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The theory of higher symmetries and the dispersion sum rules made it possible to calculate the magnetic moments (m.m.) of various baryons and mesons.

The aim of the present paper is to show that the sidewise dispersion relation [1] is the most useful approach for obtaining the transition m.m.

Let us consider an arbitrary hadron electromagnetic vertex, where the ingoing hadron is off the mass shell. If we assume that the electromagnetic form factors $F_{ij}(k^2, m_i^2, W_i^2)$ of such a vertex are analytical functions in W_i^2, as is shown in [2], one may get some system of linear, in the form factors, algebraical equations, which connect each of the form factors with all the others. The concrete form of such a system depends, to some extent, on the model, which is used for the calculation of $Im\, F_{ij}(k^2, m_i^2, W_j^2)$ but the existence of such a system is a direct consequence of the analyticity of form factors in the invariant hadron mass and unitarity.

In this paper such an approach is chosen, in which the elastic $(i=j)$ and transition $(i\neq j)$ m.m. $\mu_{ij} = F_{ij}(0, m_i^2, m_j^2)$ both of baryons and mesons are expressed in terms of the nucleon m.m. only.

Let us apply the hypothesis about the analyticity of the form factors in the hadron mass to the form factors of the nucleon vertex and vertex $N^*N\gamma$, where N^* is the resonance P_{11} with quantum numbers of a nucleon and mass $M_{N^*} = 1470$ MeV (fig.1).

Then, in the one-pion approximation, the absorptive parts of the form factors correspond to the graph of Fig. 1, and the isoscalar and isovector anomalous m.m. of those vertices satisfy the following relations [1] (see also [2-4]).

$$\langle M|\mu^{v,s}|m\rangle = -\frac{g_{\pi NN}}{\pi}\, P\int_{m+\mu}^{\infty}\frac{dW}{W}\,\frac{|P_2|}{|P_1|}\left\{\left(\frac{E_2-m}{E_1+m}\right)^{1/2}\cdot\frac{W+m}{W-M}\,M_{1-}^{v,s}(W)K^{+}(W) + \left(\frac{E_2+m}{E_1-m}\right)^{1/2}\frac{W-m}{W+M}\cdot E_{0+}^{v,s}(W)K^{+}(-W)\right\}. \qquad (1)$$

Here, $M=\{m, M_{N^*}\}$; m is the mass of the nucleon; M_{N^*} is the mass of the isobar N^*_{1470}; $M_{1-}^{v,s}$, $E_{0+}^{v,s}$ are the photoproduction multipoles; $K(\pm W)$ denotes the mesonic form factors of the nucleon; $+$ is the sign of the complex conjugation. Kinematic notations are obvious from fig. 1.

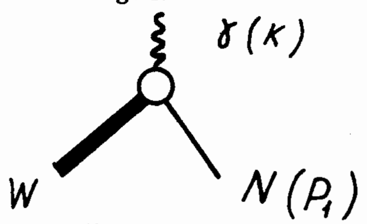

Fig. 1. The electromagnetic vertex, in which only one nucleon is off the mass shell: $p_1^2=m^2$, $k^2=0$, $P^2=W^2\neq m^2$.

The multipoles $M_{1-}^{v,s}$ and $E_{0+}^{v,s}$ have been calculated by means of a photoproduction model, in which besides the Born terms we take into account the contributions of the 3-3 resonance M_{1+} in the u-channel and in the pole approximation of the ω and ρ mesons in the t-channel [5-8] * (fig. 3).

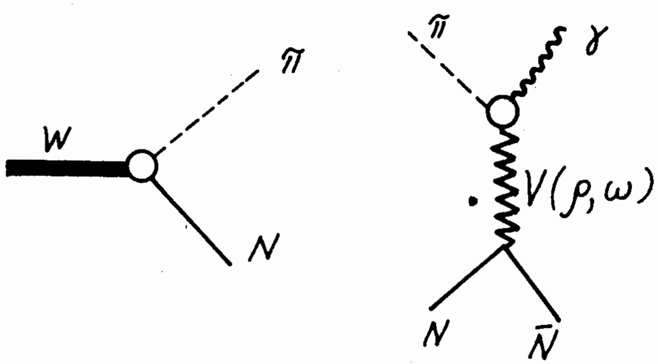

Fig. 3. The graph of the exchange by vector mesons in t-channel.

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* The ω and ϕ mesons have not been mixed, because $g_{\omega\pi\gamma} \gg g_{\phi\pi\gamma} \approx 0$ [11].

These terms introduce, into the right-hand side of the eq. (1), the anomalous m.m. of the nucleon $\mu^{v,s}$ at the expense of the multipoles $M_{1-}^{v,s(Born)}$, $E_{0+}^{v,s(Born)}$ and $M_{1+}^{3/2}$ (only for μ^{v}), and the transition m.m. $\mu_{\omega(\rho)\pi\gamma}$ at the expense of the multipoles $M_{1-}^{\omega(\rho)}$, $E_{0+}^{\omega(\rho)}$ as well as the coupling constants $g_{\pi NN}$ and $g_{\omega(\rho)NN}$.

The mesonic form factors of the nucleon in the scattering-length approximation for the S-wave and the resonant form for the P-wave, as well as the relevant phase shifts, were given in ref. [3] (model 2).

This approach allows to connect the m.m. of baryons and mesons immediately.

Thus, the eq. (1) gives the following linear relations between the m.m. of baryons and mesons.

$$\mu^{v,s} = a^{v,s} + b^{v,s}\mu_{\omega(\rho)\pi\gamma} \qquad (2)$$

$$\mu^{v,s}_{N^*N\gamma} = c^{v,s} + d^{v,s}\mu^{v,s} + f^{v,s}\mu_{\omega(\rho)\pi\gamma} \qquad (3)$$

The coefficients $a^{v,s}\ldots f^{v,s}$ are the functions of the coupling constants and the integrals containing the mutipoles $M_{1-}^{v,s}$, $E_{0+}^{v,s}$.

The eqs. (2), (3) express $\mu_{N^*N\gamma}$ and $\mu_{\rho(\omega)\pi\gamma}$ in terms of $\mu^{v,s}$ and $g_{\pi NN}$ and $g_{\omega(\rho)NN}$.

For the calculations of $a^{v,s}\ldots f^{v,s}$ we have used the corresponding experimental data for the coupling constants [9-11], indicated in the Tables 1,2. By the eqs. (2)-(3) and the experimental data [11] $\mu^{v}=1{,}85$, $\mu^{s}=-0{,}06$ we have calculated the magnitudes of the transition m.m. of the N^*_{1470} and ρ, ω (see Tables 1,2).

$$\mu^{v}_{N^*N\gamma} = 0{,}905 \pm 0{,}06, \qquad \mu^{s}_{N^*N\gamma} = -0{,}082 \pm 0{,}01 \qquad (4)$$

$$\mu_{\omega\pi\gamma} = 2{,}81 \pm 0{,}843; \qquad \mu_{\rho\pi\gamma} = 1{,}87 \pm 0{,}3 \qquad (5)$$

which are in a good agreement with the experiment:

$\mu_{\omega\pi\gamma} = 3{,}2$ [9,10], $\mu_{\rho\pi\gamma} = 1{,}78$ [17], $\mu_{N^*N\gamma} = 0{,}9$ [18].

Previously, the values $\mu^{v,s}_{N^*N\gamma}$ were estimated by different models in [12-16].

For example, identifying the nucleon with the ground state $1S_{1/2}$ and the resonance N^*_{1470} with the excitation state $2S_{1/2}$

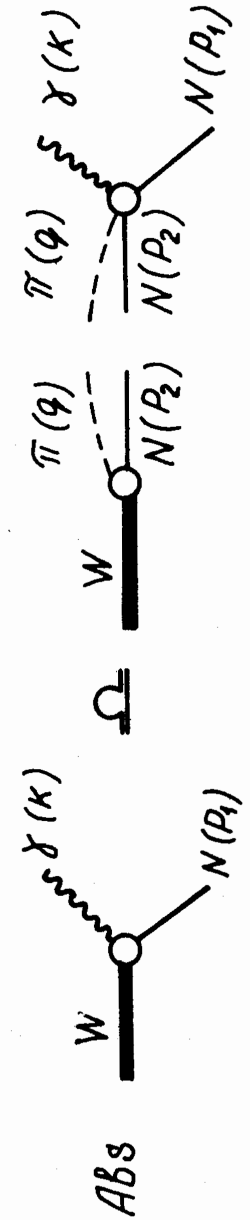

Fig. 2. The threshold graph with the $\pi-N$ contribution in the intermediate state (the sign Ω means "dominates").

of the hydrogen atom, Barut and Nagulaki [13] calculated the dipole m.m. of the relativistic transition $2S_{1/2} \to 1S_{1/2}$

$$\mu_{N^{*(+)}p\gamma} = 0{,}186\,; \qquad \mu_{N^{*(0)}n\gamma} = -0{,}156\,.$$

From the eq. (4) it follows that $\mu_{N^{*(+)}p\gamma} = 0{,}823$, $\mu_{N^{*(0)}n\gamma} = -0{,}987$, i.e. that the relation $\mu_{N^{*(+)}p\gamma} \simeq \mu_{N^{*(0)}n\gamma}$ is correct.

By calculating the values

$$\eta^{\omega(\rho)} = \frac{b^{v,s}\mu_{\omega(\rho)\pi\gamma}}{a^{v,s}}\,, \qquad \eta^{*\,\omega(\rho)} = \frac{f^{v,s}\mu_{\omega(\rho)\pi\gamma}}{c^{v,s} + d^{v,s}\mu^{v,s}}$$

one can see (Tabl. 1,2) that the contributions of the ω and ρ exchanges as compared with the contributions of the Born terms and 33 resonance are also considerable.

To sum up, we have applied the sidewise dispersion relations for the $NN\gamma$ and $N^*_{1470}N\gamma$ vertices and used the dispersion model for the calculation of the photoproduction multipoles. In this approach we have obtained the satisfactory values of the transition m.m. of the Roper resonance N^*_{1470} and the vector mesons ω and ρ .

Finally, we would like to emphasize that the principal assumption used in order to derive the eqs. (3)-(4) is that of the analyticity of the electromagnetic form factors of the hadrons in the complex plane of the invariant hadron mass.

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Table 1

$g^{(1)}_{\omega\pi\pi}$	$mg^{(2)}_{\omega\pi\pi}$	[illegible]	a^v	b^v	c^v	d^v	f^v	ϵ^ω	$\mu_{\omega\pi\gamma}$	σ^ω	$\mu^v_{A\omega\pi\gamma}$
17,32 ± 1,28	0,83 ± 0,0[illegible]	Fit to π-N scattering /11, 13/; Fit to photoproduction with exchange ω-meson	1,87 ± 0,37	−(0,13[illegible] ± 0,0[illegible])	−(0,82 ± 0,11	1,81 ± 0,1	−(0,059 ± 0,0027)	24,6 %	2,81 ± 0,043	11,3%	0,905 ± 0,061
6,0[illegible] ± 0,44	0,25 ± 0,021	Fit to π-N scattering /11,13/; Fit to photoproduction with exchange ω-meson /11/	1,87 ± 0,37	−(0,047 ± 0,[illegible])	−(0,82 ± 0,11)	1,81 ± 0,1	−(0,024 ± 0,001)	8,1 %	7,9 ± 0,79	3,9%	1,08 ± 0,067
−(30,3 ± 2,24)	−(1,48 ± 0,1)	Fit to excitation resonances /13/; Fit to photoproduction with exchange ω-meson	1,87 ± 0,37	0,2[illegible] ± 0,0[illegible]	−(0,8 ± 0,11)	1,81 ± 0,1	0,068 ± 0,0049)	42 %	1,51 ± 0,3	19,2%	1,33 ± 0,08

Table 2

$g^{(1)}_{\rho\pi\pi}$	$mg^{(2)}_{\rho\pi\pi}$	[illegible]	a^s	b^s	c^s	d^s	f^s	ϵ^ρ	$\mu_{\rho\pi\gamma}$	σ^ρ	$\mu^s_{A\rho\pi\gamma}$
6,92 ± 0,7	12,8 ± 1,3	Model of ρ-dominance /13,14/; Electroproduction π-meson on nucleon /14/	−0,0815 ± 0,0027	0,041 + 0,001	−(0,0785 ± 0,008)	0,91 ± 0,09	0,072 ± 0,006	67%	0,51 ± 0,02	69%	− 0,041 ± 0,005
1,57 ± 0,08	4,62 ± 0,3	Model of ρ-dominance; Electroproduction π-meson on nucleon	−0,0815 ± 0,0027	0,011 + 0,0031	−(0,0785 ± 0,008)	0,91 ± 0,09	0,019 ± 0,001	18%	2,28 ± 0,68	20%	−0,082 ± 0,01
1,25 ± 0,05	5,82 ± 0,1	Model of ρ-dominance; Electroproduction π-meson on nucleon	−0,0815 ± 0,0027	0,0133 + 0,001	−(0,0785 ± 0,008)	0,91 ± 0,09	0,023 ± 0,002	22%	1,87 ± 0,3	21%	−0,085 ± 0,02