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1. Introduction

In this report we briefly review the origin of singular potentials in polynomial and non-polynomial quantum field theories, summarize basic properties of the Schrödinger equation with such potentials and consider in some details a new class of potentials which have a singularity for some finite $r\ (\neq 0, \infty)$. These finite range singularity potentials (or FRSP) are of special importance for composite models of hadrons and seem to be connected with statistical bootstrap models.

The general method for reducing two-particle problems to solving some Schrödinger equation is provided by the quasipotential approach [1,2]. The partial-wave quasipotential equation is

$$u'' + [k^2 - \ell(\ell+1)r^{-2}]u = \int_0^\infty dr_1 V(r) K_\ell(r, r_1) u(r_1); \tag{1}$$

$$K_\ell(r, r_1) = (r_1/r)^{1/2} \int_0^\infty dp\, (M^2 + p^2)^{-1/2}\, p\, J_{\ell+1/2}(pr) J_{\ell+1/2}(pr_1); \tag{2}$$

$$V(r) = \int_\mu^\infty dm\, \rho(m) \exp(-mr), \quad s = 4(k^2 + M^2), \tag{3}$$

where $\rho(m)$ is the t-channel spectral function of two-particle irreducible diagrams (e.g. for the exchange of the scalar particle with mass m we have $\rho(m') \sim \delta(m' - m)$ and this, in non-relativistic limit $M \to \infty$, gives the Yukawa potential $\sim r^{-1}\exp(-mr)$). For $r, r_1 \gg M^{-1}$ the kinematical factor $(M^2 + p^2)^{-1/2}$ can be approximated by M. With this approximation, we have $K_\ell \sim (Mr)^{-1}\delta(r - r_1)$ and, instead of Eq. (1), the usual Schrödinger equation with the local potential $(Mr)^{-1} V(r)$ emerges. For $r \to 0$ this non-relativistic approximation is not valid but, in this case, non-local effects of $K_\ell(r, r_1)$ can be reproduced by using some effective local potential $V_{ef}(r)$ (see [2]). Then in super-renormalizable (SR) theories ($\mathcal{L}_{int} = \lambda : \phi^3 :$) we have $V_{ef}(r) \to const,\ r \to 0$, and in renormalizable (R) theories ($\mathcal{L}_{int} = \lambda : \phi^4 :$)

$$V_{ef}(r) \sim \lambda^2 r^{-2} \{ C_0 + \lambda [C_1 \ln(r_0/r) + C_1'] + \dots \}, \quad r \to 0.$$

In non-renormalizable (NR) theories ($\mathfrak{L}_{int}=\lambda:\phi^n:,\ n\geq 5$) the λ^2 -order diagram gives the potential $V_{ef}(r)\sim\lambda^2 r^{6-2n},\ r\to 0$. Potentials corresponding to superpropagators of non-polynomial (NP) theories will be considered later.

Quite similar classification of potential was obtained by using Bethe-Salpeter or Edwards equations (see e.g. /3/). A simple example is the Edwards equation for a scalar bound state of fermions (ψ and $\bar\psi$) interacting via some scalar gluon field ϕ :

$$\mathfrak{L}_{int}=f_1:\bar\psi\psi\phi: + \frac{1}{2}f_2:\bar\psi\psi\phi^2:+\dots . \qquad (4)$$

The equation is graphically represented by Fig. 1. Defining the wave function $u(r)$ by the Fourier transform in the 4-dimensional Euclidean space

$$\Gamma(p^2)=p^{-1}(M^2+p^2)\int_0^\infty dr\, J_{3/2}(pr)r^{1/2}\ u(r), \qquad (5)$$

we find for $u(r)$ the Schrödinger equation

$$u''-[M^2+\frac{3}{4}r^{-2}+f_1^2\Delta_F(r)+\frac{1}{2}f_2^2\Delta_F^2(r)+\dots]u=0,$$
$$\Delta_F(r)=mK_1(mr)(4\pi^2 r)^{-1} \qquad (6)$$

(The asymptotic behaviour of $V_{ef}(r)$ for $r\to 0$ in the quasipotential equation is the same as in Eq. (6)). The generalization of the above procedure which allows constructing potentials corresponding to superpropagators is quite obvious. Consider, instead of Eq. (4), the Lagrangian

$$\mathfrak{L}_{int}=f:\bar\psi\psi\sum_{n=1}^{\infty}\frac{d_n}{n!}(g\phi)^n: \qquad (7)$$

Then obvious formal manipulations give us

$$V(r)=f^2\sum_{n=1}^{\infty}c_n[g^2\Delta_F(r)]^n\equiv f^2 v(r), \qquad (8)$$

where $c_n=\frac{d_n^2}{n!}$. This expression makes sense if the series is

convergent. If it is divergent for some r (or for all r) the more refined and rigorous methods based on analytic continuations are necessary.

2. Basic Properties of Singular Potentials

The Schrödinger equation (consider for simplicity the case $k=0$)

$$u''-[\ell(\ell+1)r^{-2}+f^2v(r)]u=0 \qquad (1^*)$$

with the boundary condition $u(r)_{r\to 0}\to 0$ is equivalent to the integral equation

$$u=Zr^{\ell+1}+\frac{f^2}{2\ell+1}\int_0^\infty dr_1\, v(r_1)[r^{\ell+1}r_1^{-\ell}-r_1^{\ell+1}r^{-\ell}]\,u(r_1). \qquad (9)$$

If $I[v]=\int_0^{r_0}dr\, rv(r)<\infty$ for some $r_0>0$, the perturbation series obtained by iterations of Eq. (9) converges, i.e. the wave function u is analytic in f^2 in some vicinity of $f^2=0$. If $I[v]=\infty$, the integrals obtained by iterations of Eq. (9) diverge and some regularization is necessary (we choose the simplest regularization $v(r)\to v_\epsilon(r)=\theta(r-\epsilon)\,v(r)$). In the first case the potential is called **regular** in $r=0$, in the second case it is called **singular** in $r=0$ (or marginally singular).

In SR-theories the potentials introduced above are regular. In R and NR-theories the potentials (corresponding to any finite number of diagrams) are singular. We call a potential renormalizable (R) if the divergences in all orders of perturbation theory for u_ϵ can be eliminated order by order by using some renormalization constant Z_ϵ in Eq. (9) with regularized potential v_ϵ .

T h e o r e m 1. A potential $v(r)$ monotonic near $r=0$ is renormalizable if and only if for every $\delta>0$ there exists $r_\delta>0$ such that $|v(r)|<Cr^{-2-\delta}$, $r<r_\delta$. C o r o l l a r y: Potentials in R-theories (for any finite number of diagrams) are R-potentials. P r o o f of this theorem was given in the authors thesis in 1969. Example: $v(r)=r^{-2}\ln^\nu(r_0/r)$ (see also /4/).

Potentials (monotonic near $r=0$) which do not satisfy the conditions of the theorem are non-renormalizable (NR). The potentials of NR-theories (for any finite number of diagrams) are **NR-potentials**. In this case the perturbation theory can be used only for finite values of ϵ and the transition to the limit $\epsilon \to 0$ is possible only for the exact solution of Eq. (9) with $v \to v_\epsilon$, provided $v(r) > 0$ near $r = 0$ (repulsive singularity). The solution can be written in the form

$$u_\epsilon(r) = Z_\epsilon [w_1(\epsilon) u_1(r) + w_2(\epsilon) u_2(r)], \tag{10}$$

where $u_1(r) \to 0, u_2(r) \to \infty$ for $r \to 0$ and $w_1(\epsilon) \to \infty$, $w_2(\epsilon) \to 0$ for $\epsilon \to 0$. Here $u_1(r)$ is the solution of the original problem (8). Choosing $Z_\epsilon = Z[w_1(\epsilon)]^{-1}$ we pass to the limit $\epsilon \to 0$ and find the exact solution $Z u_1(r)$. So, NR-potentials require a summation of all the terms of perturbation theory. This problem is not solved even for rather simple potentials and approximate methods for solving the Schrödinger equation with singular potentials are of principal importance. The most general method is provided by the "asymptotic" perturbation theory.

Theorem 2. Let $v(r) = v_S(r) + v_R(r)$, where $r^2 v(r) \underset{r \to 0}{\to} +\infty$ and $\int_0^{r_0} dr\, v_R(r) [v_S(r)]^{-1/2} < \infty$. Then $v_R(r)$ is a "good" perturbation for the unperturbed problem

$$u_S'' + [k^2 - \ell(\ell+1) r^{-2} - f^2 v_S(r)] u_S = 0. \tag{11}$$

Remark: The perturbation v_R is "good" if the series in powers of v_R, in which the solution of Eq. (11) is the zeroth order approximation, is convergent at least near $r = 0$. Proof consists of arguments that n-th approximation has a majorant $C^n u_1(r)$ with $C < 1$ (see the authors thesis mentioned above). Example: $v_S(r) \underset{r \to 0}{\sim} r^{-\nu_S}$, $\nu_S > 2$; $v_R(r) \underset{r \to 0}{\sim} r^{-\nu_R}$,
$\nu_R < \frac{1}{2} \nu_S + 1$ or $v_R(r) \underset{r \to 0}{\sim} \ln^\nu (r_0 / r)$.

The main difficulty of this method is to find u_S and simpler methods are very desirable. There are two such methods: differential interpolations [5] and Pade approximations [6].

3. FRS-Potentials

Up to now we considered only marginal singularities ($r = 0$ or $r = \infty$). FRS-potentials, having a singularity at $r = b \neq 0, \infty$ require

a special treatment. FRSP naturally arise in a context of non-polynomial quantum field theories.

Let the coefficients c_n in Eq. (8) are positive and satisfy the condition $\overline{\lim_{n \to \infty}} (c_n)^{1/n} = 1$. Then the series $\sum_{n=1}^{\infty} c_n z^n$ is convergent for $|z| < 1$ and has a singularity at $z = 1$ (see e.g. [7]). Therefore, the potential (8) has a singularity for $r = b$, where $g^2 \Lambda_F(b) = 1$. The value of $v(r)$ for $r < b$ can be defined by analytic continuation in r and in g^2. First we define $v = \bar{v}$ for $g^2 \to -g^2 < 0$. For $r > b$ the potential $\bar{v}(r)$ is given by the convergent series (8), for $0 \le r \le b$ it is uniquely defined by the analytic continuation in r. For $g^2 > 0$, we define $v(r)$ in accordance with the corresponding prescription for the superpropagator [8]

$$v(g^2, r) = \frac{1}{2} [\bar{v}(-g^2 e^{i\pi}, r) + \bar{v}(-g^2 e^{-i\pi}, r)]. \tag{12}$$

Consider for example the potential

$$v(r) = \Lambda_F(r) [1 - g^2 \Lambda_F(r)]^{-\nu}. \tag{13}$$

If $\nu < 1$, then all solutions of Eq. (1*) are continuous with their first derivatives for all r including $r = b$. We will call such potentials penetrable. If $\nu > 1$, then for one of two solutions of Eq. (1*) $u'(b) = \infty$ and we must independently solve two boundary value problems: for $0 < r < b$ and for $b < r < \infty$. Such potentials will be called impenetrable. If $\nu = 1$, we introduce some symmetric regularization, e.g. $v(r) \to v_\epsilon(r) = v(r)$ for $|r - b| > \epsilon$, $= v(\epsilon)$ for $0 < r - b < \epsilon$, $v(-\epsilon)$ for $-\epsilon < r - b < 0$. It can be proved that the solution $u_\epsilon(r)$ of this regularized problem has the limit $u(r) = \lim_{\epsilon \to 0} u_\epsilon(r)$ for all r, and $u(r)$ is the solution of Eq. (1*) satisfying the condition

$$[u'(b+\epsilon) - u'(b-\epsilon)] \to 0, \quad \epsilon \to 0.$$

This solution has acceptable physical properties in spite of the discontinuity of its derivative $(u'(b \pm \epsilon) \to \infty, \epsilon \to 0)$. We call this potential semi-penetrable.

The importance of FRS-potentials for physics lies in that they provide us with infinitely high barriers which can keep particles (say, quarks) inside some region of space. For impenetrable potentials there exists an infinite number of bound states lying on infinitely rising Regge trajectories. For penetrable potentials the bound states turn into resonances with finite widths.

4. Superpropagators and Potentials

The quantities Δ_F^n are not well-defined, and so a more rigorous definition of superpropagators is necessary. A simple approach to this problem is based on using a differential equation in the momentum space (see [8] where one can also find references to different approaches of Efimov, Fradkin, Volkov, Lehmann et al.). We give here the simplest version of our method.

Consider the function $(m = 0)$

$$F_1(x) = \sum_{n=0}^{\infty} c_{n+2} \; (-x)^n \, [n!(n+1)!]^{-1}, \quad x \equiv g^2 p^2 (16\pi^2)^{-1}, \qquad (14)$$

which up to some trivial factors and the term $\sim \delta(p^2)$ is the imaginary part of the Fourier transform of the superpropagator (8) on the cut $p^2 < 0$. Defining $\delta_x \equiv x\partial_x$, we see that

$$\delta_x(\delta_x + 1) F_1 \equiv -x \sum_{n=0}^{\infty} (c_{n+3}/c_{n+2})\, c_{n+2} (-x)^n [n!(n+1)!]^{-1}. \qquad (15)$$

Now, consider the ratio $R(n+1) \equiv c_{n+3}/c_{n+2}$ and suppose there exists a function $R(z+1)$ of the complex variable z, which satisfies the Carlson's conditions (see e.g. [7]) and coincides with $R(n+1)$ for $z = n$ $(n = 1,2,\ldots)$. Then the function is unique and the operator $R(\delta + 1)$ can be uniquely defined. In accordance with Eq. (14), $F_1(x)$ is the unique solution of the equation

$$[\delta_x(\delta_x + 1) + xR\;(\delta_x + 1)] F_1(x) = 0, \qquad (16)$$

satisfying the boundary condition $F_1(x) \underset{x \to 0}{\to} c_2$. This equation has also a solution $F_2(x)$ satisfying the boundary condition $xF_2(x) \underset{x \to 0}{\longrightarrow} 1$. The function $F_2(x)$ is analytic in the x-plane with the cut $-\infty < x < 0$, the discontinuity on the cut being $-2\pi i F_1(x)$. With the aid of dispersion relations we can find the superpropagator if we know its imaginary part and use some regularizator (see [9]). This method is mathematically rigorous but the superpropagator so defined depends on the regularizator used (or on an infinite number of subtraction constants). Our definition corresponds to a particular choice of the regularizator and depends on only one arbitrary real constant. In fact $F = F_2 + CF_1$ is also a solution of Eq. (16) with the same boundary condition.

For a wide class of superpropagators the condition

$$[Re\,F(x)/\,Im F(x)] \to 0, \quad x \to -\infty \qquad (17)$$

is fulfilled for some value of C. This is the so-called **condition of minimal singularity** in the form first proposed by present author [10]. Lehmann and Pohlmeyer [11] later used the essentially stronger condition $Re\,F(x) \underset{x \to -\infty}{\to} 0$ and proved that if it is satisfied the strictly localizable superpropagator is unique. We give here an extremely simple proof of this uniqueness theorem based only on momentum space representation of the superpropagator. Suppose there exist two strictly localizable superpropagators $F(x)$ and $F^{(2)}(x)$ satisfying all analytically conditions and having the same discontinuity on the cut. Then $D(x) \equiv F^{(1)}(x) - F^{(2)}(x)$ is an integer function of order $< \frac{1}{2}$ and $\lim_{x \to -\infty} D(x) = 0$. According to the theorem of Phragmen and Lindelöf [7], $D(x) \equiv 0$. We did not find an extension of this theorem for our condition (16). Nevertheless, we use this as the most natural and general formulation of the principle of minimal singularity.

By considering simple examples one can find that the superpropagators with $Re\,F/\,Im\,F \underset{x \to -\infty}{\longrightarrow} 0$ can be obtained by some analytic continuation in coupling constant g^2. Consider Eq. (15) with $g^2 \to -g^2 < 0$. Then there exists the solution $\bar{F}(-g^2, p^2)$, for which $\lim_{p^2 \to +\infty} \bar{F}(-g^2, p^2) = 0$ and $F(g^2, p^2) = \frac{1}{2}[\bar{F}(-g^2 e^{i\pi}, p^2) + \bar{F}(-g^2 e^{-i\pi}, p^2)]$ is the correct superpropagator satisfying the minimal singularity condition in the strong form.

Here we consider the case $\overline{\lim_{n \to \infty}} (c_n)^{1/n} = 1$. Then $F_1 \underset{p^2 \to -\infty}{\sim} p^a \exp(bp)$ and $Re\,F \underset{p^2 \to -\infty}{\to} 0$ if $a < 0$. The potential $\bar{v}(-g^2, r)$ (see Eq. (12)) has no singularities for $r > 0$ whereas $v(g^2, r)$ obtained by the analytic continuation in g^2 has a singularity for $r = b = \frac{g}{2\pi}$. The simple example is $c_n \equiv 1$, when

$$F(x) \equiv F(g^2, p^2) = -g^2 (16\pi x^{1/2})^{-1} N_1(2x^{1/2}), \quad v(g^2, r) = V.P.\, 4\pi (r^2 - g^2/4\pi^2)^{-1};$$

$$\bar{F}(x) \equiv F(-g^2, p^2) = g^2 (8\pi^2 x^{1/2})^{-1} K_1(2x^{1/2}), \quad \bar{v}(-g^2, r) = 4\pi (r^2 + g^2/4\pi^2)^{-1}; \qquad (18)$$

The same expression for $v(g^2, r)$ can be obtained by immediate Fourier transformating of $F(x)$.

5. Application of FRS-Potentials and their Connection with the Statistical Bootstrap Models

The differential equation (16) for the potential (superpropagator) (13) has the form

$$[\delta_x(\delta_x + 1)(\delta_x + 1) + x(\delta_x + \nu + 1)]\,F(x) = 0. \qquad (19)$$

Its solution satisfying (17) and $\sim x^{-1}$ for $x \to 0$ is $\sim G^{20}_{13}(x|^{-\nu}_{0,-1,-1})$. In the coordinate representation this gives us the FRS-potentials. The corresponding Schrödinger equation is much more tractable in comparison with momentum space equations. If the potential is close to impenetrable we may approximately substitute it by the infinitely deep square well potential with radius $\sim b$. In this approximation the resonances are substituted by bound states (resonances of zero width). The Regge trajectories of these bound states are asymptotically $\approx b k_{k \to \infty}$ and the radius $\langle r \rangle$ of bound states (proportional to the slope of the formfactor) is $\sim b$. For the exact (penetrable) potential the slope of the trajectories and $\langle r \rangle$ somewhat increase because of ''concentrating'' the wave functions near the sufrace $r = b$. For $\nu = 1$ there exist two exact solutions. The wave function for the potential $f^2(r^2 - b^2)^{-1}$ can be expressed in terms of the solutions of the hypergeometric equation if $k = 0$ and ℓ is arbitrary. The wave functions for $v(r) = [2b(r-b)]^{-1} = (r^2 - b^2)^{-1} + [2b(r+a)]^{-1}$ is expressed in terms of the Whittaker functions if $\ell = 0$ and k is arbitrary.

Using this simple results and approximations we have estimated some basic parameters of the system composed of two particles glued together by the semipenetrable potential $(\nu = 1)$. Only a brief exposition of qualitative results is given here.

i) The finite range of forces acting between hadrons is experimentally well established. So we suppose that $b \lesssim \mu_\pi^{-1}$. Then the slope of Regge trajectories has an upper bound, which for linear trajectories (in our model the linearity is only approximately valid for small s) is $\lesssim 1\ (\mathrm{Gev})^{-1}$. 2) As $\alpha(s = 0) \leq 1$, an upper bound on f exists. The exact value of this bound depends on masses of constituent particles. For example, with the potential $f^2(r^2-b^2)^{-1}$ we have the exact result $\alpha_{max}(k=0) = \frac{1}{2} f^2 - 1$. Suppose that the hadron is composed of two particles of mass M and that for small $s = 4(k^2+M^2)$ the trajectory is approximately linear, $\alpha(s) = \alpha(0) + \alpha' s$, where $\alpha' \sim 1$. Then, from the condition $\alpha(0) \leq 1$, we find $\frac{1}{2} f^2 \leq 2 + 4M^2$ and for $M \sim \frac{1}{3}$ we have $f^2 \lesssim 5$. 3) Using the approximation of the square well potential and supposing one of two constituent particle to be neutral it is not hard to calculate the non-relativistic form factor of the composite hadron $(q^2 = \mathbf{q}^2 - q_0^2 > 0)$

$$G(q^2) = (qb)^{-1}\{Si(qb) - \frac{1}{2} Si(\frac{1}{2} qb + \pi) - \frac{1}{2} Si(\frac{1}{2} qb - \pi)\}.$$

Then $dG/dq^2 \big|_{q^2=0} = -b^2/18$ and for the values of b estimated above, we have $\langle r \rangle \sim 0.8$ fermi. This expression, being nonrelativistic, cannot be used for $q^2 \to -\infty$. The asymptotic behaviour of formfactors for penetrable potentials is, however, defined by the fact that they are regular in $r = 0$. This property of our potentials enables us to use considerations of papers /12/ leading to rapidly decreasing elastic formfactors and to scaling for deep inelastic formfactors. 4) For $k^2 \to +\infty$ the two particle approximation is evidently not valid and so our estimates of asymptotic behaviour of Regge trajectories are not exact. However, the trajectories indefinitely rise and we may hope to obtain more reasonable results by considering a generalized two particle approximation: resonance $\longrightarrow \Sigma$(resonance + resonance). 5) To roughly estimate differential cross sections for scattering of composite particles we use the approximation

$$(d\sigma / dq^2)/(d\sigma / dq^2)_{q^2=0} \sim G^4(q^2).$$

Then, comparing the exponential parametrization for the formfactor, $G(q^2) \sim \exp(-Bq^2)$, with our approximate expression Eq. (20) one finds $B \sim 10$, in qualitative agreement with experimental data. For large q^2 rescattering effects are not negligible and such a rough estimate is incorrect. It should be noted that it is probably incorrect also for very small values of q^2, because the exact wave function does not vanish for $r > b$. This means that the slope of formfactor for very small q^2 may be somewhat different from one resulting from Eq. (20). 6) The principal features of our model, such as the finite size of hadrons $(\sim \mu_\pi^{-1})$ and the finiteness of the wave function for $r = 0$, were used in parton and droplet models and in the Hagedorn-Frautschi statistical bootstrap models (SBM) /13/. We have demonstrated above how these properties follow from certain field theories generating FRS-potentials. One may simply postulate some non-polynomial interaction of gluons ϕ, such as $\mathcal{L}_{int} = f{:}\bar{\psi}\psi \exp(g\phi^{+}\phi){:}$ and obtain the semipenetrable potential acting between constituent particles. Another interesting possibility is to deduce such potentials self-consistently from SBM. The rough idea of such an approach is as follows. In SBM the density of resonances is asymptotically of the form $\rho(m) \underset{m \to \infty}{\sim} m^a \exp(bm)$ /13/. This is true also for density of resonances with fixed Q,B and Y. The angular momentum distribution of resonances has the Gaussian form $\bar{\rho}(m, \ell) \sim \rho(m)\, m^{-3/2} \ell \exp(-\ell^2/dm)$, where $d \lesssim b$ /14/. Let us suppose that the forces between constituent particles are due to resonance exchanges. Then, neglecting the widths of resonances and assuming the contribution of the resonance of mass m and spin ℓ to be $(2\ell+1)P_\ell(\cos\theta)(t-m^2)^{-1}$ where $\cos\theta \sim (1+2s/t)$, $s<0$, $t \gg |s|$, one finds the contribution of all resonances of mass m $\left(P_\ell(\cos\theta) \sim \left(\frac{\theta}{\sin\theta}\right)^{1/2} J_0((\ell+1/2)\theta)\right)$:

$$\sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta)\bar{\rho}(m,\ell) = \frac{2}{\sqrt{\pi}}\, {}_1F_1\left(\frac{3}{2}; 1; -\frac{1}{4}\theta^2 dm\right)\rho(m) \simeq$$
$$\simeq 2\pi^{-1/2}\rho(m), \qquad \text{as} \qquad \theta^2 dm \ll 1.$$

According to Eq. (3) the potential corresponding to this amplitude is $V(r) \sim \int_\mu^\infty dm\, \rho(m) \exp(-mr)$. This integral is well-defined only for $r > b$ and the potential has a singularity for $r = b$. To more accurately define the potential we must use the methods described above, finding a superpropagator with the imaginary part $\sim \rho(p)$ and then constructing the corresponding potential. In such a way one can prove that the singularity of the potential corresponding to the spectral function $\rho(m)$ is of the form $\sim (r^2 - b^2)^\gamma$ where $\gamma = -\frac{5}{2} - a$, $a \leq -3/2$. To find the superpropagator (14) with the imaginary part $\sim p^a \exp(bp)$ it is sufficient to choose $c_n \sim n^{a+3/2}$.

In SBM the parameter $T_0 \equiv (kb)^{-1}$ is interpreted as the maximum temperature of hadrons. As was pointed in /13/ the value of kT_0 is of the same order as the radius of hadrons. As soon as the radius is introduced a priori, this result seems to be accidental. In our approach the coincidence of these two apparently different parameters is unavoidable.

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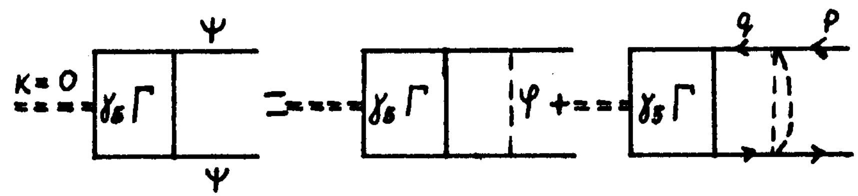

Fig. 1