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ON THE ELECTROMAGNETIC PION RADIUS COMING FROM $\Pi^{\pm}\,{}^4$He SCATTERING

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Об электромагнитном радиусе пиона из экспериментов по $\pi^{\pm}He^4$–рассеянию

Анализируется влияние неопределенностей в фазовом анализе, значений полных сечений и энергии пучка пионов на определение электромагнитного радиуса пиона из экспериментов по $\pi^{\pm}He^4$ упругому рассеянию. Показано, что из имеющихся экспериментов получается $\langle r_{\pi}^2\rangle^{1/2} =$ 0,80 ± 0,40 ферми.

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Дубна, 1973

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On the Electromagnetic Pion Radius Coming from $\pi^{\pm}He^4$ Scattering

The influence of ambiguity in phase shift analysis, of the total cross section and the beam energy on the electromagnetic pion radius, derived from $\pi^{\pm}He^4$ elastic scattering measurements, is analysed. It is shown that the pion radius obtained from existing experiments is $\langle r_{\pi}^2\rangle^{1/2}$ =0.80 ± 0.40 fm.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1973

1. Introduction

Hofstadter and Sternhein originally proposed /1/ to obtain the pion charge form factor from the analysis of elastic scattering of $\pi^{\pm}$ mesons on zero-isospin nuclei. The most appropriate nucleus for this purpose is He^4 /2/.

If one assumes that the scattering amplitude for this process is given by the sum of the nuclear amplitude and the Coulomb amplitude in the Born approximation

$$f^{\pm} = f_N \pm f_C F_a F_{\pi} \tag{1}$$

then the averaged value between the differential cross sections for π^+ and π^- will be connected only to the pure nuclear amplitude and the difference between π^- and π^+ with the Coulomb interference term, linear in the pion form factor

$$A = \left(\frac{d\sigma^-}{d\Omega} + \frac{d\sigma^+}{d\Omega}\right)/2 = |f_N|^2; \tag{2a}$$

$$D = \left(\frac{d\sigma^-}{d\Omega} - \frac{d\sigma^+}{d\Omega}\right)/2 = -2\,Re(f_N^* f_C F_a F_{\pi}). \tag{2b}$$

The problem is to find a correct amplitude for nuclear interactions and to introduce all other Coulomb corrections - the distortion amplitude $f_D^{\pm}$ /3,4/, or inner and outer Coulomb corrections /5/.

Up to now there have been made some experiments and analyses of the $\pi^{\pm}He^4$ elastic scattering /6/ which parametrized the nuclear amplitude in different ways (via the optical model, like the Kisslinger model or simple with phase shifts).

The common feature is that all other Coulomb corrections are small and do not introduce a drastic change in the electromagnetic pion radius which still remains much larger than 2.0 fm. However, in the relativistic treatment of this process the pion radius turns out to be much smaller: $r_\pi \leq 1.0$ fm/7/.

Is this a "good" result due to the relativistic terms? Below we want to show that this is not the case.

2. Ambiguities in Phase Shifts

It is well known that in the complex phase shift analysis the number or solutions may be up to 2^{L+1} /8/, where L is the number of partial waves (for 50-153 MeV energy interval are present only S , P , and D waves /10/ and therefore $L=2$). The presence of Coulomb interaction reduces the number of solutions to 4, leaving only those with a proper sign of interference. The difference between them is that the scattering amplitude has zeros in the complex $\cos\theta$ plane which are complex conjugates to each other /8/.

But in the particular case of the πHe^4 phase shift analysis it has been shown that only two solutions are in the unitarity circles /9/ for which the first zero (the scattering amplitude has two zeros) has roughly symmetrical positions in complex $\cos\theta$ plane

$$\mathrm{Re}z^I = \mathrm{Re}z^{II}$$

$$\mathrm{Im}z^I = -\mathrm{Im}z^{II} \qquad \mathrm{Im}z^I > 0,$$

where z^I and z^{II} represent the first zero of the scattering amplitude for the solutions I and II , respectively.

The existence of these two solutions turns out to be important in electromagnetic pion radius calculations. The real part of scattering amplitude which gives a large contribution to the D (eq. 2b) fitting has different values particularly at large scattering angles. For example in the simplest method with the Coulomb amplitude in the Born approximation and without any distortion amplitudes (eq. 2) the results are $r_\pi^2 = 0.75$ fm^2 for the first solution and $r_\pi^2 = 1.35$ fm^2 for the second solution (75 MeV-Crowe experiment). Figure 1 shows the real part of the nuclear amplitude for the first and the second solutions obtained from our fit of the Crowe data (75 MeV) at $\sigma_{tot} = 100$ mb.

Now let us return to the former question. From the point of view of the zeros we find that all the other phase shift analyses (Block, Crowe, Mottershead /6/) correspond to the second solution (with a large value for r_π); but Christensen's amplitude has exactly the zeros which correspond to our first solution (with a small value for r_π)*.

To summarise the ambiguity in the nuclear amplitude is very important and has much larger effects on the pion radius than all other Coulomb corrections. The problem is to find the "good" solution from the "nuclear" point of view. The preliminary phase shift analysis /10/ shows that with the "chain method" and the ratio $x = \sigma_{el} / \sigma_{tot}$ constraints, the first solution is a good one at least from 60 MeV up to 153 MeV. The same conclusion appears from the ACE method for πHe^4 phase shift analysis of the same data /11/.

3. The Total Cross Section and the Kinetic Energy in r_π Determination

But unfortunately, the ambiguity is not simply a "large correction" (aside from small corrections like a distortion amplitude). The general opinion that the pion radius is essentially independent of the values chosen for the imaginary parts of the amplitude in the forward direction turns out to be not correct. The total cross section has a large influence on the real part of the amplitude and therefore on the pion radius. Figures 2a and 2b show the pion radius dependence on the total cross section for both solutions. Only from the differential elastic cross section it is not always possible to find the true value for the total cross section. (Don't forget that in our case there are no measurements of differential cross section at very small angles). The total cross section is an independent physical observable and therefore it is necessary to find it in a special experiment.

The next problem is connected with pion radius sensitivity to the incident energy. This dependence is roughly linear and the exact value of the beam energy becomes much more important with decreasing energy (Figs. 3a and 3b). Figures 4 (a, b and c) show the dependence of χ^2/N_{DF} for A and D at different incident energies. From these figures it is possible to conclude that the pion beam energy plays a very important role in pion radius calculations.

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* This is not valid for 51.3 MeV. For this energy Christensen used solution II, but at this energy the difference between two solutions is smaller than at higher energies (see Ref. /9/).

4. Non-Point Coulomb Phase Shifts

All these calculations were made in a simple Born approximation, but it is well known that the nonrelativistic scattering amplitude for zero-spin on zero-spin in the presence of Coulomb interaction is given by

$$f^{\pm} = \frac{1}{k} \Sigma (2\ell+1) e^{2i\zeta_\ell^{\pm}} T_\ell P_\ell(\cos\theta) \pm f_C^{pt\pm}, \qquad (3)$$

where

$$f_C^{pt\pm} = f_{C(BORN)}^{pt} \exp(2i\zeta_0^{\pm} - in_C^{\pm} \ell n(\sin^2\frac{\theta}{2})),$$

$\zeta_\ell^{\pm} = \arg \Gamma(\ell + 1 + in_C^{\pm})$ Coulomb phase shift,

$n_C = \frac{Z_1 Z_2 m a}{k}$ Coulomb parameter,

$a = 1/137.036.$

This is correct for a point charge interacting particle, but formally it is possible to write that

$$f_{C,\ell}^{pt} \sim e^{2i\zeta_\ell} - 1$$

and, therefore for particles with spatial structure

$$f_{C,\ell} = f_{C,\ell}^{pt} F_a F_\pi \sim e^{2i\zeta'_\ell} - 1 = (e^{2i\zeta_\ell} - 1) F_a F_\pi$$

and for $F_a F_\pi = 1$, $e^{2i\zeta'_\ell} = e^{2i\zeta_\ell}$,
where ζ'_ℓ -non-point Coulomb phase shifts. Now the scattering amplitude is given by:

$$f^{\pm} = \frac{1}{k} \Sigma (2\ell+1) e^{2i\zeta'^{\pm}_\ell} T_\ell P_\ell(\cos\theta) \pm f_C^{pt\pm} F_a F_\pi = f_N + f_D^{\pm} \pm f_C^{\pm}, \qquad (4)$$

where $f_N = \frac{1}{k} \Sigma (2\ell+1) T_\ell P_\ell(\cos\theta)$,

$$f_D^{\pm} = \frac{1}{k} \Sigma (2\ell+1)(e^{2i\zeta_\ell^{\pm}} - 1) F_a F_\pi T_\ell P_\ell(\cos\theta).$$

The average cross section taken from experiment was fitted by

$$A = (|f^-|^2 + |f^+|^2)/2 \qquad \text{with } r_\pi \text{ fixed.} \qquad (5a)$$

The phase shifts are introduced in D/A (which is more sensitive to r_π than D) to fit only r_π.

$$D/A = (|f^-|^2 - |f^+|^2)/(|f^-|^2 + |f^+|^2) \qquad (5b)$$

and the whole procedure is repeated with new r_π for the A fit. After 3 iteractions the value of r_π is completely stabilized. The pion radius value calculated with the above procedure (eq. 5) is shown in Fig. 3b (for 75 MeV experiment).

This more exact form for the scattering amplitude (eq. 4) shows that Coulomb corrections to point charge particles have a much smaller influence on r_π than all other effects mentioned above in this paper.

The pion radius results calculated with this procedure (Fig. 5)* are shown in the form of an ideogram. The average value of the electromagnetic pion radius calculated from the Crowe experiment is $<r_\pi^2>^{1/2} = 0.80 \pm 0.40$ fm. (the error contains the uncertainty in the x ratio and therefore in the total cross section). This result shows that a proper choice of the scattering amplitude even without relativistic corrections gives a reasonable value for the electromagnetic pion radius.

To obtain a more accurate value of the electromagnetic pion radius new $\pi^{\pm} He^4$ elastic scattering experiments are needed together with an accurate estimate of the total cross section and new estimates of relativistic corrections.

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References

1. M.M.Sternheim and R.Hofstadter. Nuovo Cimento, 38, 1854 (1965).
2. E.H.Auerbach, D.M.Fleming and M.M.Sternheim. Phys.Rev., 171, 1781 (1968).
3. L.I.Schiff. Progr. Theoret. Phys. (Kyoto) Suppl., Extra Number 400 (1965).

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* Using the $x = \sigma_{el}/\sigma_{tot}$ ratio constraint /10/ the following was found for the total cross sections: 64.4, 87.1, 99.6 and 123.4 mb for 51.3, 59.7, 67.6 and 75.0 MeV experiments, respectively (Crowe /6/).

4. M.Ericson. Nuovo Cimento, 47, 49 (1967).
5. G.C.Oades and G.Rashe. Nucl. Phys., 320, 333 (1970).
6. M.M.Block, I.Kenyon, J.Keren, D.Koetke, P.Malhotra and H.Winzler. Phys. Rev., 169, 1074 (1968) (Experiment and Analysis).
K.M.Crowe, A.Fainberg, J.Miller and A.S.L.Parsons. Phys. Rev., 180, 1349 (1968) (Experimental and Analysis).
C.T.Mottershead. Phys. Rev., D6, 780 (1972) (Analysis).
7. R.A.Christensen. Phys. Rev., D1, 1469 (1970).
8. A.Gertsen. Nucl. Phys., B12, 537 (1969).
9. F.Nichitiu. Ambiguity in πHe^4 Physe Shift Analysis, JINR preprint E2-6890, Dubna, 1973.
10. I.V.Falomkin, M.M.Kulyukin, V.I.Lyashenko, A.Mihul, F.Nichitiu, G.Piragino, G.B.Pontecorvo, Yu.A.Shcherbakov. JINR preprint, E1-6534 Dubna, 1972.
11. V.O.Dumbrais, F.Nichitiu, Yu.A.Shcherbakov. ACE Method for πHe^4 Physe Shift Analysis JINR Preprint P1-6981,Dubna (1973).

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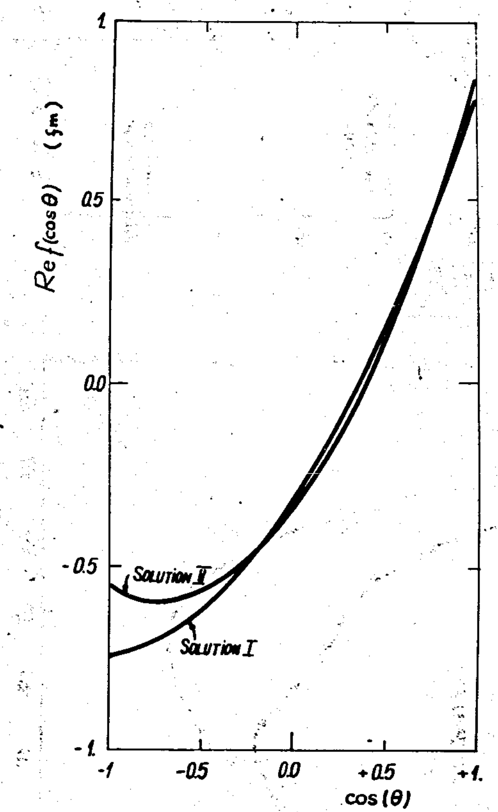
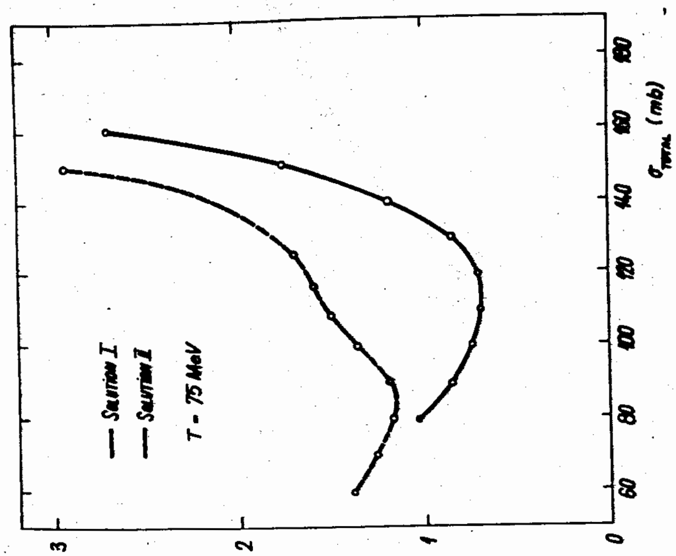
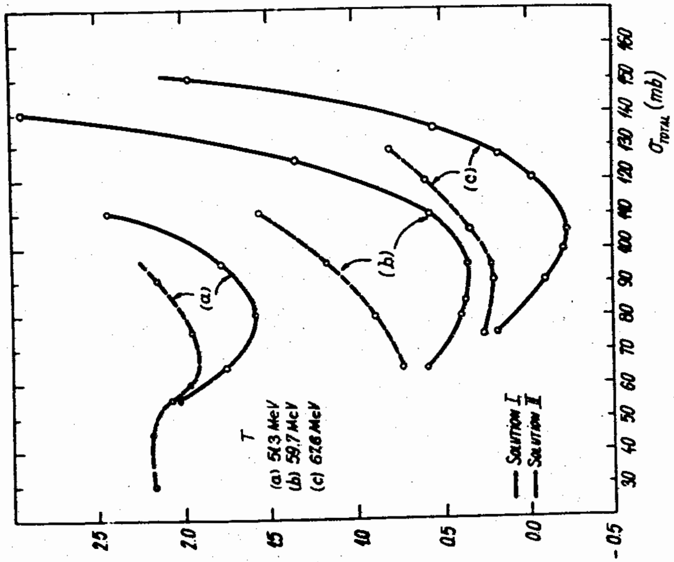


Fig. 1. Real part of scattering amplitude for solutions I and II of the phase shift analysis (Crowe et al. experiment at 75 MeV).

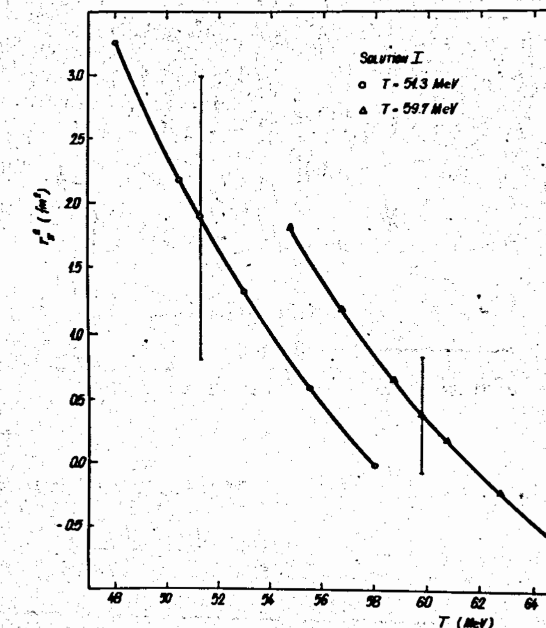


a



b

Fig. 2a and 2b. Electromagnetic pion radius dependence upon the total cross section for both solutions of the phase shift analysis.



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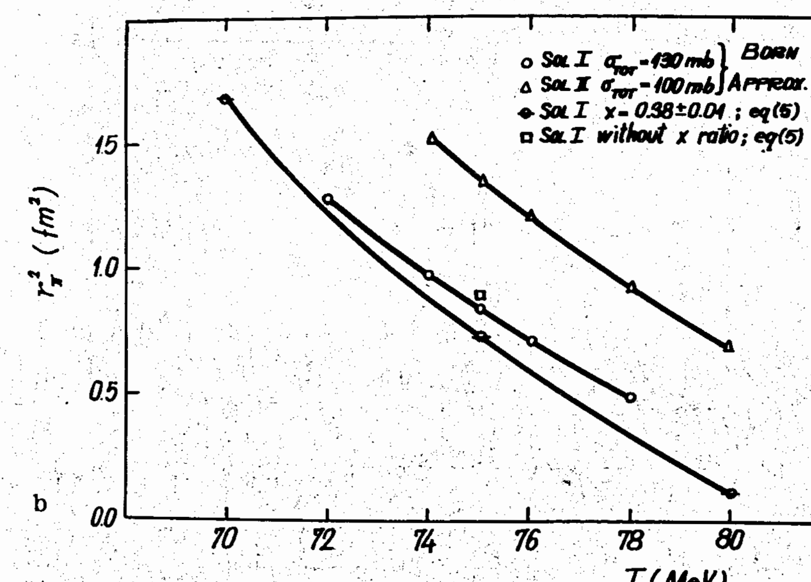

b

Fig. 3a and 3b. Electromagnetic pion radius dependence upon incident energy.

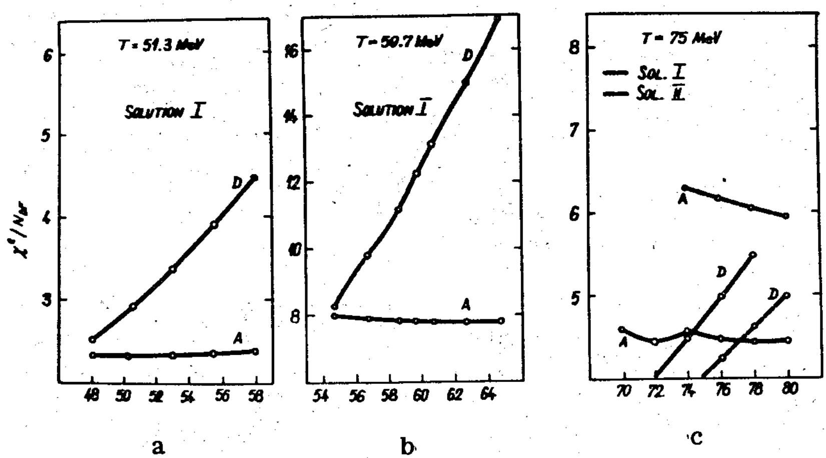

Fig. 4a, b and c. χ^2/N_{DF} from A and D fiting vs. incident energy.

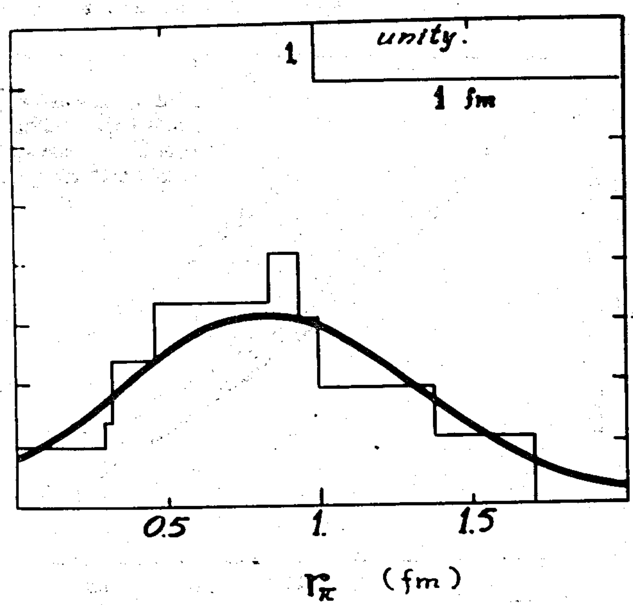

Fig. 5. Ideogram of electromagnetic pion radius results for Crowe et al. experiment (51.3, 59.7, 67.6 and 75.0 MeV) with solution I of the phase shift analysis.