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Z.Koba

MULTIPLICITY DISTRIBUTION OF PARTICLES PRODUCED IN VERY HIGH ENERGY HADRONIC COLLISIONS

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Z.Koba*

MULTIPLICITY DISTRIBUTION OF PARTICLES PRODUCED IN VERY HIGH ENERGY HADRONIC COLLISIONS

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* The Niels Bohr Institute, Copenhagen, Denmark.

Коба З.

E2 - 6918

Распределение по множественности частиц рожденных в адронных столкновениях при очень высокой энергии

Данная статья является обзором работ по инклюзивным адрон-адронным столкновениям при очень высоких энергиях, содержащим новейшие экспериментальные и теоретические исследования в этом актуальном направлении. Рассматриваются следующие вопросы: основные особенности столкновения адронов при высоких энергиях, топологические кросс-сечения, масштабная инвариантность.

Сообщение Объединенного института ядерных исследований
Дубна, 1973

Koba Z.

E2 - 6918

Multiplicity Distribution of Particles Produced in Very High Energy Hadronic Collisions

This paper reviews the works on inclusive hadron-hadron collisions at very high energy and containes the newest data on experimental and theoretical studies in this field. The following problems are considered: the main features of high energy collisons, topological cross sections, scale invariance.

Communications of the Joint Institute for Nuclear Research.
Dubna, 1973

In this talk I am going to discuss some global features of inelastic hadron-hadron collisions at very high energy, which can be obtained from one of the simplest and most direct results of the bubble chamber experiments, namely the multiplicity distributions of produced charged particles (topological cross sections). The talk will include:

1. Introduction. Main features of high energy hadron collisions.
2. What can be learned in general from the multiplicity distribution?
3. "Scaling property" of multiplicity distribution. (Work of Koba, Nielsen and Olesen).
4. Comparison with recent experimental data. (Work of Slattery).
5. Remarks.

1. Introduction. Main features of high energy hadron collisions x)

As the general background of our discussion, let me remind you of main features of very high energy proton-proton collisions. For other combinations of hadrons (e.g.,p-n, πN, KN, $\bar{N}N$,etc) the data available at present do not go up to so high energy as in p-p collision: but the general tendency is expected to be similar.

Figure 1 shows the p-p total cross section.This is taken

x) These remarks are meant for non-high-energy physicists and can be shipped. For more details, see, for instance, the most recent review [2].

from ref.1 presented at the Batavia conference, Sept. 1972. Although the new data above 100 GeV are not yet accurate, we find that they are compatible with the assumption of constant total cross section.

Out of this total cross section about 4/5 is due to inelastic processes which lead to particle production. The remaining 1/5 is the elastic cross section, but this is also regarded as the shadow of inelastic reactions. Thus study of particle production is quite essential for understanding the nature and properties of high energy hadron interactions.

There are a few points concerning the hadronic particle production process which are fairly well established empirically.

1) Average number of produced charged particles, $\langle n_{ch} \rangle$ increases slowly with the incident energy. See fig.2 (ref.1) . This is usually put in the form

$$\langle n_{ch} \rangle \sim a \log s \,, \tag{1.1}$$

where s is the c.m.s. energy squared, but the power dependence $\langle n \rangle \sim s^{k}$ with small k is not excluded, particularly at lower energy.

The number $\langle n \rangle$ is much smaller than the maximum value allowed by energy-momentum conservation. (For example, at 200 GeV/c, the total number produced[x] is ~ 12, while the maximum allowed number, which increases as $\sim s^{1/2}$, is

x) Here I have assumed that the number of neutral particles is approximately 1/2 of the charged particles.

ca 125). This means that at least some of the final state particles carry large kinetic energy in the c.m.s.

2) The distribution of transverse momentum of the final particles is limited to a small region, ca 0.4 GeV/c. This is independent of the incident energy, angle of emission, energy, multiplicity of secondaries and nearly independent of the kind of particles. Notice, however, that recent experiments at ISR have indicated the presence of high transverse momenta [3].

3) From the above two points it follows that some of the final particles must carry large longitudinal momentum. In fact, the spectrum of longitudinal momentum covers nearly the whole range of kinematically allowed values, which of course increases with incident energy. The spectrum depends on the kind of particles and on the energy . A simplification takes place, however, owing to the so-called "scaling" property, emphasized by Feynman and by Benecke, Chou, Yang and Yen but can be traced back to the work of Amati, Fubini, Stanghellini and Tonin [6] and that of Wilson [7].

To explain the "scaling" property, it is convenient to define the normalized inclusive cross section for an experiment $A + B \to C(\vec{p})$ + anything,

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{inclus.}}{d\vec{p}} = \varphi(\vec{p}; s) = \varphi(p_{\perp}, x; s) \,, \tag{1.2}$$

where we have used the notation x)

$$d\!\!\!/p = \frac{d^3\vec{p}}{\sqrt{\vec{p}^2+m^2}} \qquad , \quad x = \frac{p_{\parallel}^*}{\sqrt{s}/2} \qquad (1.3)$$

$p_{\parallel}^*$ being the longitudinal momentum in the c.m.s. Then the scaling hypothesis predicts the asymptotic behaviour

$$\mathcal{G}(p_\perp , x, s) \underset{s\to\infty}{\longrightarrow} \mathcal{G}(p_\perp , x)$$

This prediction is borne out in general. (The approach to the limit takes place in some oases already at relatively low energies, while in other cases the limit is not yet reached even at the highest accelerator energy available at present. We do not enter here into details).

4) Most of produced particles are pions. Kaons, hyperons and baryon pairs are much less. Notice, however, that ISR experiments near $x \approx 0$ show a remarkable increase of heavier particles.

2. What can be learned from the multiplicity distributions?

As mentioned befor3, the multiplioity distribution of charged particles (for example, see Fig3 taken from ref.1) is one of the first and most direct result from the bubble chamber experiment.

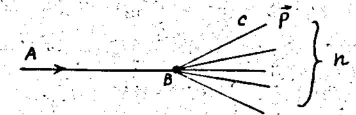
x) The notation x (Feynman's scaling variable) is a standard one, while notation $d\!\!\!/\vec{p}$ for invariant differential (introduced by de Groot) is not so generally used but is employed here for simplicity of writing down expressions.

For simplicity of writing down expressions, I shall here treat the case of a single kind of particles in the final state. But all the arguments can be applied (with slight modifications) to the actual cases of charged particle or negatively charged partbles.

Consider the inclusive cross section for

$$A + B \rightarrow C(\vec{p}) + \text{anything} \qquad (2.1)$$

This includes all the final states like Fig.4



$n = 1, 2, 3, \cdots$

Fig.4

When we integrate the inclusive cross section over the whole region of $\vec{p}$, we pick up each final particle, so that this event with n final particles is connected n times. Thus

$$\int d\!\!\!/p \, \frac{d\sigma_{inclus}}{d\!\!\!/p} = \sum n\sigma_n = \langle n \rangle \, \sigma_{tot} , \qquad (2.2)$$

where

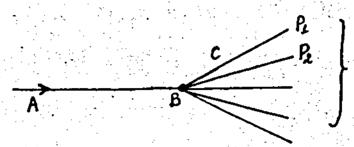
σ_n : cross section for producing n particles

$$\langle n \rangle = \frac{\sum n\sigma_n}{\sum \sigma_n} = \frac{1}{\sigma_{tot}} \sum n\sigma_n . \qquad (2.3)$$

So we conclude that the average multiplicity is the integral of the normalized single particle inclusive cross section

$$A + B \rightarrow C(p_1) + C(p_2) \quad + \text{anyting}$$

which includes final states as Fig.5



$n = 2, 3, \ldots$

Fig.5

When we integrate over $\vec{p}_1$ and $\vec{p}_2$ we pick up each pair in the final state twice, so that the above event with n final particles is counted $2 \cdot \binom{n}{2} = n(n-1)$ times. Thus

$$\frac{1}{\sigma_{tot}} \iint dp_1\, dp_2 \frac{d^2\sigma_{inclus}}{dp_1\, dp_2} = \frac{\sum n(n-1)\sigma_n}{\sum \sigma_n} = \langle n(n-1) \rangle \tag{2.4}$$

or

$$\langle n(n-1) \rangle = \iint dp_1\, dp_2\; \mathcal{S}_2(\vec{p}_1, \vec{p}_2; s), \tag{2.5}$$

where $\mathcal{S}_2(\vec{p}_1, \vec{p}_2; s)$ is the two-particle distribution function (i.e. normalized inclusive cross section). The figure 6 (ref.1) shows the energy- dependence of this quantity.

It is, however, often more interesting to consider 2-particle correlation function $\Psi_2(\vec{p}_1, \vec{p}_2; s)$ instead of the two-particle distribution itself [x]. It is defined by

x) I am using the notation which appears now to be more often in use, although we have been using a different set of letters in our works on many particle distributions and correlations.

$$\Psi_2(\vec{p}_1, \vec{p}_2; s) = \mathcal{S}_2(\vec{p}_1, \vec{p}_2; s) - \mathcal{S}_1(\vec{p}_1; s) \cdot \mathcal{S}_1(\vec{p}_2; s) \tag{2.6}$$

and extracts, so to say the essentially new information included in the two-particle distribution. Then we ask, after we have detected a particle of momentum $\vec{p}_1$, what is the chance of finding another particle with momentum $\vec{p}_2$. The answer is essentially given by

$$\frac{\mathcal{S}_2(\vec{p}_1, \vec{p}_2; s)}{\mathcal{S}_1(\vec{p}_1; s)}.$$

If the first detection of $\vec{p}_1$ does not influence the second one at all, the result will be the same as in the case of single particle detection, and we get

$$\frac{\mathcal{S}_2(\vec{p}_1, \vec{p}_2; s)}{\mathcal{S}_1(\vec{p}_1; s)} = \mathcal{S}_1(\vec{p}_2; s) \tag{2.7}$$

or

$$\Psi_2(\vec{p}_1, \vec{p}_2; s) = 0 \; . \tag{2.8}$$

But in general this will not be the case and the l.h.s. of (2.7) will be either larger (positive correlation)or smaller (negative correlation) than the r.h.s.

Now the integral of the two particle correlation function usually denoted by f_2 is easily obtained from (2.6), (2.5), (2.2).

$$f_2 \equiv \iint dp_1\, dp_2\, \Psi_2(p_1, p_2; s) = \langle n(n-1) \rangle - \langle n \rangle^2 =$$

$$= (\langle n^2 \rangle - \langle n \rangle^2) - \langle n \rangle = D^2 - \langle n \rangle \; , \tag{2.9}$$

where

$$D^2 = \langle (n-\langle n\rangle)^2\rangle = \langle n^2\rangle - \langle n\rangle^2 \qquad (2.10)$$

is the dispersion of the distribution. Notice that the Poisson distribution has the property $D = \sqrt{n}$ so that it leads to vanishing value of f_2.

When we compare $\langle n\rangle$ and $\langle n(n-1)\rangle$ shown in Fig.2 and Fig.6 we find that experimentally

$$\begin{array}{ll} f_2 < 0 & \text{below 30 GeV/c} \\ f_2 > 0 & \text{above 50 GeV/c} \end{array} \qquad (2.11)$$

This result can be interpreted as follows. At lower **energies kinematical correlations due to the energy-momentum conservation are predominant. (They are essentially negative because two** energetic particles going in the same direction are forbidden, for example). At higher energies positive correlations of non-kinematical origin (they may or may not be dynamical ones) appear and overwhelm the kinematical ones.

A further insight into the structure of the correlation function can be obtained if we take into account the kinematical constraint (sometimes called "sum rules") which it must satisfy because of energy-momentum conservation ^{x}. For the two-particle correlation function ψ_2, it can be written as

x) For the derivation, see, e.g., the review [8], where original references are given.

follows.

$$\iint dp_1\, dp_2 \left\{\sqrt{p_1^2+m^2} + \sqrt{p_2^2+m^2}\right\} \psi(\vec{p}_1,\vec{p}_2;s) = -2\sqrt{s} \qquad (2.12)$$

This integral is different from the definition of f_2, (2.9), only through a positive factor $\left\{\sqrt{p_1^2+m^2}+\sqrt{p_2^2+m^2}\right\}$ in the integrand. The high energy empirical result

$$f_2 = \iint dp_1\, dp_2\, \psi_2(\vec{p}_1,\vec{p}_2;s) > 0 \qquad (2.13)$$

can be made compatible with (2.12) in a natural and simple way by assuming that at $\geq$ 50 GeV

$$\begin{array}{ll} \psi_2(\vec{p}_1,\vec{p}_2;s) > 0 & \text{at } p_1^2 \sim p_2^2 \sim 0 \\ \psi_2(\vec{p}_1,\vec{p}_2;s) < 0 & \text{at } p_1^2, p_2^2 \gg m^2 \end{array} \qquad (2.14)$$

namely the 2-particle correlation is supposed to be positive in the central region and negative at boundary regions. (Such a behaviour is in reality verified at lower energy).

The asymptotic behaviour (at $s \to \infty$) of f_2 depends sensitively on types of models, so that it will be in principle able to distinguish between various models, all of which make similar predictions as far as the total cross sections and single particle distributions are concerned. See table 1 (from ref. 8). It should be remarked that if we take the integral of 2-particle inclusive cross-section, $\langle n(n-1)\rangle$ instead of correlation integral f_2, then we can distinguish only the last type (diffractive excitation model) from the rest, but not among the latter.

So far we have utilized only $\langle n\rangle$ and $\langle n^2\rangle$ and if the data are accurate we can certainly extract more infor-mation by means of $\langle n^3\rangle$ and still higher moments. One generalize the foregoing arguments and investigate the uti of 3- and more particle inclusive cross sections and correlati There exists a theoretical framework of generating function (and generating functional /11/) which allows a fairly conci and unified treatment of higher moments and many partic inclusive cross sections and correlations. I would also mention that this generating function has an interestin formal analogy to the grand partition function in stat tical mechanics. See for more details ref. /12/,/13/).

If we get more detailed experimental information, the multiplicity distribution in the subset of events whi one particle with a specified momentum $\vec{p}$ is detecte – this is called the associated multiplicity /14/ – then w can get partial integral of inclusive cross sections and correlations, which give us more detailed know-ledge of these functions /4,9,15/.

3. "Scaling property" of multiplicity distribution

Nielsen, Olesen and I made a speculation on w would be the limiting behaviour, if any, of the multiplicity distribution at the asymptotic energy region /16/. We made two strong assumptions (and other smaller ones, too) namely (i) the Feynman scaling is valid for all the many-particle inclusive cross sections, and (ii) quantities of the order (1/log s) can be

neglected compared to one. The outline of our original arguments is given in the appendix.

When we plot [x]

$$P(n;s) = \frac{\sigma_n}{\sum \sigma_n} \tag{3.1}$$

as a continuous function of n (i.e., we make an interpolation for non-integer n), for given s, we get for instance, curves like Fig.7.

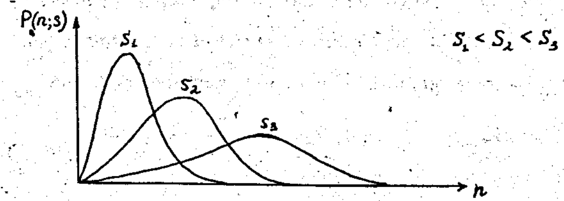


Fig.7

Each curve encloses unit area, because of normalization:

$$\int P(n;s)\,dn = 1 .$$

Now the oonslusion of our speculation is as follows. When we rescale each curve by multiplying the horizontal axis by $\frac{1}{\langle n\rangle}$ and the vertical axis by $\langle n\rangle$ thus maintaining the normalization, then at sufficiently high energy, the curves will coincide with each other, as schematized in Fig.8.

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x) In the summation $\sum \sigma_n$, the elastic cross section is usually not included, because the elastic scattering has a somewhat different property.

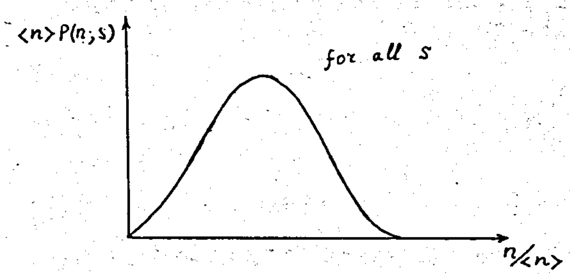

Fig.8

This asymptotic "scaling" behaviour includes, as a special case, the Poisson distribution or similar distribution predicted by the short range correlation models. In these models one has

$$\frac{D}{\langle n\rangle} \sim \frac{1}{\sqrt{\langle n\rangle}} \tag{3.2}$$

and, since $\langle n\rangle \to \infty$, the width of the limiting distribution in the rescaled plot becomes infinitely narrow, thus yielding

$$\delta\left(\frac{n}{\langle n\rangle} - 1\right)$$

in thelimit. Our "scaling" hypothesis is, however, more general and admits a limiting rescaled curve with a finite width which would correspond **to the presence of long range correlations.**

4. Comparison with recent experimental data

Slattery [7] has made a comparison of the above-mentioned "scaling" behaviour with experimental data of charged prong distribution of p-p collision at 19, 50, 69, 102, 205, 303 GeV/c and has found that except the low energy data at 19 GeV, the plots of $\langle n\rangle \frac{\sigma_n}{\Sigma \sigma_n}$ vs $n/\langle n\rangle$. are indeed very well represented by a single curve. See Fig.9 and Table 2 (taken from ref.17).

The universality of the curve for $\langle n\rangle \cdot P(n;s)$ vs $n/\langle n\rangle$ in the energy region 50-300 GeV is equivalent to the statement that

$$C_q \equiv \frac{\langle n^q\rangle}{\langle n\rangle^q}, \quad q = 2, 3, \ldots \tag{4.1}$$

are independent of energy in this region. The tables 3-4 (ref.17) show values of these parameters. As a special case the values of

$$\frac{\langle n\rangle}{D} = \frac{1}{(C_2 - 1)^{\frac{1}{2}}} \tag{4.2}$$

at varieus energies are shown in fig.10 (taken from ref.1). It starts from 2.2-2.3 at low energy and appears to become stable ~ 2 . above 50 GeV.

The remarkable stability of the parameter $\langle n\rangle/D$ even at lower energy was noticed already in 1970 by Czyzewski and Rybicki [18] and more recently a linear empirical relation of $\langle n\rangle$ and D was presented by Wroblewski.

5. Remarks

Thus we have seen that empirically the parameters

$$C_q = \frac{\langle n^q\rangle}{\langle n\rangle^q}, \quad q = 2, 3, \ldots$$

are within the present experimental accuracy energy-independent in the energy region 50-300 GeV for multiplicity distribution of charged prongs from p-p collision. Any realistic model of particle production has to be able to reproduce at least approximately this fairly remarkable relation.

As has been carefully discussed by Slattery [17], the data are still compatible with the short range correlation hypothesis[10]. See fig.11 taken from ref. 17. Therefore the empirical evidence is not yet conclusive for the early setting in of the limiting scaling behaviour. (In other words, the parameters c_q may again become energy dependent at still higher energies).

Nevertheless, it seems to us very attractive to assume that the "scaling" behaviour of the multiplicity distribution is real. Our original derivation (see Appendix) is based on two strong assumptions and may not be convincing in the energy region available at present. But this is only one of many possible **ways of derivating** "scaling" behaviour. Other approaches, other models which lead to the constant values of the parameters c_q are to be studied. ("Early scaling" seems to indicate that a certain factor in the production mechanism is already stable). Works along this line are going in Copenhagen [20].

Summary

After introductory remarks on main aspects of the high energy hadron collisions, I have firstly discussed some global features of the production process which can be immediately extracted from the multiplicity distribution of charged prongs (i.e., topological cross sections). The latter gives us namely information on normalized inclusive cross sections and correlations of charged particles integrated over the whole phase space. Experimental data show that the 2-particle correlation integral is negative at lower energy and becomes positive at higher energy.

With the help of kinematical constraint a possible behaviour of the 2-particle correlation function (i.e. positive in the central region, negative in the boundary region) is inferred. Further generalisation can be done to analysis of 3- and more-particle inclusive cross sections and correlations in terms of higher moments of multiplicity distribution.

Next I have discussed a more specific feature, the "scaling" behaviour of multiplicity distribution predicted by us for the asymptotic energy region. Slattery has made a detailed comparison with recent data and have shown that at 50-300 GeV/c p-p collision, the $\langle n\rangle \frac{\sigma_n}{\sigma_{inel}}$ vs $\frac{n}{\langle n\rangle}$ plot for the charged prongs lie indeed on a universal curve, or equivalently, $c_q \equiv \frac{\langle n^q\rangle}{\langle n\rangle^q}$ $(q=2,3,4..)$ are energy independent in this region. Any realistic model of particle production should be able to reproduce these relations at least approximately. Although the data are not yet conclusive it is attractive to assume the validity of "scaling of multiplicity distributions". Our original derivation is based on two main assumptions which are not very realistic in the present energy region. More realistic approach or models for "scaling" behaviour are desirable.

Acknowledgement. The content of this talk is largely based on the collaboration works with H.B.Nielsen and P.Olesen, to whom I owe much. I also thank P.Slattery for informing us of his analysis prior to publication. Last but not least, I express my hearty gratitude for the hospitality of JINR where this talk has been given and this manuscript has been written.

APPENDIX

Original derivation of scaling behaviour of multiplicity distribution

We assume

i) The Feynman scaling is valid for all the many particle inclusive cross sections.

ii) The energy is so high that terms of order $1/\log s$ can be neglected compared to 1.

It is convenient to introduce the rapidity variable y defined by

$$\sinh y = \frac{p_{\parallel}}{\sqrt{m^2+p_{\perp}^2}} \quad . \tag{A.1}$$

One of the nice properties of this variable is

$$d\tilde{p} = \frac{d^3p}{\sqrt{p^2+m^2}} = d^2p_{\perp}\frac{dp_{\parallel}}{\sqrt{p^2+m^2}} = d^2p_{\perp}\,dy \tag{A.2}$$

and the allowed region for y is, in the c.m.s. system

$$-\frac{Y}{2} \le y \le \frac{Y}{2} \quad , \quad Y \sim \log s \quad . \tag{A.3}$$

Integrating the normalized inclusive cross section over the transverse momentum and denoting the result by $\bar{\varphi}$.

$$\frac{1}{\sigma_{tot}}\int d^2p_{\perp}\,\frac{d\sigma_{inel}}{d\tilde{p}} = \bar{\varphi}\,(y;s) \tag{A.4}$$

we can express the Feynman scaling in the form

$$\langle n\rangle = \int_{-\frac{Y}{2}}^{+\frac{Y}{2}} dy\;\bar{\varphi}(y;s) = \bar{\varphi}(0)\,\log s + O\left(\frac{1}{\log s}\right) . \tag{A.5}$$

Notice that $\bar{\varphi}\,(y=0;\,s) = \bar{\varphi}\,(x=0)$

and by the assumption of Feynman scaling it does not depend on s .

Therefore the increase of the integral of inclusive cross section with the incident energy is, to the accuracy of neglecting $O\left(\frac{1}{\log s}\right)$, only due to the increase of the domain of integration in the rapidity variable, as illustrated in Fig.12.

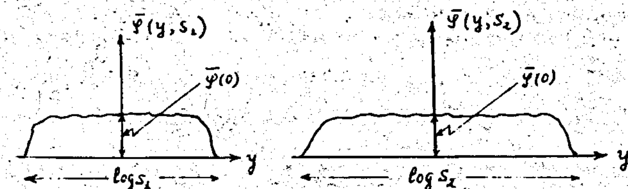

Fig.12

For 2-particle inclusive cross section, we can apply the same argument, and the Feynman scaling leads to

$$\langle n(n-1)\rangle = \iint \bar{\varphi}_2(y_1,y_2;s)\,dy_1\,dy_2 =$$

$$= \bar{\varphi}_2(0,0)\,(\log s)^2\left\{1 + O\left(\frac{1}{\log s}\right)\right\} . \tag{A.6}$$

Here the region of integration is essentially $(\log s)^2$ and $\bar{\varphi}_2(0,0)$ is energy independent. Since we know from (A.5) that $\langle n\rangle$ is of the order $\log s$, (A.6) can be rewritten as

$$\langle n^2\rangle = \bar{\varphi}(0,0)(\log s)^2\left\{1+O\left(\frac{1}{\log s}\right)\right\} \tag{A.7}$$

(A.7) and (A.8) give

$$C_2 \equiv \frac{\langle n^2\rangle}{\langle n\rangle^2} = \frac{\bar{\varphi}_2(0,0)}{[\bar{\varphi}_1(0)]^2}\cdot\left\{1+O\left(\frac{1}{\log s}\right)\right\} \to \frac{\bar{\varphi}_2(0,0)}{[\bar{\varphi}_1(0)]^2} \tag{A.8}$$

We can repeat the same reasoning and obtain more generally

$$C_q \equiv \frac{\langle n^q\rangle}{\langle n\rangle^q} = \frac{\bar{\varphi}_q(0,0,\ldots,0)}{[\bar{\varphi}_1(0)]^q}\left\{1+O\left((\log s)^{-1}\right)\right\} \to \frac{\bar{\varphi}_q(0,0,\ldots,0)}{[\bar{\varphi}_1(0)]^q}\,. \tag{A.9}$$

The r.h.s. is energy independent.

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Table 1 (Ref.8)

Asymptotic behaviour of two-particle correlation integral in various models

Model	$f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$
Uncorrelated jet model (with $p_\perp$ cut off)	$\sim A$ (const) $(A < 0)$
Short range correlation models Multiperipheral model Dual resonance model (tree approx.)	$\sim B \log s$ $\left(\begin{array}{c} B>0 \\ \text{or} \\ B<0 \end{array}\right)$
Multiperipheral model with absorption (Caneschi-Schwimmer) Diffraction and pionization Non-equilibrium model	$\sim c(\log s)^2$ $(c>0)$
Diffractive excitation model (Limiting fragmentation model)	$\sim D s^{\frac{1}{2}}$ $(D>0)$

Summary of Data
pp→n charged particles

$\langle n \rangle \quad \frac{\sigma_n}{\sigma_{inel}}$

n	19 GeV/c [a]	50 GeV/c	69 GeV/c	102 GeV/c	205 GeV/c	303 GeV/c
2	1.227 ±0.020	1.08 ±0.11	0.940 ±0.068	0.88 ±0.10	0.82 ±0.17	0.50 ±0.14
4	1.783 ±0.027	1.583 ±0.090	1.611 ±0.048	1.589 ±0.091	1.299 ±0.079	1.348 ±0.076
6	0.789 ±0.021	1.345 ±0.084	1.476 ±0.046	1.489 ±0.096	1.622 ±0.096	1.589 ±0.090
8	0.186 ±0.010	0.848 ±0.069	1.013 ±0.038	1.140 ±0.087	1.354 ±0.085	1.504 ±0.086
10	0.0314 ±0.0039	0.344 ±0.037	0.513 ±0.024	0.692 ±0.066	1.031 ±0.074	1.315 ±0.084
12	0.00150 ±0.00088	0.081 ±0.017	0.238 ±0.015	0.399 ±0.054	0.801 ±0.064	1.168 ±0.083
14	0.00050 ±0.00050	0.036 ±0.011	0.0740 ±0.0079	0.137 ±0.030	0.397 ±0.039	0.605 ±0.058
16		0.0032 ±0.0032	0.0198 ±0.0039	0.037 ±0.016	0.204 ±0.026	0.388 ±0.045
18			0.0023 ±0.0013	0.019 ±0.011	0.071 ±0.016	0.241 ±0.035
20					0.042 ±0.012	0.142 ±0.026
22					0.0129 ±0.0066	0.0189 ±0.0092
24						0.028 ±0.011
26						0.0142 ±0.0085
$\langle n \rangle$	4.038 ±0.022	5.32 ±0.11	5.888 ±0.066	6.38 ±0.12	7.65 ±0.16	8.86 ±0.15

[a] Not plotted in Figure 7.

Table 2 (Ref. 17).

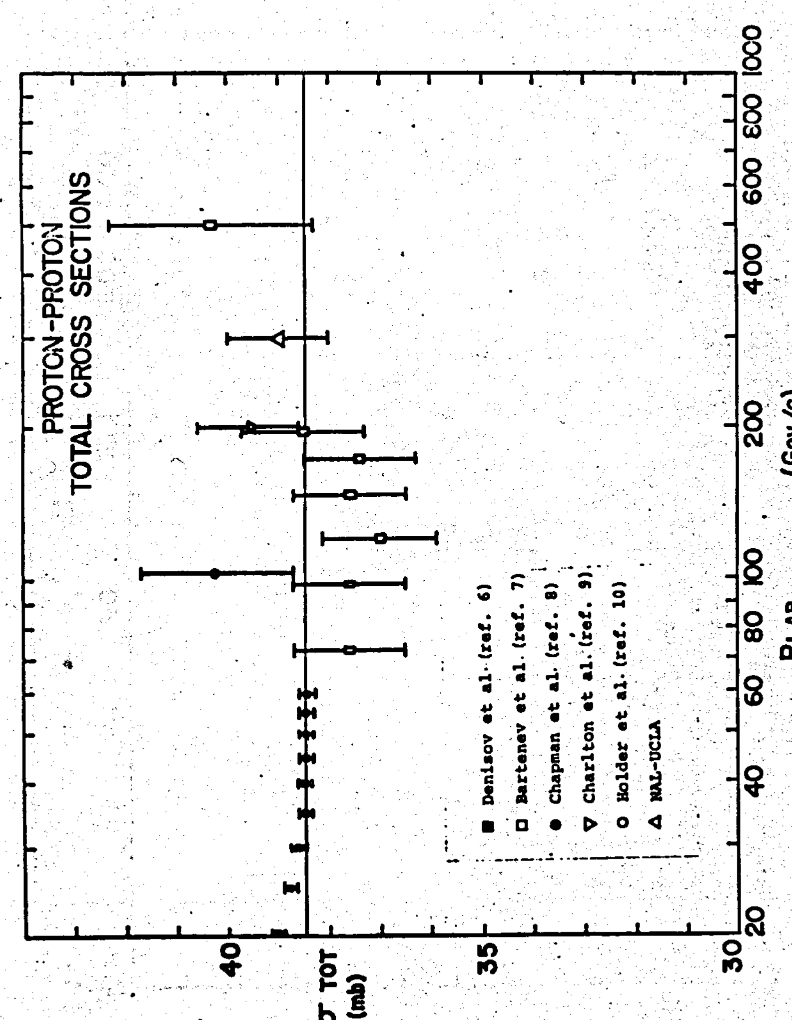

Fig. 1. (Ref. 1)

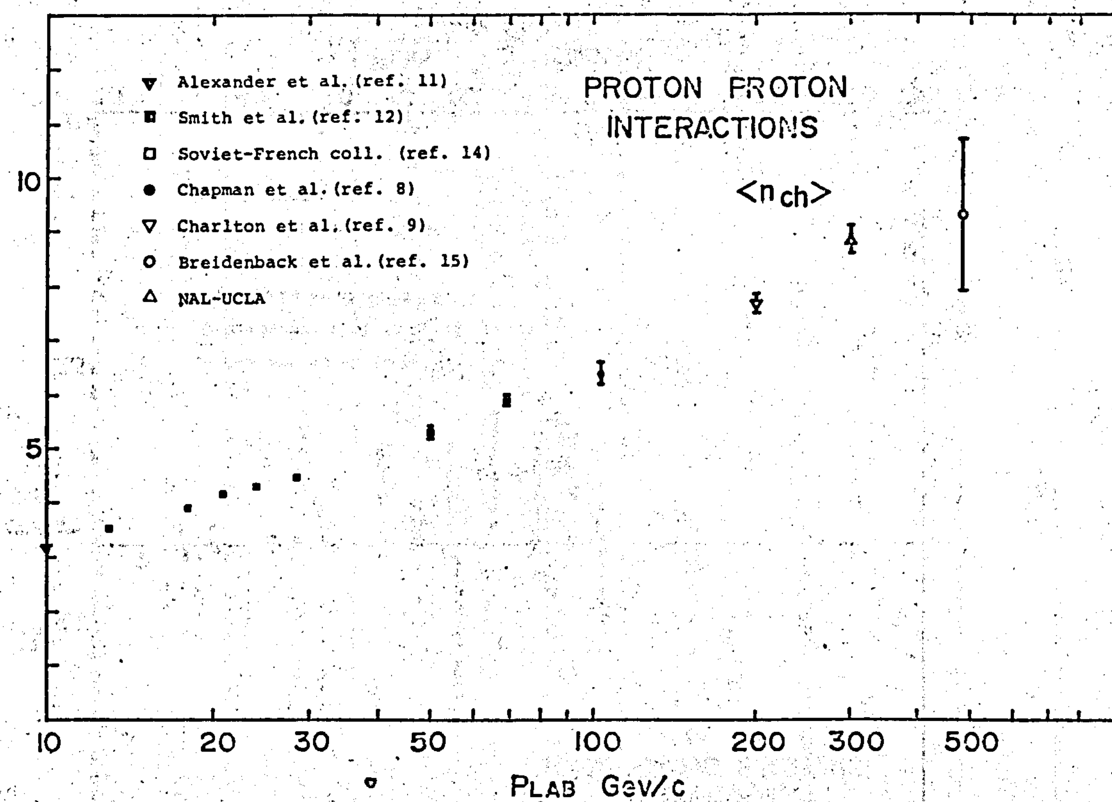

Fig. 2. (Ref. 1).

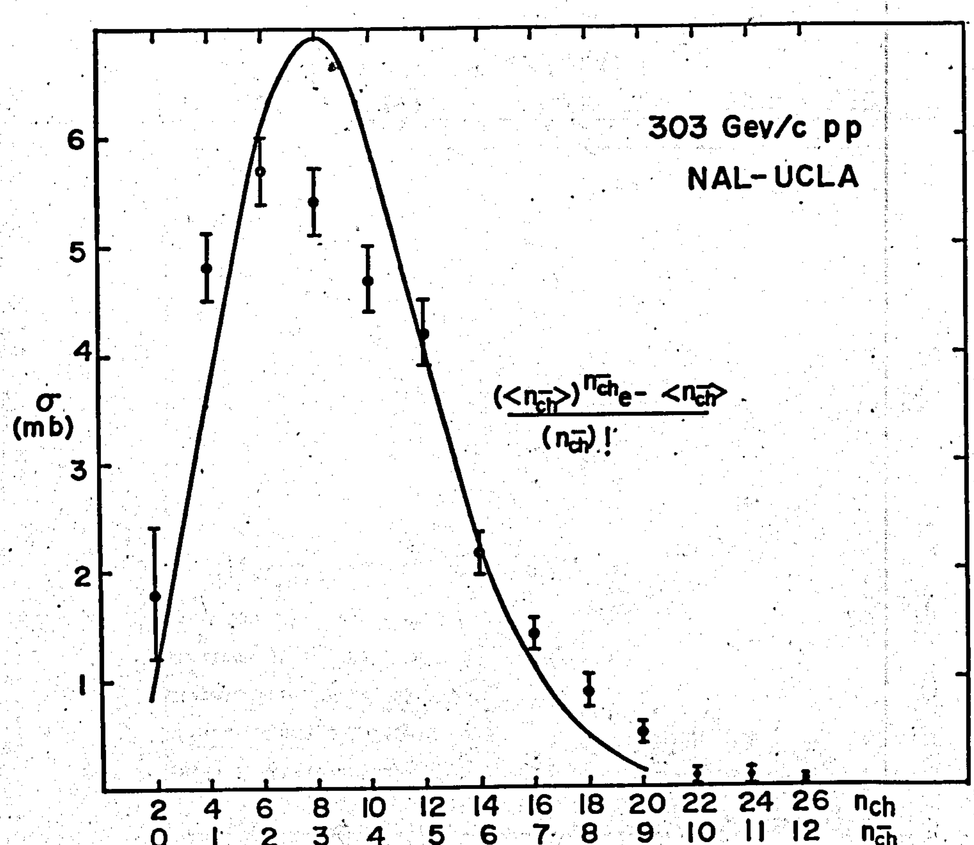

Fig. 3. (Ref. 1).

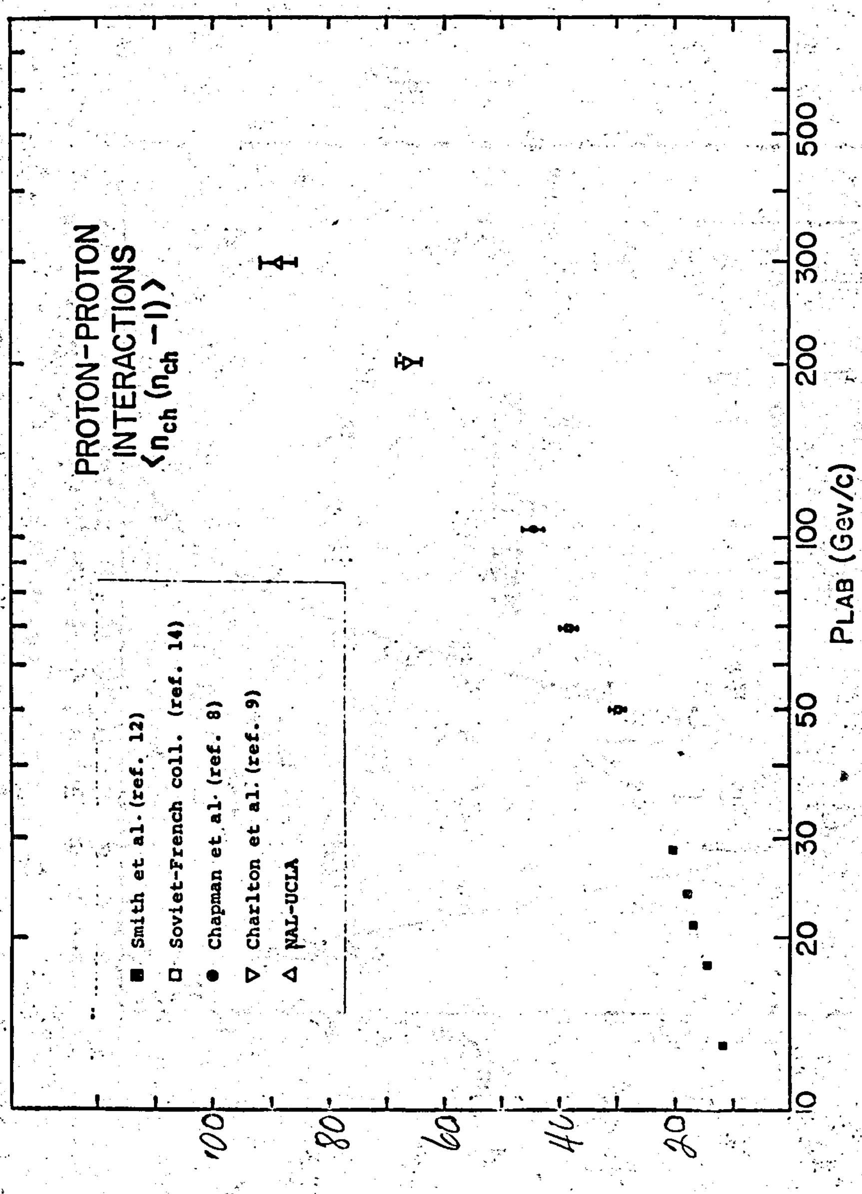

Fig. 6. (Ref. 1).

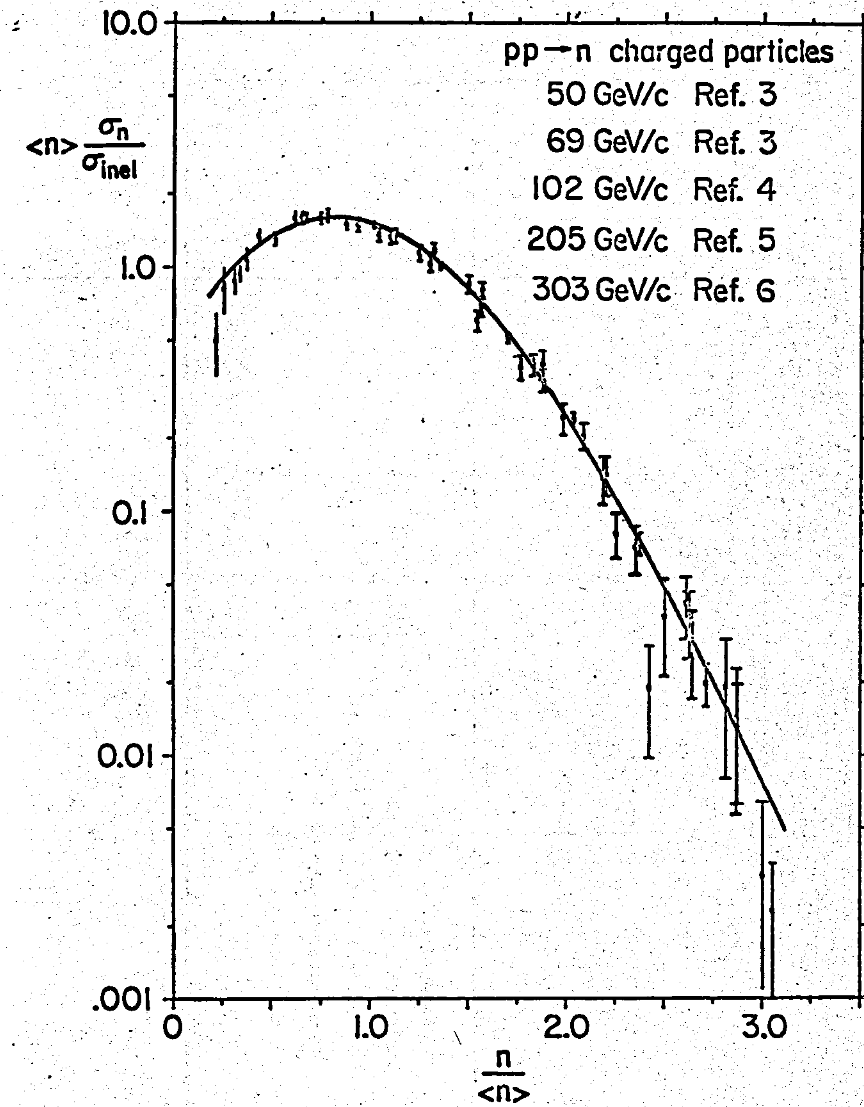

Fig. 9. (Ref. 17).

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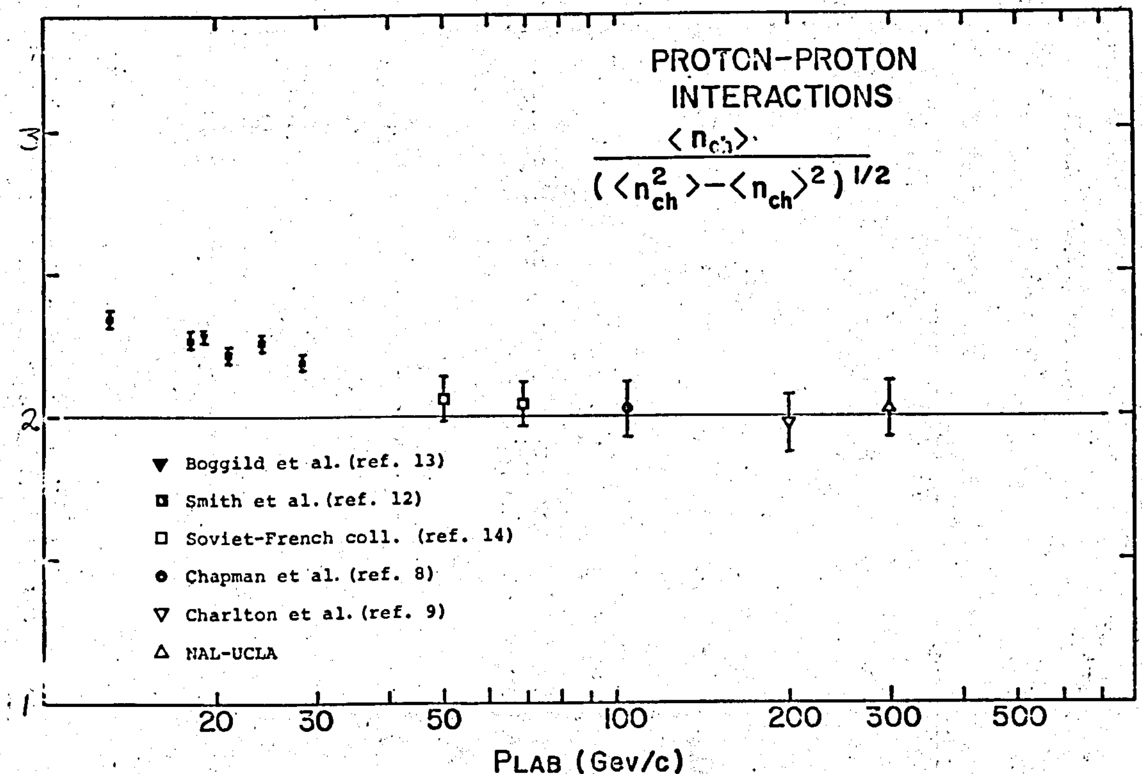

Fig. 10. (Ref. 1).

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Mueller Correlation Functions

pp → n charged particles

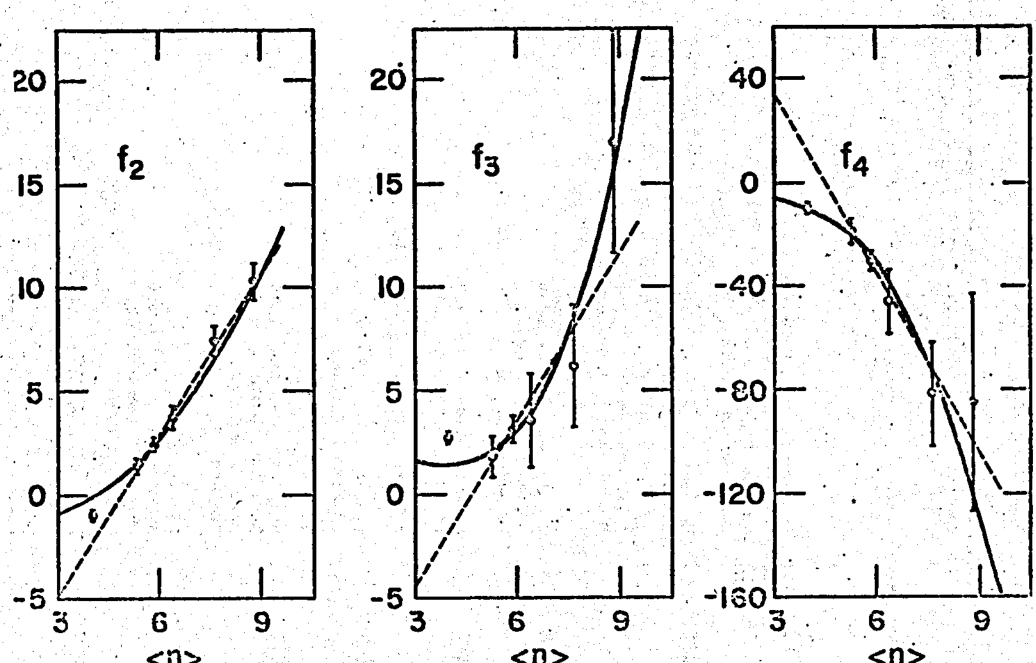

Fig. 11. (Ref. 17).