# СООБЩЕНИЯ <br> ОБЬЕАИНЕННОГО <br> ИНСТИТУТА <br> ЯАЕРНЫX <br> ИССАЕАОВАНИЙ 

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OF CHARGED PARTICLES
IN THE INTERACTIONS OF $\Pi$-MESONS
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## 1. Introduction

The study of inelastic interactions of elementary particles is important in order to understand the dynamics of strong interactions. In the last few years much experimental data were obtained regarding these interactions. Now it is important to investigate the creation of particles in the interactions with nuclei. Such works are important for checking the theoretical models. For, example, it is essential to pay attention to the intranuclear cascade model $/ 1 /$ to explain existing inclusive experimental data. In addition to this the practical applications (calculation of protection against the high energy radiations, beams of secondary particles in accelerators, etc.) are of great significance.

The problem is tackled on the following way: from the known data on the elementary particle interactions, the characteristics of secondary particles produced in nuclei are calculated. The interaction is considered as a successive two-particle intranuclear collision (usual cascade model). The corresponding calculations are very complicated and are carried out on large computers by the Monte-Carlo method.

The intranuclear cascade model was used for the description of some average characteristics of particle production in emulsions $/ 2,3 /$. But the emulsion experiments have some ambiguities (complicated structure of emulsion, small statistics of events, etc.). It is interesting to verify the model by experiments with large statistics in order to find some fine features of the interaction mechanism.

In the present paper we consider the interactions initiated by $\pi$-mesons at 40 GeV in carbon nucleifrom the propane bubble chamber ${ }^{/ 4 /}$. It is shown that the cascade model agrees qualitatively with the experiment. The possibility to improve the intranuclear cascade model has also been discussed. For completeness we have reported the results of calculations at a lower energy of 17.2 GeV . We have used the program from ref. ${ }^{5 /}$ with some modifications $/ 3 /$.

## 2. Method of Calculations

We have used the Fermi-gas model for nuclei. The distribution of the nuclear density was described by

$$
\begin{equation*}
\rho=\rho_{0} /[1+\exp [(\mathrm{r}-\mathrm{c}) / a]] \tag{1}
\end{equation*}
$$

where parameters $c$ and $a$ were taken from the experiments concerning electron scatterings with $a=4.2 \times 10^{-14} \mathrm{~cm}$ and concerning electron/scan

The nucleus was divided into series of spherical zones with constant density. The momentum distribution of nucleons in the zones was assumed to be

$$
\begin{equation*}
d N \approx p^{2} d p \tag{2}
\end{equation*}
$$

Before the simulation of every new cascade the coordinates of nucleons were defined in an accidental form. The distance between the centres of nucleons was considered to be greater than twice the radius of the 'core'' ( $0.8 \times 10-13 \mathrm{~cm}$ ). Processes like elastic, inelastic, charge exchange and absorption of pions were taken into account. The interaction cross-sections of particles used for simulation were taken from the experiment $/ 6 /$.

We assume that the particle may interact with any nucleon met along its trajectory with the distance $\leq r_{s}+\lambda$. The probability, $p_{i}, 9_{7}$ the interaction with the nucleon in the nucleus is given by 7

$$
\begin{equation*}
p_{i}=p(1-p)^{i-1}, \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
p=\exp \left[-\sigma \sigma_{t o t} \quad \pi\left(r_{s}+\lambda\right)^{2}\right] \tag{4}
\end{equation*}
$$

$r_{s}=(1.3-1.4) 10^{-13} \mathrm{~cm}$. and $\lambda$ is the wavelength of the incoming particle. If the nucleon was produced in the final state with a momentum less than

$$
\begin{equation*}
p_{F}=\left(3 \pi^{2} \rho\right)^{1 / 3} \tag{5}
\end{equation*}
$$

the interaction could not proceed.
We supposed that the fastest particle interacted first on each stage of the intranuclear cascade development and the ejected nucleon became the usual cascade particle. In the space
occupied by the hit nucleon, a hole was created inside the nucleus. Since the velocities of intranuclear nucleons are much smaller than those of cascade particles, the hole once created remained empty during the entire development of the cascade. Hence the particles moving after the fastest particles met less number of nucleons on their way (trailing effect $/ 5-8 /$ ).
3. Simulation of the Intranuclear Cascade ${ }^{/ 2,3,5-7 /}$

In order to calculate the particle interactions with nuclei, one proceeds in the framework of different models from the particle interaction amplitude with the nucleon. The elastic scattering amplitude is given by

$$
\begin{equation*}
f(q)=A(T) e^{-\beta(T) q^{2} / 2} \tag{6}
\end{equation*}
$$

For inelastic interactions with many-particle production the amplitude of the process depends on a large number of variables. It is impossible to write the amplitude on an analytical form with parameters defined from experimental data. In the present paper we have used a parametrization of the experimental data on the inclusive reactions of the $\pi$-production (we have neglected the production of strange particles and antinucleons).

This approach is very restricted, of course, but the available experiments do not allow a more detailed analysis to be made. However, such consideration permits one to get some important consequences about average characteristics of the nuclear interactions (the multiplicity distributions, the angular distributions, the energy distributions, etc.).

For the calculation of any characteristic $x$ by the MonteCarlo method one defines a random number $\epsilon$ uniformly distributed in the interval (0-1):

$$
\begin{equation*}
\epsilon=\int_{0}^{x} W(y) d y \prime^{\prime} \int_{0}^{x_{\operatorname{ma}} x} W(y) d y, \tag{7}
\end{equation*}
$$

where $W(y)$ is the differential distribution of $x$. In our case it is convenient to obtain $x$ from the approximation of a function $F$ given by

$$
\begin{equation*}
x=F\left(\epsilon, a_{1}, a_{2}, \ldots ., a_{n}\right), \tag{8}
\end{equation*}
$$

where $a_{1}, a_{2}, \ldots a_{n}$ are the numbers obtained by comparing the calculated distribution with the experimental data.

The production of leading particles is a feature of inelastic
interactions at high energies $T \geq 10 \mathrm{GeV}$. This effect is usually taken into account by introducing the coefficient of inelasticity $k$, which is a part of total energy spent in the production of particles, i.e.

$$
\begin{equation*}
k=E_{\mathrm{s}} / E_{\mathrm{c}} . \tag{9}
\end{equation*}
$$

The distribution of the coefficient of inelasticity has been approximated with the expression

$$
\begin{equation*}
k=\epsilon^{a}\left[\sum_{n=0}^{N} a_{n} \epsilon^{n}+\left(1-\sum_{n=0}^{N} a_{n}\right) \epsilon^{N+1}\right] \tag{10}
\end{equation*}
$$

The values of $a_{n}$ are given in Table I.
It is assumed that in inelastic $\pi-N$ interactions in c.m.s. there are two leading particles, i.e. the $\pi-$ meson and the nucleon with energies

$$
\begin{equation*}
E_{L 1}=(1-k) E_{\mathrm{c}} / 2, \quad E_{L 2}=E_{\mathrm{c}}-E_{\mathrm{s}}-E_{L 1}, \tag{ll}
\end{equation*}
$$

where $E_{s}=k E_{c}$ :
There is no information about the angular distribution of leading particles. It is known that the primary particle moves at a very small angle in the forward direction and the nucleon in the opposite direction. We assume that the leading $\pi$-meson moves at an angle $\theta \sim 0^{\circ}$ and the nucleon at $\theta-180^{\circ}$. A more accurate definition of the leading particle angular distribution will not change the theoretical results appreciably.

The angles of secondary particles are given by

$$
\begin{align*}
& \cos \theta=2 \epsilon^{1 / 2}\left[\sum_{i=0}^{M} d_{i} \epsilon^{i}+\left(1-\sum_{i=0}^{M} d_{i}\right) \epsilon^{M+1}\right]-1,  \tag{12}\\
& d_{i}=\sum_{j=0}^{M} d_{i j} \ln ^{i} T, \tag{13}
\end{align*}
$$

where the values of $d_{i j}$ are given in Table II.
The kinetic energies of secondary particles in the c.m.s. are calculated by the formulae

$$
\begin{align*}
& \mathfrak{J}=\mathcal{J}_{\text {max }} \epsilon^{1 / 2}\left[\sum_{n=0}^{N} b_{n} \epsilon^{n}+\left(1-\sum_{n=0}^{N} b_{n}\right) \epsilon^{N+1}\right],  \tag{14}\\
& b_{n}=\sum_{i=0}^{N} b_{n i} \ell_{n}^{i}\left(T+T_{0}\right), \\
& \mathcal{T}_{\text {max }}=\sum_{i=0}^{N} c_{i} P_{n}^{i}\left(T+T_{0}\right), \tag{16}
\end{align*}
$$

where $T$ is the energy of the primary particle in the rest system of the nucleon and $T_{0}=10 \mathrm{GeV}$. The value of coefficients $b_{n i}$ and $c_{i}$ are given in Table III.

After determining the energies of leading particles (eqs. 10 , 11) and energies and momenta of secondary particles (eqs. 12-16), the energy of the last particle (as permitted by the law of conservation of energy) is given by

$$
\begin{equation*}
E_{j}=E_{c}-\Sigma E_{i}-E_{L 1}-E_{L 2} \tag{17}
\end{equation*}
$$

The account of the law of conservation of energy is given in refs. ${ }^{10,11 /}$. The angle between the last two particles is defined by the law of conservation of momentum. The multiplicity distribution of secondary particles $(N=j+2)$ is obtained directly from the above consideration. It is assumed that the number of created charged pions is twice the neutral ones and the probability of having the proton or the heutron in the final state is equal.

The angles for elastic scatterings are calculated by eq. 6. The quantities $A(T)$ and $(T)$ are taken from ref. ${ }^{9 /}$. Results of ref. ${ }^{10 /}$ are used for the calculation of the intranuclear cascade at $T<10 \mathrm{GeV}$. It may be noted that the coefficients in Tables I-III differ slightly from those used in ref. $/ 2,11 /$. It is due to the fact that in this analysis we have much more statistics to define the elementary interaction precisely.

Figures $1,2^{*}$ show the multiplicity and angular distributions at 40 GeV and $60 \mathrm{GeV} \pi-N$ interactions calculated by our method and the corresponding experimental data.

## 4. Results of Calculations and Comparison with Experimental Data

Figures 3-5 show some results of calculations and the experimental data $/ 4 /$. The multiplicity distributions of secondary charged particles are given (all pions and protons having momenta greater than $550 \mathrm{MeV} / \mathrm{c}$ ) for pion interactions with carbon nuclei having different number of slow protons, i.e protons with momenta ranging between 160 and $550 \mathrm{MeV} / \mathrm{c}$. Some average characteristics are presented in Table IV.
*All distributions are normalized to unity.

Figures 6 and 7 show the multiplicity and angular distributions in the case of pion interactions with light emulsion nuclei at 17.2 GeV . It is seen that the model satisfactorily explains the general features of secondary particles up to an energy of 17 GeV . However, we do not have a quantitative agreement when the model predictions are compared with the experimental data with higher statistics. For example, the calculation of the average number of secondaries at 40 GeV is equal to $7.99 \pm 0.04$ while the experiment gives $6.98 \pm 0.04 / 47$ (The analogous difference is also for the average number of $\gamma$-quanta). This difference is obviously larger than errors on the multiplicity distribution used in the initial data (see fig. 1). It is interesting to note that the theoretical dependence of $N_{s}$ on $N_{h}$ (Table IV) is more stronger than the experimental one.

In order to improve the theoretical results, one must take into account such effects like resonance production and many-particle interactions, i.e. the simultaneous absorption of several secondary particles with one intranuclear nucleon. These effects decrease the number of intranuclear collisions. Many-particle interaction is very important at higher energies as most of secondary particles fly at a very small solid angle. These effects will be taken into consideration in our subsequent papers.

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Table I
Values of the Coefficients in Equation (10)

|  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $a_{0}$ | $a_{1}$ | $a_{2}$ | $a_{3}$ |
| 0.48 | $0.39 €$ | 0.3791 | -0.8513 | I. 0762 |


| －－－ | －－－ | 6¢2•0－ | $87560 \cdot 0$ | 6TLEO•0 | 88970．0 | $\varepsilon$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 ¢TLO•O | $6650 \cdot 0$ | оธ8・ャ | 9T99L．$\tau$－ | 6I8LL．0 | 608Tで－ | 2 |
| 2199．0－ | 08ちを．0－ | ร¢6＊6z－ | 09カカカ・TI | 0699 ${ }^{\circ} \mathrm{t}$ | 9¢Z¢2． | โ |
| と90¢＊て＇ | $6 \varepsilon \tau z \cdot \tau$ | 0ZL．¢9 | OTELO＊$\angle 2$ | ટ¢отt•0 | ¢ヵ¢L6．${ }^{\text {－}}$ | 0 |
| $0^{8>}>\theta$ | 08＜日 | ${ }^{18} q_{q}$ | ${ }^{2} q_{q}$ | ${ }^{19}$ | ${ }^{10} q^{9}$ | ！ |



$$
\begin{aligned}
& 7.40 \pm 0.05 \\
& 8.50 \pm 0.99 \\
& 8.99 \pm 0.17 \\
& 9.84 \pm 0.34 \\
& 9.25 \pm 0.36 \\
& 7.99 \pm 0.04
\end{aligned}
$$

experiment/4/

$$
\begin{aligned}
& 6.47 \pm 0.06 \\
& 7.4 I \pm 0.09 \\
& 7.58 \pm 0.12 \\
& 7.83 \pm 0.20 \\
& 7.44 \pm 0.30 \\
& 6.98 \pm 0.04
\end{aligned}
$$

$$
6.47 \pm 0.06
$$

$$
\tau \quad \rho \quad \circ \text { u }
$$



Fig. 1. Multiplicity distribution of secondary charged particles in $\pi-N$ interactions at $40 \mathrm{GeV} / \mathrm{c}$. The solid circles are the experimental data $12 /$, the continuous line is due to the theoretical predictions. The average multiplicities obtained by the experiment and theory are $N_{\text {exp }}=5.42 \pm 0.03$ and $N_{T H}=5.45 \pm 0.06$.


Fig. 2. Angular distribution of pions in the c.m.s. in $\pi-N$ interactuons, at 60 GeV . The solid circles are the experimental data fis a and the continuous line is due to the theoretical predjetions


Fig. 5. Multiplicity distribution of shower particles in $\pi^{-} \mathrm{C}$
interactions at 40 GeV with two (a), three (b) and more than



Fig. 7. The angular distribution of shower particles produced in r-CNO interactions at 17.2 GeV in lab. system.

