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AMBIGUITIES IN π^4He PHASE SHIFT ANALYSIS

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Pion scattering on ^4He is a very good test for basic problems which appear in the simplest case of the scattering of zero spin on zero spin. One of the most interesting problems is the construction of the scattering amplitude from exact knowledge of the differential cross section and the total cross section at a given energy, which in fact, is a nontrivial problem.

Ambiguities in the case of elastic spin-zero scattering were first considered by Crichton /1/ in a numerical example.

The existence and the uniqueness theorems for the solutions of the nonlinear integral equation for the phase of the scattering amplitude were obtained by Newton /2/ and Martin /3/.

Some improvements of the Newton-Martin results were made together with the treatment of the case of the inelastic spin-zero scattering by Atkinson et al. /4/. Above the inelastic threshold there is a "continuum ambiguity", a fact that has been illustrated by Bowcock and Hodgson /5/ in a specifical example.

On the other hand, the ambiguities in the complex phase shift analysis (inelastic scattering) with a truncated series for amplitude were treated by Gersten /6/ in a different way. For the case of the scattering of spinless particles he has found 2^L nontrivial sets of complex phase shifts (L is maximal partial wave), which correspond to the same number of the partial waves (note that in the Gersten's treatment there is a finite number of the partial waves). The scattering amplitude is given by

$$f(x) = f(\ell) \prod_{i=1}^{L} (x - x_i)/(1 - x_i), \tag{1}$$

$$f(x) = \frac{1}{k} \prod_{\ell=0}^{L} (2\ell+1) P_\ell(x)(\eta_\ell e^{2i\delta_\ell} - 1)/2i, \tag{1'}$$

where $x = \cos\theta$. Each set of the phase shifts was found by replacement of one or more zeros (x_i) of the amplitude by their complex conjugates. For instance for Chrichton example we have found two zeros: x_1 and x_2 with the first set of S,P,D waves and x_1^* (complex conjugates of x_1) and the same x_2 with the second set of phase shifts.

There is another ambiguity (trivial) which corresponds to

$$\begin{matrix} \delta_\ell \to -\delta_\ell \\ \eta_\ell \to \eta_\ell \end{matrix} \quad \text{or} \quad f(x) \to -f^*(x) \tag{2}$$

and therefore there are 2^{L+1} solutions.

By checking the unitarity condition one can eliminate the unacceptable sets and in fact the real number of the different solutions which correspond to the same physical observable will be $\leq 2^{L+1}$.

In the practical case of the $\pi^{\pm}$ scattering on ^4He for one set of physical observables only two sets of the phase shifts were found /7/. With one solution for the phase shifts we can find a set of zeros of the complex amplitude (in the complex $\cos\theta$ plane)

$$f(x)=\frac{1}{k}\sum_{\ell=0}^{L}(2\ell+1)T_{\ell}P_{\ell}(x)=\sum_{i=0}^{L}a_i\cos^i(\theta) \qquad (3)$$

and with (1) we can find another 2^L-1 amplitudes from different combinations of zeros and their complex conjugates. For $\pi^{\pm}\,{}^4He$ elastic scattering Coulomb interference has a strong influence and therefore the trivial ambiguity (2) is simple to remove. In all these calculations the unitarity conditions were imposed. With these sets of phase shifts the experimental data were fitted once more.

The final result is that the only two solutions are compatible with the input conditions. The difference between them is such that the first zero of one of the solutions is roughly the complex conjugated of the first zero of the other solution. For both solutions the second zero is roughly in the same position

$$x_2 \approx 0.27 - i\,0.18 \qquad (4)$$

This stable zero for which $Im x_2$ is not too large is a consequence of the dip in the differential cross section around $\theta \approx 74^o$.

The scattering amplitude from Block, Crowe, and Motterhead /8/ phase shift analysis has the first zero in the lower half plane ($Im x_1 < 0$) and in our work /7b/ the first zero is in the upper half plane ($Im x_1 > 0$). From the phase-shift point of view the large difference between solution I (with $Im x_1 > 0$) and solution II (with $Im x_1 < 0$) is in the S and P waves for the kinetic energy larger than 60 MeV. (See table 1 for example of 51.3MeV and 67.6 MeV - Crowe experiment).

It is possible to resolve this kind of ambiguity by using the shortest path method for zeros and obtaing continuous trajectories of zeros *.

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* See Ref. /9/ for an example of such kind of analysis for K^+P scattering.

Figure 1 shows the $Im f(x)$ and $Re f(x)$ for solution I and II for 59.7 MeV (Crowe experiment). In physical region the real part of scattering amplitude is practically the same. $Im f(x)$ for solution I and II have the same optical point ($x=1$) but large difference appears in backward direction. The backward dispersion relation calculations may help to discriminate this ambiguity, but for that purposes is necessary, of course, a good set of experimental data at 180^o .

Unfortunately, the continuity in zeros it is impossible to do at present because the total cross section is not known well enough and this experimental information turns out to be important in such kind of analysis.

Following the same procedure for finding the zeros of the scattering amplitude in the complex $\cos\theta$ plane (via phase shift analysis), but with different imput for the total cross section, a dependence of these zeros upon total cross section was found.

Figure 2 shows the dependence of the first zero for each solution upon the total cross section (these results are for the average between π^+ and π^- differential cross section of the Crowe experiment). All zeros are inside an ellipse with foci -1 and +1, and semi-major axix $X_0 = 1+4m_\pi^2/(2k)^2$ (Lehmann-Martin ellipse where the scattering amplitude is analytic) but for 51.3 MeV only for $\sigma_{tot} > 60$ mb.

The second zero has a high stability upon the total cross section and for all energies /7b/ is inside the circle:

$$|x-x_2| < 0.05. \qquad (5)$$

The zeros which are independent of total cross section are obtained from a fit of experimental data with eq. 1 for scattering amplitude (Fig. 2). It is impossible to use the shortest path method for this kind of zeros (the first one) because they are exact complex conjugates to each other (the second zero is inside the circle (5)). In the same figure is shown the first zero of scattering amplitude for 97, 110, 153 MeV calculated from phase shift analysis /7b/. In table 2 we present like an example two sets of phase shifts for each solution and for different total cross sections (75 MeV).

For very low energy - 24 MeV, the Nordberg and Kinsey experiment /10/, only the S and P waves are enough. The zero of this amplitude is situated in 0.37$-i$0.35 stable upon total cross section. Additional D wave introduces another zero with high sensibility to the total cross section but very far from the convergence ellipse. (This is the so-called "statistical zero" /12/).

But even with only one zero for scattering amplitude (S and P waves) there is another important quantity which is sensitive to the total cross section - the scattering length. Figure 3 shows this dependence for the average between $\pi+$ and π^- differential cross section. The arrows in fig. 3 indicate the value of scattering length calculated from Norberg and Kinsey phase shift analysis and used in new forward dispersion relation for π ^4He /11/.

In conclusion, in the practical case for analysis of scattering of zero spin on zero spin ($\pi^\pm\, ^4He - \pi^\pm\, ^4He$) with a finite number of partial waves and taking into account unitarity conditions and the total cross section (and of course χ^2 criterium) we obtain only two solutions for phase shifts.

The preliminary phase shift analysis for π ^4He elastic scattering /7b/ has shown that with the "chain method" and $x = \sigma_{el}/\sigma_{tot}$ ratio constraint, the first solution is preferable at least from 60 MeV up to 153 MeV. For the low energy interval (<60 MeV) the difference in phase shifts between solution I and II is smaller than at larger energy and therefore is simple to confuse the first solution with the second one.

The total cross section information is very important (even at very low energy) for a good continuity of zeros trajectories and therefore for removing ambiguities in the phase shifts.

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TABLE 1

	T - 51.3 MeV SOLUTION I	T - 51.3 MeV SOLUTION II	T - 67.6 MeV SOLUTION I	T - 67.6 MeV SOLUTION II
δ_0	-7.87 ± 0.09	-8.31 ± 0.12	-11.48 ± 0.69	-7.94 ± 0.13
η_0	0.919 ± 0.007	0.837 ± 0.03	0.733 ± 0.022	1.00 ± 0.05
δ_1	8.84 ± 0.06	8.43 ± 0.06	11.19 ± 0.32	12.65 ± 0.09
η_1	0.948 ± 0.005	1.00 ± 0.09	1.00 ± 0.09	0.816 ± 0.04
δ_2	1.03 ± 0.04	0.98 ± 0.04	1.83 ± 0.06	2.24 ± 0.06
η_2	1.00 ± 0.04	0.984 ± 0.02	0.940 ± 0.014	1.00 ± 0.02
σ_{el}	28.8 mb	28.6 mb	39.3 mb	39.3 mb
σ_{TOTAL}	65.3 mb	65.2 mb	99.2 mb	97.6 mb
X_1	-3.285 + i·1.153	-3.530 - i·0.821	-1.765 + i·1.209	-2.117 - i·1.087
X_2	0.309 - i·0.198	0.308 - i·0.198	0.284 - i·0.183	0.282 - i·0.184
χ^2	23.081	23.043	24.748	25.490

TABLE 2

T - 75.0 MeV

	SOLUTION I	SOLUTION II	SOLUTION I	SOLUTION II
δ_0	-11.58 ± 0.26	-8.79 ± 0.17	-14.70 ± 0.28	-8.69 ± 0.23
η_0	0.715 ± 0.008	0.927 ± 0.014	0.662 ± 0.028	1.00 ± 0.02
δ_1	13.29 ± 0.11	14.68 ± 0.06	12.21 ± 0.28	14.23 ± 0.12
η_1	1.00 ± 0.03	0.859 ± 0.016	0.979 ± 0.016	0.739 ± 0.005
δ_2	2.46 ± 0.04	2.76 ± 0.06	2.29 ± 0.12	2.72 ± 0.08
η_2	0.952 ± 0.005	1.00 ± 0.016	0.899 ± 0.005	0.977 ± 0.004
σ_{el}	45.1 mb	45.1 mb	45.8 mb	45.1 mb
σ_{TOTAL}	94.3 mb	92.8 mb	130.0 mb	126.7 mb
X_1	-2.072 + i·0.806	-2.092 - i·0.784	-1.297 + i·1.059	-2.084 - i·0.785
X_2	0.291 - i·0.188	0.271 - i·0.187	0.279 - i·0.190	0.271 - i·0.187
χ^2	70.219	69.839	44.513	70.138

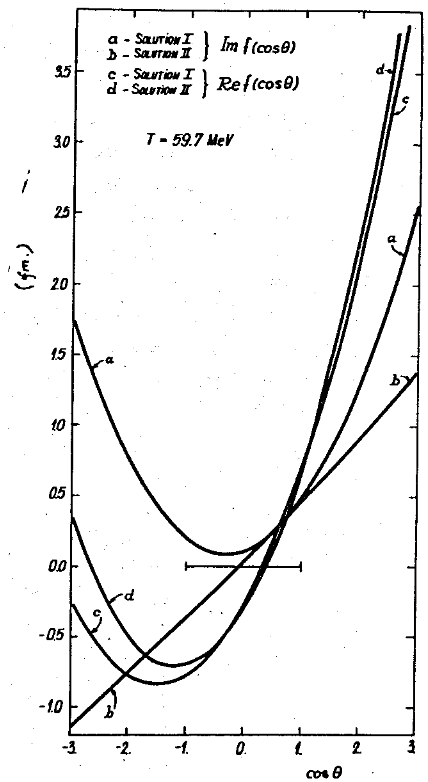

Fig. 1. $Imf(x)$ and $Ref(x)$ vs. $x = \cos\theta$ for solution I and II (59.7MeV Crowe experiment).

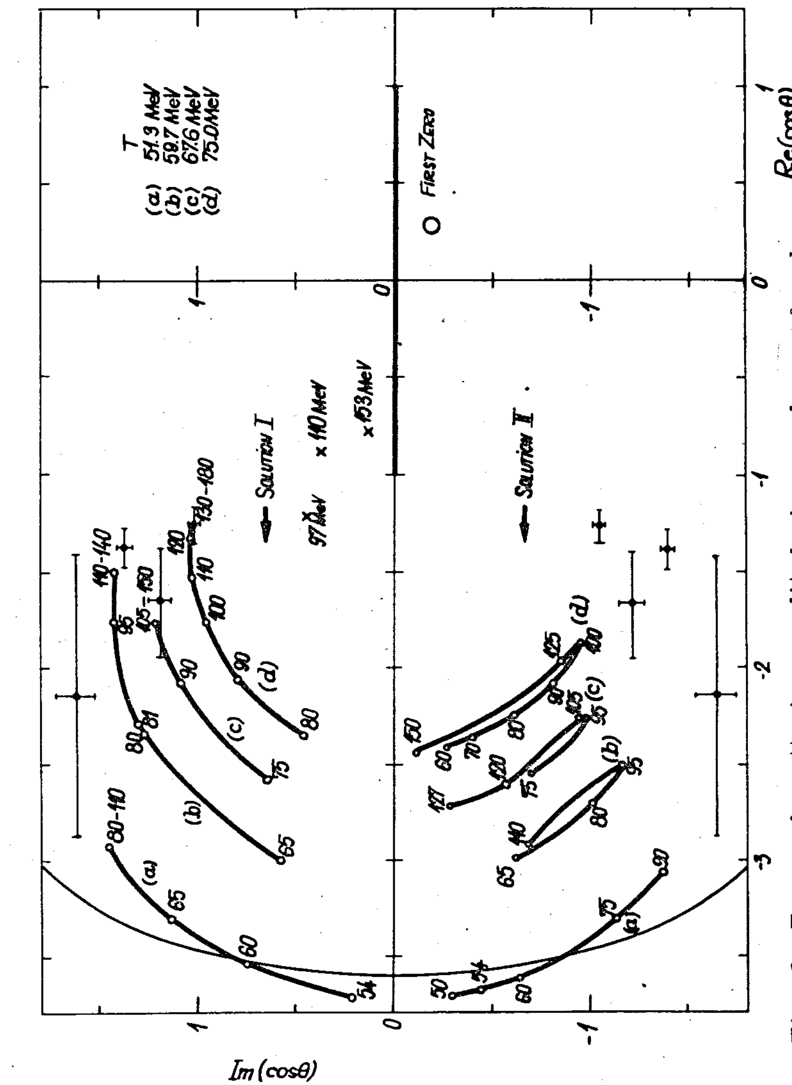

Fig. 2. Zeros of scattering amplitude in complex $\cos\theta$ plane for solution I and II (from phase shift analysis) o (a) 51.3 MeV, (b) 59.7 MeV, (c) 67.6 MeV, (d) 75.0 MeV; ● zeros which are independent of total cross section (eq. 1); ✗ zeros for 97, 110 and 153 MeV - from preliminary phase shift analysis for $\pi\,{}^4He$ elastic scattering /7b/. The line is a part of Lehmann-Martin ellipse for 51.3 MeV.

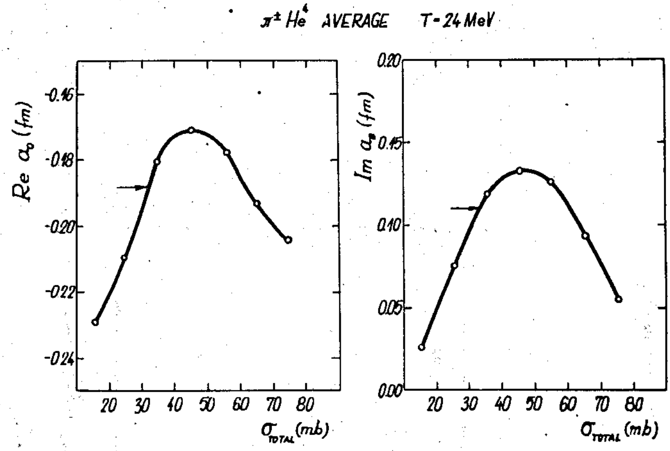

Fig. 3. Real and imaginary part of scattering length calculated for different total cross section (24 MeV Nordberg and Kinsey experiment).