terms and their first corrections in the limit $\nu \to \infty$, ξ fix:

$$\begin{aligned} w_1 &\simeq f_1(\xi) + \frac{1}{\nu} g_1(\xi), \\ w_2 &\simeq \frac{4\xi}{\nu} f_1(\xi) + \frac{1}{\nu^2} g_2(\xi). \end{aligned} \tag{16}$$

A comparison of formulas (10) and (16) gives

$$\begin{aligned} h_1(\xi) &= -\frac{1}{\xi} g_1(\xi) + \frac{1}{4\xi^2} g_2(\xi), \\ h_2(\xi) &= -4 f_1(\xi). \end{aligned} \tag{17}$$

According to experiment

$$\int_0^1 d\xi \xi [f_1^p(\xi) - f_1^n(\xi)] \neq 0 ,$$

so that condition (14) appears as a relation between leading and non leading terms in formulas (16).

Finally we remark that the sum rule (14) may be expressed in terms of the quantities d_t and d_ℓ which are directly connected with σ_t and σ_ℓ

$$\begin{aligned} \sigma_{t,\ell} &= \frac{8\pi^2 a}{\nu + q^2} d_{t,\ell} , \\ d_t &= w_1 \simeq -\frac{1}{4} h_2(\xi), \\ d_\ell &= -w_1 + w_2 (1 + \frac{\nu}{4\xi}) \simeq \frac{1}{\nu} \xi [h_1(\xi) - h_2(\xi)], \end{aligned} \tag{18}$$

such that

$$c = \lim_{\nu \to \infty} \{ \nu \int_0^1 d\xi \, d_\ell^{p-n}(\xi, \nu) - 2 \int_0^1 d\xi \xi \, d_t^{p-n}(\xi, \nu) \} = 0. \tag{19}$$

Another possibility to obtain finite δm is the occurence of non-canonical dimensions /7/. As we shall see an infinitesimal change of the scaling power

$$v_i \sim \frac{1}{\nu^{2+\epsilon}} g_i(\xi), \quad \epsilon > 0 \tag{20}$$

is sufficient for convergence. In this case the weight functions fulfill /5/

$$\begin{aligned} &\lim_{\lambda^2 \to \infty} \psi_{i(s)}(\vec{u}, \lambda^2) = \psi_{i0}(\vec{u}); \quad s = 2 + \epsilon \\ &\psi_{i(s)}(\vec{u}, \lambda^2) = \frac{1}{\Gamma(s)} \int_0^{\lambda^2} d\lambda'^2 \psi_i(\vec{u}, \lambda'^2)(\lambda^2 - \lambda'^2)^{s-1} \\ &= f_s * \psi_i \quad . \end{aligned} \tag{21}$$

Returning to furmula (5) we have to discuss the typical integral

$$\int d\lambda^2 \psi(\vec{u}, \lambda^2) \frac{1}{Q^2 - \lambda^2} = \frac{1}{Q^2} * \psi(\vec{u}, Q^2)$$

$$= \frac{1}{Q^2} * f_{-s} * f_{s} * \psi(\vec{u}, Q^2) = \frac{1}{Q^2} * f_{-s} * \psi_{(s)}(\vec{u}, Q^2), \qquad (22)$$

$$Q^2 = [q_0^2 - (\vec{q} - \vec{u})^2].$$

(The rules for handling convolutions of generalized functions may be found in [8]). Explicit evaluation of the convolution gives

$$\int dy_1 dy_2 \psi_{(s)}(\vec{u}, y_1) f_{-s}(y_2 - y_1) \frac{1}{Q^2 - y_2} =$$

$$= \int dy_1 \psi_{(s)}(\vec{u}, y_1) \int_0^\infty dt\, t^{-s-1} \Gamma^{-1}(-s) (Q^2 - t - y_1)^{-1} = \qquad (23)$$

$$= -\Gamma(s+1) \int d\lambda^2 \psi_{(s)}(\vec{u}, \lambda^2)(\lambda^2 - q_0^2 + (\vec{q} - \vec{u})^2)^{-s-1}$$

If we apply these formulas for $s = 2 + \epsilon$ then the q integral corresponding to equation (1) converges and behaves as $\lambda^{-2(1+\epsilon)}$ for $\lambda^2 \to \infty$ such that the λ^2-integration is finite too.

So there are at least two different mechanisms to obtain finite mass corrections.

Let us remark that in the so-called anomalous cases (compare ref. [9] and [6]) condition (14) would be changed. So a violation of that condition could be traced back to an occurence of an anomalous case if this is not ruled out by other experimental information.

The foregoing considerations show that the connection between automodelity in virtual Compton scattering and the finiteness of electromagnetic mass corrections is of special physical interest. In fact the existence of finite non-zero contribution to δm from the region of large momenta q means the occurence of dimensional quantities in that region. Therefore the non-asymptotic contributions are responsible for finite mass corrections which are proper dimensional quantities. In this manner the vanishing of the logarithmically divergent contribution to δm can be understood as a consistency condition of the automodel character of the virtual Compton scattering in the considered asymptotic region.

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