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ON THE POSSIBILITY OF DETERMINATION OF THE COUPLING CONSTANT $\Pi^- He^3 H^3$

Submitted to ЯФ

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О возможности определения константы связи $\pi^- {}^3He\,{}^3H$

Исследована возможность определения константы связи $\pi^- {}^3He\,{}^3H$ на основе экспериментальных данных по дифференциальному сечению $\pi\,{}^3He$ – рассеяния при 100 Мэв. Метод основывается на экстраполяции $d\sigma/d\Omega$ к ${}^3H$ – полюсу и использовании конформных преобразований. Точность существующих экспериментальных данных по упругому $\pi^+ {}^3He$ – рассеянию недостаточна для извлечения сведений о величине этой константы связи.

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Дубна, 1973

Dumbrais O.V., Nichitiu F., Shcherbakov Yu.A., **E2 - 6962**

On the Possibility of Determination of the Coupling Costant $\pi^- {}^3He\,{}^3H$

The possibility of determination of the $\pi^- {}^3He\,{}^3H$ coupling constant on the basis of the existing experimental data on the differential cross section of the $\pi^+ {}^3He$ scattering at 100 MeV has been investigated. The method is based on the extrapolation of $d\sigma/d\Omega$ to the ${}^3H$ - pole exploiting the conformal mapping techniques. The accuracy of the existing data is insufficient for the estimation of the magnitude of this coupling constant.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1973

The question about the magnitude of the coupling constant $\pi^- He^3 H^3$ is interesting because of the possibility to determine it, on the one hand, by means of models and, on the other hand, in a phenomenological way on the basis of data on the $\pi^\pm He^3$ interactions.

If one considers the nucleus to consist of an assembly of weakly bound nucleons, sufficiently extended that the forward π -nuclear amplitude is simply the coherent sum of the individual πN amplitudes, then the nuclear analog of the ordinary πNN coupling constant should be the same, namely $f^2_{\pi^0 He^3 He^3} = f^2_{\pi^0 pp} \approx 0.08$ /1/. More refined estimations based on the Goldberger - Treiman relation lead to the ratio $f^2_{\pi^- He^3 H^3} / f^2_{\pi^- pn} \approx 1.5$ /2/.

How to determine the coupling constant $f^2_{\pi^- He^3 H^3}$ dispensing with model-dependent considerations and experimental data on other than $\pi^\pm He^3$ scattering processes?

The most accurate phenomenological determination of coupling constants can be achieved via dispersion relations. However, even the use of forward dispersion relation encounters, as a rule, the well known difficulty connected with the treatment of the unphysical cut and the indispensability of introducing the model describing the asymptotic behaviour of the amplitude. In the case of the $\pi^\pm He^3$ scattering the calculations on the basis of forward dispersion relations cannot be carried out at all because of the lack of the data on the total cross sections. On the other hand the use of dispersion relations in the $\cos\theta$ variable cannot be expected to give accurate results for the pole residues because of the unphysical cuts. However, in the latter case not everything is lost in the sense that the hypothesis about the analyticity of the scattering amplitude in $\cos\theta$ can be used not only for the establishing of dispersion relations but for a more modest goal as well: one can assume that the differential cross section is an analytic function in $\cos\theta$ and, analytically continuing it from a physical region to the pole, to determine the coupling constant /3/.

The analytic structure of the differential cross section of the $\pi^+ He^3$ -scattering in the $\cos\theta$ -plane at fixed pion energy is shown in Fig. 1, where the scale corresponds to the value of the kinetic energy $T = 100$ MeV. One can see that the conditions for extrapolation of the cross section from the physical region $-1 \le \cos\theta \le 1$ to the H^3 -pole at $\cos\theta_{H^3} = -19.56$ is highly unfavourable: first of all the pole is a way off from the physical region and, secondly, the presence of the nearby right-hand cut * deprives us of the possibility to represent the differential cross section by the series which would give us hope to be convergent at the position of the H^3 -pole. One could expect that the way out of such a bad situation might be the use of conformal mappings.

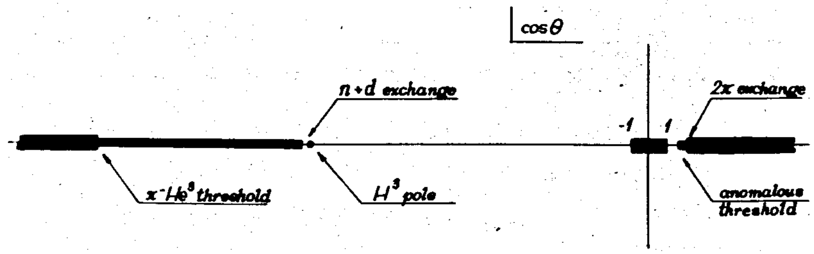

Fig. 1. The analytic structure of the differential cross section in the $\cos\theta$ -plane.

We map the entire cut $\cos\theta$ plane onto the unifocal ellipse and its interior in the z -plane /4,5/ . In particular the pole is transformed to the point $z_{H^3} = -5.95$ and the cuts on the ellipse. The physical region is mapped onto the interval $-1 \le z \le 1$. The semimajor axis of the ellipse is 6.53 which makes, in fact, our ellipse to be very close to the circle. The resulting analytic structure of $d\sigma/d\Omega$ in the z -plane is shown in Fig. 2.

Such a mapping is the optimal one /6,7/ with respect to the analytic properties of the function. It, on the one hand, makes the distance to be extrapolated through significantly smaller and, on the other hand, is expected to accelerate as much as possible the

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* In the case under consideration, owing to the loosely bound structure of He^3, the situation becomes even worse: the so-called, anomalous cut emerges (see, e.g. /1/), which causes the stretching of the cut in the direction of the physical region: the distance $\cos\theta = 2.13$, corresponding to the 2π exchange shortens to $\cos\theta = 1.91$.

convergence of the series representing the quantity $(z - z_{H^3})^2 \times \frac{d\sigma(z)}{d\Omega}$ inside the ellipse even at the position of the pole.

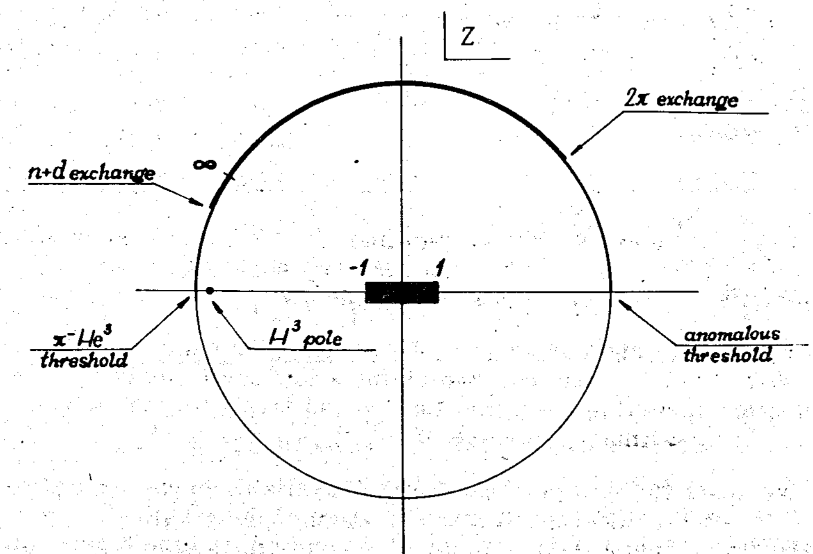

Fig. 2. The analytic structure of the differential cross section in the z -plane.

Therefore, if we write down the expression

$$(z - z_{H^3})^2 \frac{d\sigma(z)}{d\Omega} = \sum_{n=0}^{\infty} a_n z^n , \tag{1}$$

then the value

$$B \equiv \lim_{z \to z_{H^3}} (z - z_{H^3})^2 \frac{d\sigma(z)}{d\Omega} = \sum_{n=0}^{\infty} a_n z_{H^3}^n \tag{2}$$

with an accuracy to the coefficient will represent the coupling constant $f^2_{\pi^- He^3 H^3}$ /8,9/*:

$$B = f^4_{\pi^- He^3 H^3} \frac{16M^6[4M^2(\omega + t_p/4M^2) + t_p(\mu^2 - \omega^2 - s\,t_p/4M^2)]}{s k^4 \mu^4 (4M^2 - t_p)} \qquad (3)$$

$$\left(\frac{dz}{d\cos\theta}\right)^2_{z=z_{H^3}},$$

Here μ and M denote pion and He^3 masses respectively, $t_p = M^2 + 2\mu^2 - s$, s is *cms* energy squared, k is *cms* momentum and ω is pion laboratory energy.

Fitting the right-hand side of Eq. (1) to the experimental data on $d\sigma(z)/d\Omega$ one can determine a few coefficients $a_0, \ldots, a_N$, and after inserting them into Eq. (2) and truncating the series at $n = N$, to estimate the value of $f^4_{\pi^- He^3 H^3}$.

We used for this purpose the only available experimental data /10/ on the differential cross section of the elastic $\pi^+ He^3$ scattering. These data consist of 14 points at pion kinetic lab. energy 100 MeV covering the range $32^\circ \lesssim \theta_{cm} \lesssim 162^\circ$. It turned out that the χ^2 arguments indicated that it was sufficient to retain five terms in the series (1) to obtain a good fit **. This gave the value $f^4_{\pi^- He^3 H^3} = 49 \pm 26$. These numbers, however, have not very much to do with the actual value of the coupling constant because the check showed that the series (1) with five terms diverge at the position of the H^3 pole. In the table we give the values of the individual terms of the power series (1) for the 5-parameter fit at various values of z.

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* Due to the equivalent spin and isospin structure of proton, neutron and He^3, H^3, one can make use of the expression for the pole term of the $\pi^+ p$ scattering replacing the proton mass by that of the He^3.

** The value of χ^2 is 18.6 for 9 degrees of freedom. This points to the insufficient accuracy of the experimental data.

Z	a_0	$a_1 Z$	$a_2 Z^2$	$a_3 Z^3$	$a_4 Z^4$
-0.2	15.8	-5	17.4	-4.4	0.3
-0.4	15.8	-10	70	-35	5
-0.6	15.8	-15	157	-119	24
-0.8	15.8	-20	278	-283	77
-1.0	15.8	-25	435	-553	187
-2.0	15.8	-50	1740	-4423	2995
-3.0	15.8	-75	3914	-14928	15162
-4.0	15.8	-100	6959	-35386	47918
-5.0	15.8	-125	10874	-69114	116989
Z_{H3}	15.8	-148	15650	-119400	242550

One can see that the convergence of the series (1), being satisfactory in the physical region, becomes worse with z increasing and already at $z = -3$ the value of the last term is larger than that of the preceding term. Hence, it is obvious that one cannot truncate the series (1) at $N = 4$ if one wants to continue analytically to the pole. On the other hand, we are unable to determone more terms in the series (1) due to the insufficient accuracy of the experimental data which exist at the present time. How to estimate the error of the value of the coupling constant under such circumstances?

No simple answer exists to this question *, therefore, we merely indicate that with six terms in the series (1) the value of χ^2 remains the same but that of the coupling constant becomes quite different $f^4_{\pi^- He^3 H^3} = -91 \pm 342$.

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* In much broader sense this question can be put in the following way: "What is the relation between the analytical and statistical nature of the experimental data?" This problem in particular is discussed in details in the review article of one of the authors /11/, devoted to the analysis of the analytic continuation in physics of strong interactions.

For the purpose of checking the obtained numbers and with the vague hope to improve them we investigated also the following relation /14/:

$$(z - z_{H^3})\left[\frac{d\sigma(z)}{d\Omega} - \frac{B(z)}{(z - z_{H^3})^2}\right] = \sum_{n=0}^{N} a_n P_n(z) . \qquad (4)$$

Here $P_n(z)$ are the Legendre polynomials. In this representation the pole term is subtracted from the differential cross section and the remainder * is expanded in the series. Due to the condition that the remainder is an analytic function in the whole ellipse only in the case when the pole term has been subtracted correctly, i.e. when the parameter $f^4_{\pi^- He^3 H^3}$ in eq. (3) coincides with the actual value of the coupling constant, one expects the series (4) with the fixed number of terms to represent the remainder the better the closer is the chosen value of $f^4_{\pi^- He^3 H^3}$ to the actual value of the coupling constant. In other words, if one constructs the curve $\chi^2(f^4_{\pi^- He^3 H^3})$ then the minimum of it will correspond to the searched value of the coupling constant and the values for which $\Delta\chi^2 = \pm 1$ should be taken as the upper and lower limits of the coupling constant.

Unfortunately, in our case this representation produced improvements neither in the values of χ^2 nor $f^4_{\pi^- He^3 H^3}$.

Therefore our final conclusion consists of the statement that in spite of the principal possibility of the determination of the coupling constant $f^4_{\pi^- He^3 H^3}$ using the method discussed above its practical determination cannot be carried out on the basis of the existing experimental data. It is necessary to improve significantly the accuracy of measurements of the differential cross section of the $\pi^+ He^3$ scattering.

However, the calculations which have been done allow us to make some significant conclusions with respect to the future measurements ** . First of all, it is necessary to cover the region

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* It is expected that the additional factor $(z - z_{H^3})$ will cause the decrease of N /4/.

**The detailed discussion about the requirements to the experimental data on the differential cross section in order to extract the values of the coupling constant may be found, e.g. in /12,13/.

of large angles completely and accurately as much as possible because the pole is situated to the left from the physical region. Even the mere information about the backward scattering would help very much * .

The socond remark is connected with the choice of the optimal energy at which the measurements should be performed. It can be seen in Fig. 3 that with the increasing energy the H^3 pole appro-

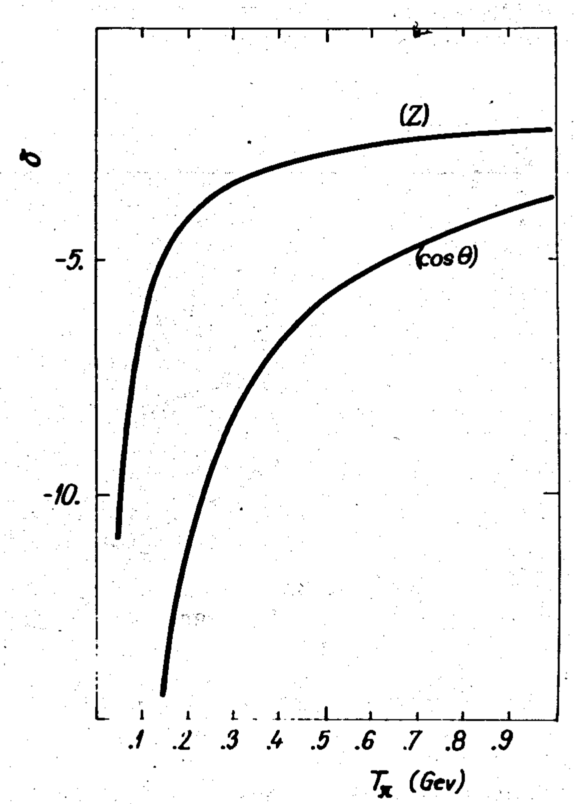

Fig. 3. The distance δ of the pole of the differential cross section from the origin in the $\cos\theta$ -plane and in the z -plane as a function of the pion kinetic energy.

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* It turned out that the mean values of $f^4_{\pi^- He^3 H^3}$ are quite sensitive with respect to the assumed artificial experimental value of $d\sigma(180^\circ)/d\Omega$.

aches the physical region, therefore one might think that the energy should be as large as possible. However, it is not so due to two factors: the larger the energy, the larger is the number of waves which must be taken into account and consequently, more terms one will need in the series to describe the experimental data. This will lead unevitably to the increase of the errors. Secondly, in Fig. 4 one can see that with energy increasing the distance between the cut and the pole decreases or, in other words, the position of the pole approaches the edge of the ellipse and that worsens the convergence of the series (1) or (4).

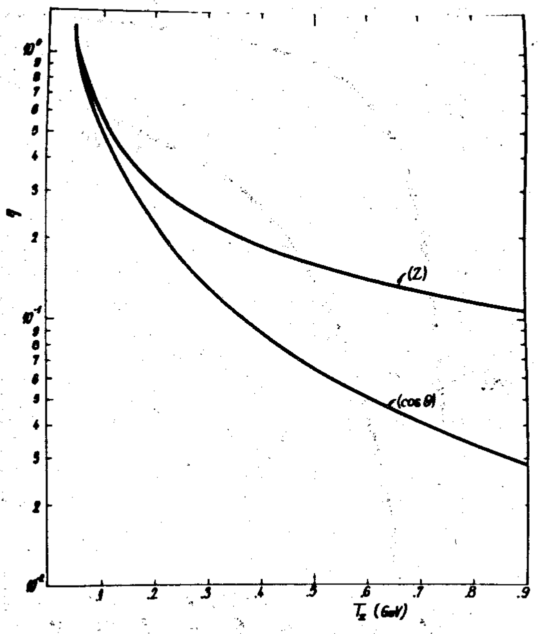

Fig. 4. The distance η between the pole and the cut in the $\cos\theta$ - plane and the z -plane as a function of the pion kinetic energy.

To summarize what has been said above we think that the existence of accurate and dense measurements of the differential cross sections of the $\pi^+ He^3$ scattering at large angles and at energies of the order of 200 - 500 MeV should make it possible

to determine the coupling constant $f^4_{\pi^- He^3 H^3}$ with a reasonable accuracy *.

The determination of the coupling constants $g^2_{He^3 He^3 \pi^0}$ and $g^2_{He^3 dp}$ on the basis of the existing much better data on differential cross section of the pHe^3 scattering is in progress.

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* In any case this method allows to extract the value of this coupling constants from any theoretical model for the differential cross section of the $\pi^+ He^3$ scattering, where the Coulomb corrections (being ignored in present analysis because of the large angles at which the data exist) should be taken into account properly.