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THE DYSON EQUATION FOR THE TWO-PARTICLE GREEN FUNCTION AND THE BOUND STATE PROBLEM

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The equations of quantum field theory [1], e.g. the equations for the Heisenberg field operators, are of essentially nonlinear nature. The Dyson equations for the one particle mass and polarization operators bear quite a similar nature. At the same time the Bethe-Salpeter (B.-S.) equation for the two-particle Green function is often treated as a linear one. This lack of correspondence is directly connected with the usual definition of the kernel of the B.-S. equation within the perturbation theory in the form of an infinite sum of ''irreducible'' (in the sense of two-particle sections) Feynman diagrams.

However, such a representation often proves to be unfit, e.g. in quantum electrodynamics (QED), because of the presence of the ''infrared'' divergences closely associated with the zero photon mass. In the case of the bound state problem the ''infrared'' divergences do not arise even in the ''scattering'' approximation if we explicitly take into account the effects of ''binding'' in the intermediate virtual states. To this end it is necessary to sum up selectively an infinite sequence of ''irreducible'' diagrams in the kernel of the B.-S. equation [2]. Such a summation can be accomplished in the simplest way if there is an additional equation for the B.-S. interaction kernel (e.g. of the type of the Dyson equation [3] for the one-particle mass operator).

Now we try to deduce an appropriate equation in the framework of QED taken as an example. Consider the Green function of two distinct particles with masses m_1 and m_2 and spin 1/2 (say an electron and a positive muon):

$$G(x_1 x_2; y_1 y_1) = \langle 0 | T\{\psi_1(x_1)\psi_2(x_2)\bar{\psi}_1(y_1)\bar{\psi}_2(y_2)\} | 0 \rangle . \quad (1)$$

In QED the Heisenberg field operators $\psi_{1,2}$ satisfy the equation

$$(\hat{p}_x - m)\,\psi(x) = e\,\hat{A}(x)\,\psi(x)\,, \qquad (2)$$

where

$$\hat{p}_x = i\frac{\partial}{\partial x_\mu}\gamma_\mu \quad ; \qquad \hat{A}(x) = A^\mu(x)\,\gamma_\mu$$

and $A^\mu(x)$ is the Heisenberg operator of the photon field. We apply the operator $(\hat{p}-m)$ to the Green function G. Then using the definition of the T-product in terms of the θ-functions, eq. (2) and the canonical equal-time commutation relations for the operators ψ we obtain the relation

$$(\hat{p}_{x_1} - m_1)\,G(x_1 x_2\,; y_1 y_2) = -\delta^4(x_1 - y_1)\,S_2(x_2 - y_2) +$$

$$+ e_1\gamma_{1\mu} <0|\,T\{A^\mu(x_1)\psi_1(x_1)\psi_2(x_2)\bar{\psi}_1(y_1)\bar{\psi}_2(y_2)\}\,|0>, \qquad (3)$$

where

$$S(x-y) = -i<0|\,T\{\psi(x)\bar{\psi}(y)\}\,|0>$$

is the complete one-particle Green function (propagator) of a fermion. The term with the δ-function comes from differentiation of the θ-function.

Now we define the generalized irreducible vertex function Γ of the two-particle system /4/ similar to the case of one particle. To this end we consider the five-point Green function

$$R^\mu(x_1\,x_2\,; y_1\,y_2\,|\,z) =$$

$$= <0|\,T\{\psi_1(x_1)\psi_2(x_2)A^\mu(z)\bar{\psi}_1(y_1)\bar{\psi}_2(y_2)\}\,|0>\,. \qquad (4)$$

and set

$$R^\mu = \mathfrak{D}^{\mu\nu}\,G * \Gamma_\nu * G\,, \qquad (5)$$

where

$$\mathfrak{D}^{\mu\nu}(x-y) = i<0|\,T\{A^\mu(x)A^\nu(y)\}\,|0>$$

is the complete one-photon Green function, and G is the two-particle Green function, defined by eq. (1). In the operator eq. (5) symbol $*$ denotes the "convolution" with respect to the two-particle virtual states. Definition (5) is represented with the help of the diagrams in Fig. 1.

Now eq. (3) for the Green function G can be rewritten, using eq. (4), in the following way:

$$(\hat{p}_{x_1} - m_1)\,G(x_1 x_2\,; y_1 y_2) = -\delta^4(x_1 - y_1)\,S(x_2 - y_2) +$$

$$+ e_1\gamma_{1\mu}\,R^\mu(x_1 x_2\,; y_1\,y_2\,|\,x_1)\,. \qquad (6)$$

We pass with the help of the Fourier transform to the momentum space. Then, with the account of definition (5) of the generalized vertex function, eq. (6) takes on the following operator form:

$$S_{f1}^{-1}\,G(\mathcal{P}) = -I_1 S_2 +$$

$$+ e_1\gamma_{1\mu}\int d^4k\,\mathfrak{D}^{\mu\nu}(k)\,G(\mathcal{P}-k) * \Gamma_\nu(\mathcal{P}-k, \mathcal{P}) * G(\mathcal{P})\,, \qquad (7)$$

where $\mathcal{P}$ is the total four-momentum of the two-particle system and S_f is the Green function (propagator) of a bare particle (without the account of interaction with the photon field), satisfying the equation:

$$(\hat{p} - m)\,S_f(p) = 1\,. \qquad (8)$$

Now we introduce according to Dyson /3/ the one-particle mass operator M:

$$S = S_f + S_f\,M\,S\,, \qquad (9)$$

or

$$M = S_f^{-1} - S^{-1}\,.$$

The B.-S. equation for the two-particle Green function looks like:

$$G = G_0 + i G_0 * K * G , \tag{10}$$

where

$$G_0 = - S_1 S_2 ,$$

from where the following operator expression for the kernel K can be obtained

$$i K = G_0^{-1} - G^{-1}. \tag{11}$$

Inserting expressions (9) and (11) into eq. (7) we find a suitable representation for the kernel K:

$$K(\mathcal{P}) = - i M_1 S_2^{-1} +$$

$$+ i e_1 \gamma_{1\mu} S_2^{-1} \int d^4 k\, \mathfrak{D}^{\mu\nu}(k) G(\mathcal{P} - k) * \Gamma_\nu (\mathcal{P} - k , \mathcal{P}) . \tag{12}$$

In terms of the Feynman diagrams eq. (12) is displayed in Fig. 2, where symbol —✶— denotes the factor S^{-1}. On looking at eq. (12) and Fig. 2 it stands out at once the formal resemblence of this representation for the kernel K and the Dyson equation for the one-particle mass operator M.

Eq. (12) is unsymmetric with respect to both the particles. However, it is quite easy to get in a similar way the "mirror" equation (with the substitution $1 \leftrightarrow 2$):

$$K(\mathcal{P}) = - i S_1^{-1} M_2 +$$

$$+ i e_2 \gamma_{2\mu} S_1^{-1} \int d^4 k\, \mathfrak{D}^{\mu\nu}(k) G(\mathcal{P} - k) * \Gamma_\nu (\mathcal{P} - k , \mathcal{P}) , \tag{13}$$

which in the graphic form is presented in Fig. 3. Combining eqs. (12) and (13) it is possible to obtain the symmetric expression for the kernel K.

In application of eqs. (12) and (13) it is convenient to represent the quantities G and Γ in the following way:

$$G = G_0 + i G_0 T G_0 , \tag{14}$$

$$\Gamma = - i \frac{\delta G^{-1}}{\delta A^{ext}} \Big|_{A^{ext}=0} = - i \Gamma_{01} S_2^{-1} - i S_1^{-1} \Gamma_{02} + \Lambda , \tag{15}$$

$$\Lambda = - \frac{\delta K}{\delta A^{ext}} \Big|_{A^{ext}=0} , \tag{16}$$

where T is the off-mass-shell scattering amplitude and Γ_0 is the one-particle vertex function, corresponding to the diagramm in Fig. 4. Now the terms containing T and Λ should be considered small in eqs. (14) and (15).

Then solving eqs. (12) or (13) by iterations in the modified perturbation theory we find as an initial approximation for the kernel K the one-photon exchange diagram, shown in Fig. 5. The corresponding approximation for the Green function $G^{(1)}$ obtained from eq. (10) is obviously the sum of the ladder diagrams, presented in Fig. 6. Now we are able to construct the next, improved approximation for the kernel K. Inserting the function $G^{(1)}$ into eqs. (12) and (16) we find the set of terms shown graphically in Fig. 7.

The main merit of the expounded procedure lies in the possibility of the explicit and straightforward account of binding in the intermediate virtual states of the interaction kernel. In fact even the first (ladder) approximation $G^{(1)}$ for the Green function reproduces the existing bound states. For loosely bound system in QED binding is important only in the intermediate states with low-frequency photons. In this region we may use the nonrelativistic expression for the two-particle Green function. The high-frequency region may be treated by usual Born approximations. The sum of both the contributions yields a comlpetely infrared convergent result for the bound state energy shifts.

The renormalization of eq. (12) for the B.-S. kernel K presents no special difficulties and can be accomplished in a standard way (cf. the renormalization of the Dyson equations).

It consists in attributing the vertex renormalization constant Z_1 to the bare (point) vertex in eq. (12) and introducing renormalized quantities such as charge, masses, propagators and vertices.

The most convenient method of relativistic description of the two-particle bound system is the quasipotential method of Logunov and Tavkhelidze [5,6]. The kernel of this three-dimensional equation-quasipotential is determined in terms of the off-mass-shell scattering amplitude T which can be obtained from the equation

$$T = K + K * G_0 * T = K + K * G_0 * K + \dots \ .$$

This approach considerably facilitates calculation of the energy levels of bound systems /6/.

We hope that the proposed equation (12) for the B.-S. interaction kernel may prove to be helpful also in some other problems.

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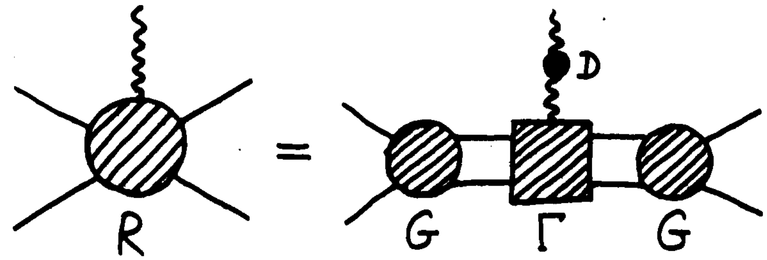

Fig. 1

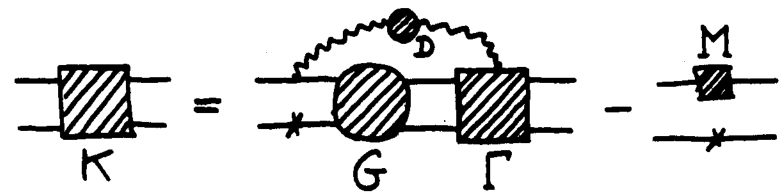

Fig. 2

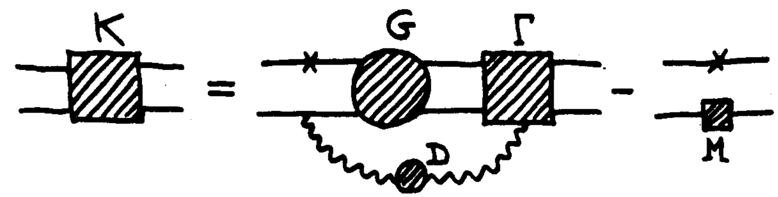

Fig. 3

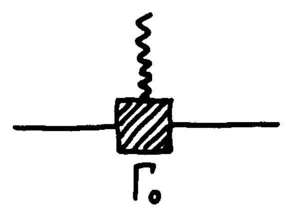

Fig. 4

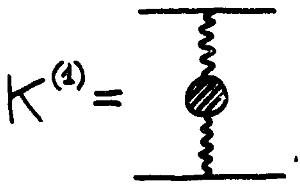

Fig. 5

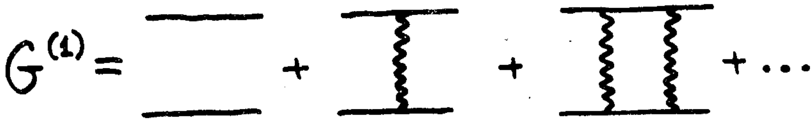

Fig. 6

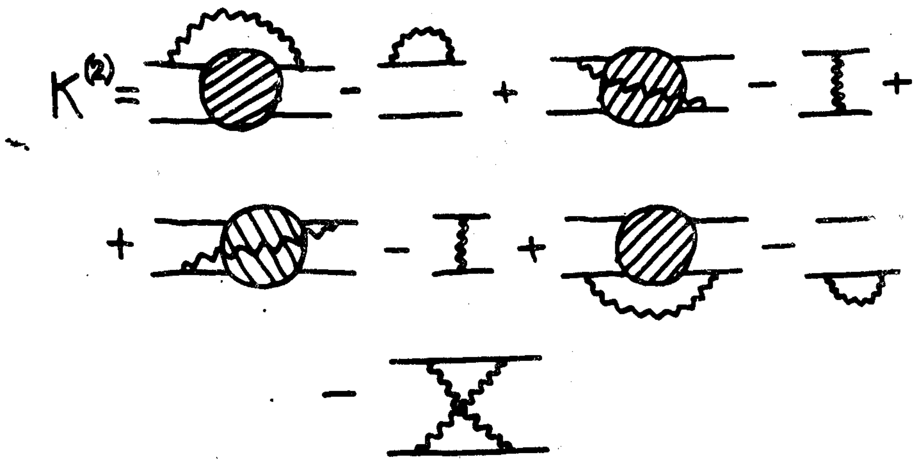

Fig. 7