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GLOBAL PROPERTIES OF COLLAPSING MATTER (Black Holes)

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S U M M A R Y

In the light of recent theoretical studies it has been found that in the process of collapse matter tends to minimize the variety of its global properties.

Of much importance are the considerations according to which, in the external space, black holes do not excite some fields (e.g., scalar, massive vector, neutrino fields). However, the electromagnetic fields are excited if the photon does not possess any small rest mass.

One of the criteria of the existence of fields outside black holes, namely the incompatibility of the presence of the sources of a given field (global nonzero "charge") with the possibility of formation of a closed metric, is considered.

For example, the presence of the total electric charge, the source of the generalized Maxwell field with potential of the type $\frac{1}{r^{l(l+1)}}$, $l>1$; and, in particular, $\sim 1/r^5$ and the source of the Yang-Mills field are incompatible with the formation of the closed metric.

There are some grounds to expect that the long-range neutrino forces would also be incompatible with the closed metric. Then this would mean that similar external fields of black holes should exist.

The sources of scalar and massive vector fields are incompatible with the closed metric.

However the inverse theorem is wrong: the possible formation of a closed metric in the presence of sources cannot yet testify in a decisive manner in favour of the fact that black holes have no external long-range scalar and vector fields.

The presently available proofs for the disappearance of the fields outside black holes are given which assume, in particular, the existence of the event horizon and the finiteness of the potentials and fields on the horizon. It is also assumed that the fields influence weakly the metrics.

However the long-range scalar field, e.g., in the form $\Box \varphi + \frac{1}{6} R \varphi = j$ is in contradiction with this assumption. There are some arguments in favour of the existence of the external massive vector (baryon) fields of black holes. These arguments are associated with the generalized Gauss theorem for the massive vector field.

A rigorous consideration of the problem under discussion requires however that the nonstatic complete (internal and external) solution should be found for black holes. This consideration would then result in the fact that the situation with the presence of horizons, bare singularities and the bahavior of fields outside and inside black holes would be clear.

In the light of recent theoretical studies, one has gathered the impression that in the process of collapse, in the process of gravitational closing, matter tends to minimize the variety of its glôbal properties, the variety of the parameters characterizing the system as a whole. Indeed, many global characteristics are stripped in the process of collapse when a matter falls below the surface of the "event horizon" forming a "black hole". In this way, outside the black hole (collapsar) there is observed the lose of the magnetic dipole moment, of the highest gravitational multipoles and, possibly, the capability of exciting some external fields, etc.

This situation was figuratively defined by Wheeler as follows. "A black hole has no hair". It is interesting to understand which properties of systems may be denoted by this term "hair", how proceeds the disappearance of this hair in the space around the blak hole, in which cases, in the language of this terminology, this hair comes out (i.e.,something is lost by a system in the course of radiation before gravitational closing), in which cases this hair is "dressed", say, à la Schwarzschild sphere and becomes inaccesible for an exterior observer.

A microscopic material system, for example, a celestial body, may possess a variety of global characteristics such as total mass, total electric charge, total angular momentum,etc. A celestial body consisting of, e.g., hydrogen gas has huge baryon and lepton charges.The system may also possess strangeness. From the point of view of the electron-neutrino weak interactions, a celestial body may be a source of a neutrino-antineutrino field decreasing as $1/r^5$. In principle, the macroscopic material system may be a source of a scalar field. It may possess magnetic dipole moment, highest gravitational momenta, etc. The stripping of the global characteristics, this peculiar "gravitational striptease", can go much far.

There exist such final states of material systems that are deprived of all their global characteristics; here we imply systems with closed metric for which, in particular, total mass*, total angular momentum and total electric charge are zero. The black holes and the systems with closed metric are two extreme and not transforming to each other (at least in classical physics) states of systems with minimized characteristics. As will be seen, the discussion of the global characteristics which violate the metric of closed systems and comes with it in contradiction/2/ is very valuable for understanding gravitational striptease in the production of black holes. We recall that the studies of Ginzburg/3/ (1964) and Ozernoj/4/ showed that the magnetic dipole moment measured at a certain distance from the collapsar tends to zero as the collapsar matter falls below the surface of the event horizon in the process of gravitational closing, i.e. when the star surface approaches the Schwarzschild surface**. Dorosh-

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*It is supposed that the so-called Λ-term in the Einstein equations is absent.

**In the process of gravitational collapse, when the surface of a star is approaching the Schwarzschild surface ("event horizon"), there also proceeds a stripping of free electrons down to a minimal value n : $\frac{\varkappa M m_e}{r_{gr}} \sim \frac{n e^2}{r_{gr}}$. That is, for a critical mass $M \sim M_{\odot}$, $n \sim \varkappa M_{\odot} m_e / e^2 \sim 10^{17}$. In other words, there remains one electron per 10^{16} gr $\sim 10^{10}$ ton of matter. The matter density is, in this case, $\mu \sim 10^{18}$ gr/cm^3.

kevich, Zel'dovich and Novikov[5] (1965) showed that collapsars do not possess highest gravitational multipoles: they are irradiated in the process of collapse. (In this case the hair comes too).

Price[4] (1971) et al.[17,18] concluded that the matter carrying long-range scalar field sources which has fallen below the Schwarzschild sphere induces no scalar field outside the black hole. This case is not associated with any radiation: the sources of the scalar field are hid in a black hole. Hartle[7,8], concludes that the collapsar too, exerts no neutrino forces in the exterior space. He gives the expression for the potential of neutrino forces from sources localized near the collapsar[7] at the point a

$$V \sim \frac{G}{r^5}\left(1-\frac{M}{a}\right)+\dots \tag{1}$$

If the place of localization of the neutrino field sources approaches the Schwarzschild surface ($a \to M$), the expression (1) is equal to zero. The black hole has no neutrino hair. The interpretation of this case is more complicated and raises some questions. It is asserted that the black holes have no vector meson (baryon) field[17,18]. It is interesting to answer the question as to which global properties of black holes are conserved. There exists the assertion that the black holes may possess only mass (M), electric charge ($\mathcal{E}$) and angular momentum ($\mathcal{J}$) since these quantities obey conservation laws. However, another question arises what is the situation with enormous baryon (or lepton) charges of a collapsing star which are also conserved.

Ruffini and Wheeler[9] try to explain this situation in the following way: "Electric charge is a distinguishable quantity because it carries a long-range force (conservation of flux; Gauss's law). Baryon number and strengeness carry no such long-range force. They have no Gauss's law ... Nor has anyone ever been able to give a convincing reason to expect a direct and spontaneous violation of the principle of conservation of baryon number. In gravitational collapse, however, that principle is not directly violated; it is transcended. It is transcended because in collapse one loses the possibility of measuring baryon number, and therefore this quantity cannot be well defined for a collapsed object. Similarly, strengeness is no longer conserved".

In an analogous way, Hartle et al. interprets the impossibility of establishing the presence of the neutrino charge in a black hole by means of an experiment performed outside this hole. In the most recent book by Zel'dovich and Novikov /10/ this situation is formulated as follows:

"The disappearance of signals from particles buried in the process of collapse is not the death of these particles: indeed, we do not consider the man who turned the corner of a building to be perished".

Further it would be interesting to discuss in more detail to what extent the interpretation of Ruffini-Wheeler is adequate to the situation in question in collapsing systems.

It is also interesting to discuss arguments in favour of the fact that black holes have no external scalar, massive vector and neutrino fields. But beforehand it is advisable to consider an assertion (say,lemma) concerning systems described by a closed metric. As will be seen below the results of this consideration are very important for the problems under discussion. This assertion is formulated as follows:

If a system containing sources (specific charges) of some field is found to be incompatible with the closed metric then the corresponding black hole has outside it the field of the given sources.

The characteristics of the system (critical matter density, etc.) are assumed to be such that when the values of the charges in question tend to zero the metric becomes closed.

As is known, the Friedmann line element/11/

$$ds^2 = a^2(\eta)d\eta^2 - a^2(\eta)d\chi^2 - a^2(\eta)\sin^2\chi(d\theta^2 + \sin^2\theta\, d\varphi^2) \quad (2)$$

describes one of the models of the closed world.

Here the variable χ changes in the limits

$$0 \leq \chi < \pi . \quad (3)$$

The variable η is connected with the time t by a simple relation $c\,dt = a\,d\eta$.

The same eq.(2) describes the internal metric of a black hole provided that the matter of the system is distributed in such a manner that the region of χ is filled only to $\chi_0 \leq \frac{\pi}{2}$. Then the external solution, which is Euclidean at infinity, must in an appropriate fashion, be sewed with the internal solution. The metric as a whole is nonstatic but, in a certain approximation, the external metric may be the Schwarzschild metric. Finally, if the matter fills the region $\frac{\pi}{2} < \chi_0 < \pi$, then there arises a system with semi-closed metric. If spheres with some values are circumscribed around the point $\chi = 0$, then the surface of the sphere is

$$S = 4\pi a^2(\eta)\sin^2\chi . \quad (4)$$

The surface of the sphere S increases with increasing $\chi < \frac{\pi}{2}$. However for $\chi > \frac{\pi}{2}$ the dimensions of the sphere decrease and for $\chi = \pi$ the sphere reduces to a point. The world becomes closed. For $\chi_0 < \frac{\pi}{2}$ (black hole)[12] the spheres increase monotonously with increasing radius at a given time moment (η): the quantity $r = a\sin\chi$ assumes in the external metric the meaning of a monotoneously increasing radius. The semi-closed metric (in the case $\frac{\pi}{2} < \chi_0 < \pi$) is characterized by the presence of the minimal value of $r \left(\frac{\partial r}{\partial \chi} = 0, \frac{\partial^2 r}{\partial \chi^2} > 0\right)$. In other words, by the presence of a specific throat (in the Wheeler terminology "wormhole") which links the internal and external metrics. The matter density $\mu(t)$ integrated over the whole space of the closed world gives the "bare" mass of the system, i.e., the total mass disregarding the gravitational defect

$$M_0 = 2\pi^2 \mu(t) a^3(t) . \tag{5}$$

This value of the bare mass defines the radius of the closed world (a_0) at the moment of its maximum expansion

$$a_0 = \frac{\varkappa M_0}{3\pi c^2} . \tag{6}$$

The latter expression is directly obtained from the Einstein equation[11]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2\pi \varkappa \mu}{3} - \frac{c^2}{a^2} , \tag{7}$$

if in eq.(7) it is put $\dot{a} = \frac{da}{dt} = 0$ and according to Eq.(5) $\mu_0 = \frac{M_0}{2\pi^2 a_0^3}$.
The total mass of a part of the closed world localized in the domain from $\chi = 0$ to χ_0 (i.e., the bare mass minus its gravitational defect) is given by

$$M_{tot} = \frac{c^2}{\varkappa} a_0 \sin^3\chi_0 . \tag{8}$$

Thus, the total mass of the closed system ($\chi_0 = \pi$) is zero. The total electric charge of the closed world is also zero. The latter fact is due to the law of conservation of electric charge. The attempts to place the electric charge in the closed world result in a contradiction between the Gauss theorem ($\int E_n dS = 4\pi e$) and the closed metric[11] which illustrate the above lemma. The character of deformation of the metric by a small electric charge is discussed in refs.[13,14]. For an arbitrary small electric charge, a noticeable deflection from the closed metric occurs only when χ is arbitrarily close to π. In other words, if spheres with $\chi > 0$ are circumscribed around a small charge $\mathcal{E}$ localized at $\chi = 0$, then these spheres are characterized by the expression (4). With increasing of χ to $\chi = \frac{\pi}{2}$ the spheres increase. For $\chi > \frac{\pi}{2}$, if the charge is very small, the spheres decrease. But in the domain $\chi > \frac{\pi}{2}$ the electric strength lines ("hair" of the electromagnetic field) become more dense. The spheres (4) for $\chi > \frac{\pi}{2}$ plays the role of peculiar condensing lenses for the strength lines of an electrostatic field.

A detailed consideration[14] shows that when the density of strength lines is such that the electrostatic potential reaches the value

$$\varphi = \frac{c^2}{\sqrt{\varkappa}}, \tag{9}$$

then with further increase of χ the spheres begin again to grow, and the metric transforms to the well-known Nordström-Reissner metric which, in the given case, is characterized by the value of the Schwarzschild mass

$$M_{tot} = \frac{\mathcal{E}}{\sqrt{\varkappa}} \tag{10}$$

and

$$ds^2 = \Phi c^2 dt - \frac{dr^2}{\Phi} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{11}$$

where

$$\Phi = \left(1 - \frac{\mathcal{E}\sqrt{\varkappa}}{c^2 r}\right)^2. \tag{12}$$

The radius of the minimal sphere which is allowed by the electric charge is proportional to the magnitude of the charge

$$r_{min} = \frac{\mathcal{E}\sqrt{\varkappa}}{c^2} . \tag{13}$$

The metric ceases to be closed for an arbitrary small electric charge. Figuratively speaking, the electric strength lines ("hair") become more dense for χ close to π so that they "punch" in the metric a "wormhole" (throat) into which the electric vector flux rushes forming outside the given material system the Nordström-Reissner metric. For an arbitrary small electric charge, the metric as a whole resembles the metric of a semi-closed $(r' = 0,\ r'' > 0)$ world*. In the hairdresser language, outside the metric there arises a bun "horse tail". But if the charge $\mathcal{E}$ is large enough then the minimal sphere may be absent. In this case, the closed metric can be violated already at $\chi \leq \frac{\pi}{2}$. Then this metric describes a black hole obligatorily with external electrostatic field.

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*If our Universe had a matter density $\mu \sim 10^{-29}$ gr/cm^3 , which in the case of the electric neutrality of the matter could lead to a closed metric, then the presence of a single redundant electron in the Universe would render it unclosed with throat

$$r_{min} \sim \frac{e\sqrt{\varkappa}}{c^2} \sim 10^{-33} \text{cm},$$

and the total mass for the exterior observer would be found to be

$$M_{tot} \sim \frac{e}{\sqrt{\varkappa}} \sim 10^{-6} \text{ gr}.$$

The system which becomes a system with closed Friedmann metric when the electric charge tends to zero was called "electrostatic fridmon"/2/, and the external metric (11) - the fridmon metric, when Φ is given by eq. (12).

It should be noted, however, that the external metric of this type may have another internal continuation which describes the Papapetrou model with the same relation $M = \mathcal{E}/\sqrt{\varkappa}$. But in this case, the internal solution describes the static system in which the gravitational and electrostatic forces equilibrate each other. Then the sizes of the material system are necessarily larger than its gravitational radius /15/. The friedmon metric is the extreme case of the semi-closed metric

$(M_{tot} > \mathcal{E}/\sqrt{\varkappa})$ for $M_{tot} \to \mathcal{E}/\sqrt{\varkappa}$.

It is possible to perform a "test" of the lemma suggested without constructing a self-consistent solution which takes into account the effect of the sources of the field considered. It is possible, e.g., for methodical purposes, to find and discuss the solutions of the Maxwell equation for the electrostatic case with $\varepsilon \neq 0$ under the conditions of a strictly closed metric, assume, for simplicity, in the Einstein world or in the Friedmann world at the moment of the maximum expansion. Under these conditions, the contractions of the requirement $\mathcal{E} \neq 0$ with closed metric result in the appearance of divergences for the potential, as $\chi \to \pi$.

It is easy to see that in this case the solution for the electrostatic potential is

$$\varphi = \frac{\mathrm{const}}{a \sin\chi} . \tag{14}$$

For the energy density we naturally have

$$T_0^{\,0} \sim \frac{1}{a^2 \sin^4\chi} . \tag{15}$$

At $\chi \to \pi$ the expressions (14) and (15) diverge even if the source (charge $\mathcal{E}$) in the region near $\chi = 0$ is smeared out over a certain finite sphere. In other words, for $\chi = \pi$ there apears a singularity which is characteristic of the point source which we have not expected in this place. This is just the contradiction with closed metric.

We apply the test suggested to the massive vector field. Then the question is as to what situation arises in the closed metric if in the baryon-neutral matter of this system there will be, say, one redundant neutron at $\chi = 0$, and this neutron will be the sourse of, say, a ρ -meson vector field.

The equations for the massive (m_ρ) vector field in an arbitrary curved space are (c = 1)

$$F^{\mu\nu}_{,\nu} - m_\rho^2 \varphi^\mu = -4\pi j^\mu . \qquad (16)$$

We consider central symmetric solution for this equation in the absence of free waves. We solve the problem under the same assumtions as the previous one concerning the moment of the maximum expansion of the Friedmann world ($a = a_0$, $\dot{a} = 0$).

Let $a_0 m_\rho > 1$, and the metric be given in the form (2) so that $\sqrt{-g} = a_0^4 \sin^2\chi \sin\theta$. In this case the system of equations (16) reduces to a single equation

$$\frac{1}{\sin^2\chi}\frac{d}{d\chi}\left(\sin^2\chi \frac{d}{d\chi}\varphi_0\right) - m_\rho^2 a_0^2 \varphi_0 = -4\pi a_0^2 j^0 . \qquad (17)$$

For small χ_0 the charge g and the charge density ρ are connected by the relation $g = \frac{4}{3}\pi\rho_0 a^3 \chi_0^3$; $j^0 = \frac{\rho_0}{a_0}$, provided $\chi \leq \chi_0$, and $j^{\prime}$ = 0 provided $\chi > \chi_0$.

Using the standard receipt, it is easy to get for g outside the charge location the expression

$$\varphi_0 \sim \frac{\beta e^{-\lambda\chi}}{\sin\chi} , \qquad \text{here } \lambda = \sqrt{a_0^2 m_\rho^2 - 1} , \qquad (18)$$

which naturally may be regarded as an analogue of the ordinary expression $\varphi \sim \frac{1}{r} e^{-m_\rho r}$ in the Euclidean space. According to eq. (18) we could draw the conclusion that for $\chi \to \pi$ the potential φ_0 diverges similarly to the case of electrodynamics and that the presence of the sources of the massive vector field with nonzero total charge is incompatible with the closed metric.

But this conclusion is really wrong; it is based on the error. We have used, by force of habit, the boundary conditions which are ordinary for the Euclidean space. These conditions of the finiteness of the solution select naturally the solutions which decrease exponentially with increasing χ .

In the case of the closed world there is no space infinity, and therefore a more general type of solution is possible [15]

$$\varphi_0 = \beta \frac{e^{-\lambda\chi}}{\sin\chi} + \gamma \frac{e^{+\lambda\chi}}{\sin\chi} . \qquad (19)$$

The requirement for the solution to be finite and continuous outside the source is satisfied by the condition imposed on the coefficients β and γ :

$$\beta e^{-\lambda\pi} + \gamma e^{+\lambda\pi} = 0 .$$

Thus, outside the point source we have the following solution

$$\varphi_0 = \frac{g \operatorname{sh} \lambda(\pi-\chi)}{\operatorname{sh} \lambda\pi \sin\chi} . \qquad (20)$$

Now φ_0 for $\chi \to \pi$ is finite and the field $\left(\frac{\partial\varphi_0}{\partial\chi}\right)$ for $\chi \to \pi$ vanishes.

We are led to the conclusion that the presence of the sources of massive vector field is compatible with the closed metric, that the closed world can contain the nonzero total baryon charge (the source of the massive vector field). In ref. [15] a closed metric taking into account the massive vector field effects is constructed. The essential difference between the massless and massive vector fields has thus been established.

The result we have obtained does not contradict the assertion that the massive vector field is absent outside black holes. But, at the same time, this result is not a proof of the validity of this assertion. The matter is that the lemma inverse to that suggested above is not definitely valid. It is enough to recall that the mass, the source of the gravitational field, allows the closed metric and the black hole excite in the external space a gravitational field.

As was shown by R.Asanov [16], the sources of the massless scalar field are also compatible with the Friedmann closed metric. In the metric $ds^2 = e^{\gamma(r,t)}dt^2 - e^{\alpha(r,t)}dr^2 - e^{\beta(r,t)}d\Omega$ the equation for the scalar field

$$\nabla_\sigma \nabla^\sigma u = -4\pi j \tag{21}$$

if of the form

$$e^{-\alpha}\left[u'' + \left(-\frac{\alpha'}{2} + \beta' + \frac{\gamma'}{2}\right)u'\right] - e^{-\gamma}\left[\ddot{u} + \left(\frac{\dot{\alpha}}{2} + \dot{\beta} - \frac{\dot{\gamma}}{2}\right)\dot{u}\right] = 4\pi j, \tag{22}$$

j is the invariant density of the scalar field sources.

In this metric, the Bianchi identity $\nabla_\sigma T_1^\sigma = 0$, using eq. (22), yields the relation $\gamma'\mu + 2ju' = 0$; μ is the mass (23) density. Here the signs $'$ and $\cdot$ denote the derivatives with respect to r and τ , respectively. If it is possible to introduce comoving synchronous coordinates (i.e., when γ is independent of r) we have the consequence

$$ju' = 0. \tag{24}$$

Then the scalar field must be either free ($j = 0$) or

$$u' = 0. \tag{25}$$

In the Friedmann metric the condition (25) is a direct consequence of the world homogeneity and in the scalar field, eq. (22), there remain only time derivatives.

In ref.[16] a model of the closed world is considered under the assumption

$$\dot{u} = \frac{\dot{\alpha}}{2\sqrt{\varkappa}}, \tag{26}$$

where $e^{\alpha} = a_0(1-\cos\eta)^2$, $0 \le \eta \le 2\pi$. That is, one implies a dust-like model of the Friedmann world[11] in a synchronous coordinate system.

It is remarkable that the expression for the density of the scalar field sources is then

$$j = \frac{2\sin^4 \eta/2 - 1}{32\pi\sqrt{\varkappa}\, a_0^2 \sin^8 \eta/2}. \tag{27}$$

The density changes in a specific manner with time($cdt=ad\eta$). $\dot{j}$ vanishes when $\sin^4 \eta/2 = 1/2$ and at certain time moments changes even the sign. This behaviour of the scalar field sources may be explained by the fact that, contrary to the electric charge, the scalar field charge does not obey the conservation law. Thus, the closed world may have the nonzero total charge of the scalar field. The situation with the neutrino field (B) is far more complicated.

The preliminary consideration shows that placing a point neutrino field source at the point $\chi = 0$ we are led to the image (with opposite sign) of this source at $\chi = \pi$

$$-B(\chi-\pi) = B(\chi). \qquad (27')$$

If further consideration shows that this result is valid this will mean that the nonzero leptonic charge is incompatible with the closed metric. In this case, there will appear necessarily an external continuation of the metric and the existence of neutrino hair outside black holes will be inevitable. In other words, there will be a contradiction with the Hartle result/7,8/.

Now we consider the existing proofs of the absence of scalar ($\mathcal{U}$), massive-vector (φ) and neutrino (B) fields outside black holes. All the available proofs of this kind/6,7,17,18/ are based on a number of suppositions. The most important of them are the following.

i) The system under consideration possesses an event horizon $g_{11} \to \infty$; $r \to r_{gr}$.

ii) The potential of the field under consideration has a finite value on the event horizon.

iii) The influence of the field on the metric may be neglected in the case of an arbitrarily weak field.

We consider in more detail the situations which happen for different fields.

I. The Scalar Long-Range Field

We recall that the scalar field has a large variety of peculiar properties which were mentioned, in particular, by Dicke[20]. Further we shall discuss some of them. First of all, we note that the scalar field equation can be written in the form (21): $\nabla_\sigma \nabla^\sigma \mathcal{U} = -4\pi j$ and in the form[22,23]

$$\nabla_\sigma \nabla^\sigma \mathcal{U} + \frac{R}{6}\mathcal{U} = -4\pi j \,. \tag{28}$$

The latter is conformal invariant and has a number of advantages compared to (21). The problem of eq.(21) which is similar to the Nordström-Reissner problem was solved by Fisher[19] more than twenty years ago. The metric obtained in this work strongly differs from the Nordström-Reissner and Schwarzschild metric: it is surprising that in the case of the scalar field the event horizon is absent. More correctly, the appropriate Schwarzschild sphere reduces to a point and this occurs for any small charge of the scalar field source.

This result in itself seems to be so surprising and unlikely that one wants automatically to find any calculation errors or some natural restrictions of the range of applicability of the static metric for just the scalar field. Rirstly we give the results of ref.[19]. The metric obtained by Fisher is

$$ds^2 = \left(\frac{z - z_0}{z + z_1}\right)^{\rho} dt^2 - \frac{r^2}{z^2}\left(\frac{z - z_0}{z + z_1}\right)^{\rho} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{29}$$

Here $z_{0,1} = \sqrt{\varkappa^2 m^2 + \varkappa G^2} \mp \varkappa m$; G is the scalar charge (30)

$$P = \frac{\varkappa m}{\sqrt{\varkappa^2 m^2 + \varkappa G^2}} , \tag{31}$$

$$z(r) = r \exp\left(\frac{\nu-\lambda}{2}\right) \to r, \; r \to \infty ; \; g_{00} = e^{\nu}, \; g_{11} = -e^{\lambda} . \tag{32}$$

and

$$(z - z_0)^{1-P} (z + z_1)^{1+P} = r^2 . \tag{33}$$

According to (29) g_{11} turns to infinity nowhere. g_{11} Similarly to g_{00}, tends to zero for $z \to z_0$. According to (33) g_{11} and g_{00} are equal to zero at $r = 0$.

The potential obtained in this paper has, at $r = 0$, a logarithmic singularity:

$$u = \frac{G}{2\sqrt{\varkappa^2 m^2 + \varkappa G^2}} \ln \frac{z + z_1}{z - z_0} . \tag{34}$$

The Fisher's results were independently obtained more than twenty years ago by Jams, Newman and Winicour[24]. In their paper the metric was found to be

$$ds^2 = \left[\frac{2R + r_0(\mu+1)}{2R - r_0(\mu-1)}\right]^{1/\mu} dR^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2) - \left[\frac{2R - r_0(\mu-1)}{2R + r_0(\mu+1)}\right]^{1/\mu} dt^2 , \tag{35}$$

where

$$r^2 = \frac{1}{4}\left[2R + r_0(\mu+1)\right]^{1+\frac{1}{\mu}} \left[2R - r_0(\mu-1)\right]^{1-\frac{1}{\mu}} . \tag{36}$$

By means of a simple transformation, this metric transforms to the metric (29) which is more convenient for discussion since in it g_{11} is the coefficient for dr^2 rather than for dR^2 as is the case of eq.(36).

Both papers[19,24] contain some errors[25,26] in the analysis of the asymptotic behavior of the metric*, but they do not concern the form of the linear element (29) which has been calculated correctly. Thus, for the scalar field described by eq.(21) there is no condition of applicability of the theorems which provide evidence for the fact that the black hole has no external scalar field, if in the case of the scalar field the metric (29) is assumed to be correct.

In fact, in this case the system with scalar field sources has no event horizon (the condition i) is not fulfilled). In this case there is no black hole, but there is something similar to a "bare"singularity to which the Schwarzschild sphere is degenerated. But the potential turns to infinity on this degenerate Schwarzschild sphere (the condition ii) is not fulfilled). The latter remark concerns the criticism of the proofs of refs./7,17,18/ which deal with the static metric.

An analysis of the problem being a nonstationary problem in the comoving coordinate system which is given by, e.g., Price/6/ will be discussed later on.

It is obvious that the metric (29) possesses a variety of unexpected properties which are hardly reconciled to physical

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*As R.Asanov remarked/26/, in the papers/24/ in the notation $\tau_0 = 2m$ there is no gravitational constant. Therefore, there arise difficulties in interpreting the asymptotic metric for $\varkappa \to 0$.

In fact, Fisher has $\frac{1}{\rho} = \sqrt{1+\frac{G^2}{\varkappa m^2}} \underset{\varkappa\to 0}{\longrightarrow} \infty$, $\rho \to 0$.

In the papers of JNW $\quad \frac{1}{\rho} = \sqrt{1+\frac{\varkappa G^2}{\tau_0^2}} = \sqrt{1+\frac{\varkappa G^2}{m^2}} \underset{\varkappa\to 0}{\longrightarrow} 1$.

In the first case, when $\varkappa \to 0$ the metric (29) becomes Euclidean.

intuition. This is, first of all, the destruction of the Schwarzshild horizon for arbitrarily small scalar field). Then the question arises. Is it possible to bring into agreement this property of the metric (29) with the transition to the limit $G \to 0$ which necessarily leads to the Schwarzschild metric? It is found that this property is fulfilled. At $G \to 0$, according to (30) $z_0 = 0$; $z_1 = 2\varkappa m$. According to (31) $p = 1$, according to (33):

$$z + 2\varkappa m = r . \tag{37}$$

Consequently, for $G \to 0$ the metric is

$$ds^2 = \left(1 - \frac{2\varkappa m}{r}\right) dt^2 - \left(1 - \frac{2\varkappa m}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{38}$$

On the other hand, at any small G, g_{00} and g_{11} tend for $r \to 0$ monotonously to zero.

A formal analysis shows that when $r \to 0$ and $G \to 0$ the metric behaves in a nonanalytic manner: $z \to 0\ (r \to 0)$, but at the very limit ($r = 0$), according to (37) assumes by jump the value

$$z = -2\varkappa m . \tag{39}$$

It may be supposed that the static metric (29) can be applied only up to some r_{crit}. It is quite possible that there exist some physical causes which have not been analysed yet and which do not allow, in this metric, to restrict the internal solution to any small r.

The cause of this may be the following peculiar properties of the scalar field. We consider, as an example, in a certain Newton approximation, the total mass of the system distributed over a spherical domain of a radius r_0. Let the bare mass of a matter (disregarding its gravitational mass defect) be M_0 and the total scalar charge distributed over this domain be G.

By generalizing the well-known Arnowitt-Deser-Misner relation [27] for the total mass of the system we get

$$M_{tot} = M_0 - \frac{\varkappa M^2_{tot}}{2c^2 r_0} - \frac{G^2}{2c^2 r_0}, \tag{40}$$

or

$$M_{tot} = -\frac{2r_0 c^2}{\varkappa} + \left(\frac{4r_0^2 c^4}{\varkappa^2} + \frac{2r_0 c^2 M_0}{\varkappa} - \frac{G^2}{\varkappa}\right)^{1/2}. \tag{41}$$

According to (41) the total mass vanishes for

$$r_0 = \frac{G^2}{2M_0 c^2}. \tag{42}$$

The system in question cannot be localized in the domain $r < r_0$. When $r < r_0$ there arises the negative value of the total mass, the gravitational attraction is replaced, as if, by the gravitational repulsion, and the system conserves its minimal sizes r_0.

The latter may testify in favour of the fact that a continuation of the external vacuum metric (29), to any small distances (similarly to the static metric) appears to be invalid.

If we analyse the structure of the metric (29) in the light of the above considerations, we can draw the following conclusion.

As $r \to \infty$, $z \to r$,

$$-g_{11} = e^{\lambda} = \frac{r^2}{z^2}\left(\frac{z - z_0}{z + z_1}\right)^P \to 1, \quad r \to \infty. \tag{43}$$

On the other hand, using (33), e^{λ} can be expressed as

$$e^{\lambda} = \frac{(z - z_0)(z - z_1)}{z^2}. \tag{44}$$

Or, substituting the values of z_0 and z_1

$$e^{\lambda} = 1 + \frac{2\varkappa m}{z} - \frac{\varkappa G^2}{z^2} \tag{45}$$

For large, but finite r, and consequently, z, $e^{\lambda} > 1$, as in the cases of the Schwarzschild and Nordström-Reissner metrics.

However, starting from certain r or z, e^{λ} becomes smaller again and at $r \to 0$ tends monotonously to zero. But, the most unexpected fact is that e^{λ}, changing in this manner, nowhere turns to infinity. In other words, in this case the event horizon is not formed*.

But it is remarkable that with changing r, e^{λ} turns twice to unity: at $z \to \infty$ (i.e. at $r \to \infty$) and at

$$z \to \frac{G^2}{2m} = z_c . \qquad (46)$$

Following the meaning of the relation (42), the relation (46) may be interpreted as follows: if a matter is charged by a scalar charge and localized in the domain $z_c = \frac{G^2}{2m}$, then the total mass of the system in the range from $z = 0$ to $z = z_c$ is zero. In other words, if the Schwarzschild mass measured at $r \to \infty$ turns out to be m, then the whole of it is localized only in the range $z > z_{crit}$.

In eq. (33) assuming z to be equal to the critical value $z_c = \frac{G^2}{2m} = \frac{z_0 z_1}{z_1 - z_0}$ it is possible to find the critical r_{crit}.

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* In a recent paper by R.Asanov "The Static Scalar Field and the Event Horizon", P2-6564 Dubna, 1972, a model with scalar and electrostatic fields for the Schwarschild mass m with the requirement for the metric to be Euclidean at the point $r = 0$ is constructed.

The numerical solution shows that in any case if $r \gtrsim 0{,}9m$, where $e^{\lambda} > 1$ and where it is meaningful to sew the internal and external solutions, the event horizon is absent.

$$
r_{crit} = \frac{G^2}{2m}\left(\frac{\sqrt{\varkappa^2 m^2+\varkappa G^2}+\varkappa m}{\sqrt{\varkappa^2 m^2+\varkappa G^2}-\varkappa m}\right)^{\frac{1}{\sqrt{1+\frac{G^2}{\varkappa m^2}}}} \cong 2\varkappa m, \quad \text{if } \frac{G^2}{\varkappa m^2} \ll 1. \qquad (47)
$$

Summarizing the above considerations concerning the metric (29) and the specific features of the scalar field we may conclude that the problem of the scalar field in the process of collapse of systems is still (be careful) open. It will be solved when one will succeed in finding the internal solution of the collapsing system taking into account the scalar field effect on the metric and the external solution sewed with the former. It is essential that this should be done outside the framework of perturbation theory. This problem is essentially nonstatic. The trouble is that, contrary to electrodynamics, here in the case of central symmetric motion of matter there can occur a monopole radiation of the scalar field which changes the mass and the gravitational radius of the system.

Above we have considered the scalar field obeying eq.(21). The other form of the scalar field eq.(28) leads to the metric which, in a particular case, contains the event horizon /28/. This particular case is characterized by a definite relationship between the total mass (m) and the total charge (G) of the system

$$G^2 = 3\varkappa m^2. \tag{48}$$

In this case there arises an external metric which is quite analogous to the particular case of the Nordström-Reissner metric when the total mass (m) is equal to the total electric charge ($\mathcal{E}$):

$$g_{00} = e^{\nu} = e^{-\lambda} = \left(1 - \frac{a}{r}\right)^2, \quad a = \varkappa m\sqrt{\frac{\varkappa G^2}{3}}. \tag{49}$$

But, contrary to the electrostatic case, in this metric too, the scalar potential on the event horizon turns to infinity

$$\mathcal{U} = -\frac{G}{r-a}. \tag{50}$$

Here we have the counter-example when the black hole possesses an external scalar field. Because on the event horizon the potential $\mathcal{U}$ vanishes the theorem according to which the field is zero outside the black hole cannot be applied to this case (the condition ii) is violated).

On the other hand, a possible existence of the external field is proved by the general analysis of Chase[29], following which, in this case, the potential at the event horizon should have infinitely large values. Generally speaking, the Chase's result is obtained in the simple way.

In the case of electrostatics, when the potential transforms as the fourth vector component, the first integral of the equation gives the derivative of the electrostatic potential with respect to the coordinate ("field")

$$\varphi' = \frac{e}{r^2} e^{\frac{\lambda+\nu}{2}} = -\frac{e}{r^2} g_{00} g_{11}, \tag{51}$$

whereas in the case when the field $\mathcal{U}$ is scalar[19]

$$U' = -\frac{g}{r^2} e^{\frac{\lambda-\nu}{2}} = \frac{g}{r^2} \cdot \frac{g_{11}}{g_{00}} . \tag{52}$$

In the Nordström-Reissner metric we have $-g_{00}g_{11} = e^{\lambda+\nu} = 1$.

In a similar metric, or in a more general one, but also with the event horizon, when $e^{\lambda} \to \infty$, $r \to r_{gr}$ and g_{00} is bounded at the event horizon, the field U' must inevitably assume infinitely large values. This is just the peculiar feature of U, the peculiar feature of its scalar nature.

Generally speaking, some similar cases which are characterized by infinite values of certain physical quantities on the Schwarzshild surface may be "disqualified" as unphysical cases. For example, they may not be the extreme cases of the physical collapse. In the process of collapse there must not arise a singularity in the metric on the event horizon which is not removable by a coordinate transformation: the known invariants must not have a singularity on the horizon.

From this point of view the metric (49), as the one which coincides formaly with the Nordström-Reissner metric, is quite correct. Singularity does not arise for invariants (e.g. $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$) on the event horizon. Moreover, in spite of the fact that the potential U (as $r \to a$) diverges, the energy density (T_0^0)for $r = a$ vanishes. This is due to a peculiar[28] dependence of the tensor T_ν^μ with respect to U and derivatives.

In addition, the peculiar feature of the scalar field consists in that in the Lagrangian the scalar potential is added to the mass[20,21]

$$L = -\sqrt{1-\frac{v^2}{c^2}}\,(m+U).$$

This means that, from the point of view of the observer, of a system moving with velocity v *, the scalar potential is of the form[20,21]

$$u' = \sqrt{1-\frac{v^2}{c^2}}\,u. \qquad (53)$$

If we take into account then according to (49)

$$\sqrt{1-\frac{v^2}{c^2}} = \sqrt{g_{00}} = \left(1-\frac{a}{r}\right), \qquad (54)$$

and the potential (50) is given by the expression

$$u = -\frac{G}{r}\frac{1}{1-\frac{a}{r}},$$

then for an observes which crosses the horizon in an falling down freely coordinat system the potential of the scalar charge localized under the horizon remains finite.

Keeping apart the particular case (48), the general solution for which $\nu' + \lambda' \neq 0$, contains, similarly to (25), a singularity only when $r \to 0$.

Thus, it seems advisable to consider the process itself of collapse of the matter charged by the scalar field sources. This means that it is necessary to consider a nonstatic problem: to find a nonstatic internal solution (in the domain occupied by matter) and a solution in vacuum sewed with it, e.g. in a falling down coordinate system.

This task has been performed by Price[6], but in the framework of perturbation theory. Price [6] considers it possible to disregard the scalar field effect on the metric if the scalar field is assumed to be weak. He uses the model of dust-like matter. The internal metric of a star is described by the

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*In the equation $\nabla_\sigma \nabla^\sigma u = -4\pi j$ the potential u and the scalar charge density j are invariants, while the total scalar charge $G = \int j\, dV$ is not invariant and transforms as a volume: $G = \sqrt{1-v^2/c^2}\, G_0$.

Friedmann like element (2). The external solution is given by the well-known spherical symmetric line element in a comoving (synchronous) coordinate system[11]

$$ds^2 = dT^2 - \frac{(\partial r/\partial R)^2}{1+2E(R)}dR^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2).$$

The system begins to collapse from the distance $r_{sf} = 4M$.

At this initial moment the scalar field (ϕ) is supposed to be static ($d\phi/dT = 0$, $d^2\phi/dT^2 = 0$). For convenience the internal initial form of the potential is chosen with a definite χ -dependence. The particular χ -dependent solution is taken as

$$\phi_S = \frac{\sqrt{2}}{8}\cos\chi\left(-11 + \sin^2 2\chi\right). \tag{55}$$

The author shows that at the moment when the star surface crosses the event horizon the potential of the scalar field and its derivatives remain finite and non-vanishing. Thus, it is proved that the scalar field perturbs weakly the metric. The author[6] ignores completely the discussion of the metric (29) which seems not to allow the application of perturbation theory to Price's problem. Thus, for the moment it is possible only to state the contradiction between two approaches. As we have seen above, in addition there is the direct countre-example of the Price's result, the metric (49) where there is the horizon, but at the horizon the scalar potential turns to infinity. In this particular case (48) $G^2 = 3\varkappa m^2$. This means that the scalar and gravitational forces are of the same order. Moreover the zero approximation (the event horizon exists, the scalar field is absent) has no sense: G = 0 gives rise to $m = 0$. The concrete Price model is doubtful too. The matter is that, in the synchronous coordinate system, according to the Bianchi identity we have $\mathcal{U}' = 0$, the potential must not depend on χ (see eqs. (23), (24) and (25)). In general, eq. (22) for the potential in the system used by Price should not contain derivatives with respect to the spacial coordinate. It is unknown to what extent a given concrete χ-dependence is allowed in a given model and whether the initial conditions $\phi_t = 0$ and $\phi_{tt} = 0$ are admissible, bearing in mind eq.(22) which, in the synchronous system must, strictly speaking, contain only time derivatives.

Further, Price uses actually the law of conservation of scalar charge. At the same time, as the analysis of the scalar charge density behavior in the Friedmann metric shows[16], the scalar charge density can change its sign with time and even vanish (27).

If we performed a test of the above lemma for the scalar charge in the closed world assuming, by analogy with the electrostatic case, that the integration constant is unchanged then we would obtained for the scalar potential an expression similar to the electrostatic one (14), i.e. $\mathcal{U} \sim \frac{1}{\sin\chi}$ which would be incompatible with the clased metric at $\chi \to \pi$.

The scalar field possesses another surprising property: the particle mass of the scalar field source must be a function of the scalar potential. The variation of the scalar potential with time changes the mass of the system and inversely. This property was also indicated by Dicke[20]. The corresponding relation in general relativity is a consequence of the contracted Bianchi identity

$$\nabla_\sigma T^\sigma_\nu = 0, \tag{56}$$

from where[16]

$$\dot{\mu} = \check{j}\,\dot{u}\,, \tag{57}$$

where μ is the matter density, $\check{j}$ - the scalar charge density.

Thus, the study of the scalar field under the conditions of a collapsing system is a far more complicated problem than the electrodynamic problem. It seems to us that it has not been studied yet completely.

It is very important to explore the behavior of the massive scalar fields in the process of collapse. These fields have recently been discussed in the theory of elementary particles and are of fundamental importance when attempting to construct a unified theory of weak and electromagnetic interactions /30/.

2. The Vector Massive Field

Unfortunately an external solution of the type of the Nordström-Reissner solution for the massive vector field has not been obtained as yet. The example of the scalar field shows how important is the concrete form of the metric for the analysis of the state of the external field of collapsing systems. Ignoring the concrete form of the metric it is hard to garantee the absence of any surprise in this case.

Let us consider the example of a massive vector field in the form of the ρ -meson field. The sources of the ρ -meson field are nucleons, assume for simplicity, neutrons. The field $F_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$ and the potential φ_μ obey the equation

$$F^{\mu\nu}_{\ \ ,\nu} - m_\rho^2 \varphi^\mu = -4\pi j^\mu, \tag{58}$$

where m_ρ is the mass of the ρ-meson, j^μ is the baryon current vector.

In this case, contrary to the case of the scalar field, the baryon charge g obeys the conservation law. But, contrary to electrodynamics, in vector mesodynamics there is no Gauss theorem.

It is considered that this difference results in the fact that black holes have no external massive vector field.

It is remarkable that in the case of the massive vector field there is a certain peculiar analogue of the Gauss theorem, more correctly, a peculiar generalization of it. A more detailed consideration of the relations available here forces us to assume, that in the case of the massive vector field the problem of the external metric of the black hole is, carefully speaking, awaiting its solution.

Let us consider a mesodynamic analogue of the electrodynamic Gauss theorem, in the Euclidean metric, for simplicity.

Let a baryon charge ρ be localized in a certain domain in such a way that

$$\rho \neq 0 \; , \; r < r_0 ,$$
$$\rho = 0 \; , \; r > r_0 .$$

For the flux of the mesodynamic vector E_n through the closed surface surrounding the charge, using eq. (57'), we have the following expression

$$\int E_n dS = m_\rho^2 \int \varphi^0 dV - \int \rho dV . \qquad (58)$$

After a sphere of radius $r \gg r_0$ has been circumscribed around the charge, we get a vector flux through this sphere in the form

$$\int E_n dS = 4\pi m_\rho^2 \int_0^r \varphi^0(r) r^2 dr - 4\pi g . \qquad (59)$$

If $\varphi^0(r) = \frac{g}{r} e^{-m_\rho r}$, then

$$\int E_n dS = 4\pi g \left[m_\rho^2 \int_0^r e^{-m_\rho r} r\, dr - 1 \right] . \qquad (60)$$

In electrodynamics the flux of the vector E_n has the same value on the sphere of any radius which surrounds this charge. But in the case of mesodynamics the total flux of E_n decreases with increasing radius. For the sphere radius tending to infinity, the r.h.s. of eq.(60) vanishes: the flux of E_n is completely cancelled.

If it is appropriate to employ in mesodynamics the term "straight lines" then in the case of vector massive field the straight lines do not end, similarly to the case of electrodynamics, by the charges of opposite sign. Figuratively speaking, they are cancelled by the specific "field charge" which is realized by the field potential and always distributed continuously over the whole space. We imply the integral in the r.h.s. of eq. (60).

The sphere of finite radius τ is crossed by the nonzero vector flux. This is the "hair" of the baryon flux. The assertion that outside the black hole the flux vanishes means that inside the black hole it becomes zero when approaching the event horizon. This means that, in some manner, "the field charge" integrated over the internal space of the black hole increases so that it becomes able of compensating the potential vanishing which has occured outside the black hole. Without appropriate increase of the potential in the internal space it is impossible to let the flux vanish throught the Schwarzschild sphere.

Considering the situation with the discussed flux in the closed world it is easy to verify that the potential (20) provides vanishing of the flux on the world boundary for $\chi = \pi$. This possibility is due to the fact that inside the closed world in the expression for the potential one adds a term with increasing exponent. Therefore the charge density increases so that to compensate the baryon charge. In this sense, in the closed world the total "baryon charge" (the r.h.s. of eq.(58))is zero. Because of the change in the potential the "hair" of the ρ -meson field are setlled inside the closed world.

The problem is as to at the expense of what can increase the integral of the vector massive field inside the black hole. It is known that the boundary conditions on the Euclidean infinity for a collapsing system do not change. In other words, it is impossible to make the external field of the black hole equal to zero, it should be, figuratively speaking, "driven inside" the black hole, so that to increase the integral $m_\rho^2 \int \varphi^0 dV$ inside the black hole to the value which compensates the total baryon charge g . In any case, it is still unclear how in the process of collapse the generalized Gauss theorem is fulfilled. One may not assert that black holes have no external baryon field.

3. The Neutrino Field

The situation with the neutrino forces is much more complicated. In fact, if there proceeds the interaction $(e\nu)(e\nu)$, then a system consisting e.g. of hydrogen should induce in the surrounding space a neutrino-antineutrino field with potential $B \sim 1/r^5$. We imply here the vector mode of interaction for which there is the law of conservation of the sources of this field, the law of conservation of leptonic charge. The forces are here the repulsive forces. The quantity

$$L = [n(e^-) - n(e^+)] + [n(\nu_e) - n(\tilde{\nu}_e)]$$

conserves. Denoting the corresponding spinors as follows

$$\psi_e \text{ and } \psi_\nu$$

the conservation law for leptonic charge is then

$$\partial_\alpha (\bar{\Psi} \Gamma_\alpha \Psi) = 0,$$

where
$$\Psi = \begin{vmatrix} \psi_e \\ \psi_\nu \end{vmatrix}, \quad \Gamma_\alpha = \begin{pmatrix} \gamma_\alpha & 0 \\ 0 & \gamma_\alpha \end{pmatrix},$$

γ_α are the corresponding Dirac matrices.

The situation is complicated by the presence of the paper of Hartle[7], one often refers to. He asserts that outside the black hole the neutrino force potential vanishes. In the framework of the approximations of this paper, it seems to be correct. If in the collapse the whole matter of the black hole turns into neutrons then the problems we are interested in cancel since in this case the leptonic charge is irradiated.

But, in principle, the situation is possible when the matter density is very small and the mass of the object is large enough. There arises the question whether the metric may be closed (e.g. the metric of our Universe), if the total leptonic charge is nonzero[2]. Unfortunately, the neutrino field is not the solution of some equation of the Maxwell type and here the law of conservation of leptons is not connected with any analogue of the Gauss theorem. One may however attempt to construct formally a certain generalized Maxwell field for the case of electrostatic forces with the dependence $1/r^5$ and higher. Such a generalization has been made by the author and Beresin[31]. With this consistent formalism, the Gauss theorem is automatically formulated for a certain tensor. The formalism includes the static potentials

$$V \sim \frac{1}{r^{2\ell+1}} . \tag{61}$$

Similarly to the electrostatic field, the generalized vector field does not vanish outside the black hole. The external metric of this collapsar is a generalization of the Nordström-Reissner metric. This example is instructive by that the potential with such a high dependence on r does not exclude the appropriate Gauss theorem*. It might be assumed that the absence of neutrino hair outside black holes is in some way associated with the fast decrease of the potential with increasing distance. However, the example of the appropriately generalized Maxwell field shows that here the situation is more complicated.

Although the generalized Maxwell field, similarly to the neutrino-antineutrino field, is the vector field, although in

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*The classic equation of motion for the given charge is written as in ref./7/

$$m_0 \frac{du^\mu}{dt} = \frac{G}{c^2\sqrt{2}} u^\nu \left(\partial^\mu B_\nu - \partial_\nu B^\mu\right), \; F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu ,$$

where B_ν is the vector-potential, u^μ is the velocity vector. In the same way one chooses the interaction lagrangian

$$\mathcal{L}_W = -\frac{1}{c^2} j_\mu B^\mu , \quad j_\mu - \text{ the current.}$$

But the lagrangian of the free field interaction is now written in the form

$$\mathcal{L}_f = -\alpha \left(F_{\mu\nu} F^{\mu\nu}\right)^K , \; \alpha \text{ is constant.}$$

Then the generalized Maxwell equation reads

$$\frac{\partial}{\partial x_i} \left[\left(-F_{\mu\nu} F^{\mu\nu}\right)^{K-1} F^{i\gamma}\right] = \frac{1}{4\alpha K c^2} j^\gamma .$$

The Gauss theorem is fulfilled for the tensor

$$D^{i\gamma} = \left(-F_{\mu\nu} F^{\mu\nu}\right)^{K-1} F^{i\gamma} .$$

For $K = 1$ the formalism leads to the Maxwell theory.

the both cases the static potentials have the same power dependence, the analogy between the two fields is, as yet, exhausted only by these characteristics, The theorems proved for the generalized Maxwell field cannot be simply transferred to the Hartle case. The attempt to come to contradiction (or agreement) with the closed metric in the Hartle case, by analogy with the electrostatic field, by means of a direct calculation of the neutrino-antineutrino quantities at $\chi \to \pi$ seems to lead really to singularities, as in the electrostatic case . For a point source of the neutrino forces localized at the point $\chi = 0$ of the closed world there arises a mirror image of the source at $\chi = \pi$

$$B(\chi) = -B(\pi - \chi).$$

In other words it would seem that the corresponding neutrino hair should not disappear. This result needs a more thorough checking. However if the lepton charge conservation law is not violated in strong gravitational fields it is apparently hard to conceive a "mechanism" by means of which it would be possible to dress the neutrino hair in the closed world or à la Schwarzschild sphere. The neutrino vector field in its properties is in a certain sense, close to the Maxwell one. Contrary to the scalar field, this field is characterized by the inevitable existence of particles and antiparticles as the sources of this field, similarly to the case of the Maxwell field. Unlike the mesodynamic field, any boundary condition cannot let the neutrino field (as the electrostatic one) vanish on the surface surrounding the source of this field. This vanishing of the neutrino field inside the black hole, when approaching the horizon from inside the black hole, must be in some way, realized if outside the black hole the neutrino field is absent. In other words, in the case of the neutrino field, as in the previous cases of the scalar and vector meson fields, the final conclusion about the behaviour of these fields outside the black hole can be drawn after finding the sewed internal and external solutions.

As a result of this discussion of the nonstatic problems, it is possible to conceive all the situation with the presence of horizons, bare singularities (if any) and the behaviour of the fields outside and (obligatorily) inside black holes.

Leaving the question as to whether or not black holes have the scalar and baryon fields open we are able of asserting more definitely that there may exists a world with closed metric with neutral scalar field sources, There may exist a closed world with unequal number of nucleons and antinucleons.

It should be stressed that, in principle, the closed world may exist in the form of very small sizes (small $a_0 = a_{max}$)and contain a substance of very small mass M_0. But the needed homogenous matter density at the moment of maximal expansion of the system must obey the relation

$$\mu_0 \sim \frac{c^6}{\varkappa^3 M_0^2} . \qquad (62)$$

So, for the mass of the order of the solar mass ($M_\odot \sim 10^{33}$gr) the maximal sizes of the closed world (6) are $a_0 \sim 1$km and the density is $\mu_0 \sim 10^{18}$gr/cm^3. The range of applicability of the classical theory (nonquantum) for the formation of systems with closed metric lies in the region of the mass $M_0 \sim 10^{-5}$ gr and of the closed world size $\frac{\hbar}{M_0 c} \sim a_0 \sim 10^{-33}$ cm. If in our world we could build such systems artificially out of the matter surrounding us or, under some conditions, they could emerge at will then some peculiar situations would occur.

These closed systems would then be characterized outside by the complete absence of hair. All the properties of matter would be buried in these systems without external remainder (hair) and irrevocably. As we have seen above, such a closed system may, in principle, consist of neutrons alone (i.e. without the same number of antineutrons), Therefore its formation would mean that we lose some number of neutrons. This would be a direct violation of the law of conservatión of the barion number. In this situation it would unappropriate to use the term "transcended" introduced by Wheeller. This situation is adequately described by the term "violated". Following the book of Zel'dovich and Novikov, it is impossible to apply to this example the comparison with a man "who turns the corner". Now even this corner has not remained*. However, as is known, the formation of systems with closed metric out of matter surrounding us is impossible.

The black hole, the semi-closed system and the closed world can be described by the same line element (2). Although the semi-closed and the closed worlds are lower energy states of systems consisting, in principle, of the same number of, say, neutrons the transitions of black holes to the state of a semi-closed system are nevertheless forbidden. The matter is that the event horizon (e.g. the Schwarzschild sphere) is as if the sur-

*If here we can draw some analogy with the man we rather imply the lieutenant Kizhe who, as is known, "has no body".

face of a semi-conducting ("one way") membrane - it can only absorb matter and increases in this process only its sizes. The system which is fallen under this surface cannot decrease its total mass, cannot irradiate energy. The transition of the black hole to the state of the semiclosed system is impossible *. The transition of a semi-closed system to the closed one is impossible for the same reasons. The fact that these transitions are forbidden appears to be a peculiar manifestation of the law of conservation of baryon charge.

From the methodical point of view, in order to develop the above-mentioned ideas it is useful to discuss the instructive example of the collapse of very small masses the possibility of which was illustrated by Zel'dovich/32/. He shows that a given arbitrary small N number of baryons can be summed in such a manner that their total mass measured by an external observer will be arbitrary small. In fact, the total mass M for a matter at rest of density μ is

$$M = 4\pi \int_0^R \mu(r) r^2 dr \tag{63}$$

and the total number of particles N is given by

$$N = 4\pi \int_0^R n(r) e^{\lambda/2} r^2 dr, \tag{64}$$

where $n(r)$ is the particle density, $e^{\lambda/2} = \sqrt{g_{11}}$. If the distribution is $\mu = \frac{a}{r^2}$, $r < R$; $\mu = 0$, $r > R$, then in the case of an ultra-relativistic gas

$$\mu = \frac{3}{4} \hbar (3\pi^2)^{1/3} \frac{1}{2} n^{4/3}$$

and the expression for the total mass is

$$M = \mathrm{const}\, N^{2/3} a^{1/2} \left(1 - \frac{8\pi \varkappa a}{c^2}\right)^{1/3}, \text{ if } \lambda = \mathrm{const} \tag{65}$$

*Some other situation occurs in the case of anticollapse when a radiation originating from the system is possible.

According to eq.(65), for any N the mass M tends to zero, provided $a \to \frac{c^2}{8\pi\varkappa}$. Zel'dovich notice that, in principle, it would be possible to design such a machine which would perform magnificent contractions, would lead the system to the desired configuration with extremely large gravitational mass defect at which the energy released would be close to the total proper energy of the system, In nuclear reactions the energy released is about one percent of the mass of the system. However, we are interested not in the Zel'dovich fantastic machine, but the instructive occasion for discussing the extreme case of the system with zero total mass. Let we have formed a system consisting of N neutrons, or Zel'dovich has succeeded in constructing a machine, which by realizing the case $a = \frac{c^2}{8\pi\varkappa}$ bring the system to a state with zero total mass. If such a case was realized, the system would completely disappear from our experience. The construction of this machine would lead to a decrease, i.e. actually annihilation of baryons in the Universe. In this sense the result is quite adequate to the formation of closed systems with which we have dealt above.

Now using this example it is appropriate to continue the discussion of the generalized Gauss theorem for the massive vector field. Neutrons are sources, e.g. ρ- meson forces which are completely neglected in the Zel'dovich treatment. If we attempt to induce a collapse of, say, one gram of neutrons then when the neutrons are localized in a region much smaller than $\frac{\hbar}{m_\rho c}$ the value of the second term in eq.(60) becomes infinitesimal. When the region of localization is still by many orders of magnitude larger than the gravitational radius of the system $(r_{gr} \sim 10^{-28}\,\mathrm{cm})$, when the gravitational forces may still be neglected, the generalized Gauss theorem is actually

$$\int E_n dS = -4\pi g. \tag{66}$$

In other world, there arises a purely electrodynamic analogue of the Gauss theorem with its characteristic properties, in particular, concerning baryon hair. If the collapse of such a system was realized, then in virtue of applicability of eq.(66) the corresponding black hole would have baryon hair and, consequently, nonzero mass. Here we have taken the opportunity to illustrate once more the importance of the analogue of the Gauss theorem in mesodynamics. Although the appropriate calculations of the machine in the framework of the idealization suggested by the author are quite correct it should however be noted that this machine cannot, in principle, be applied to the real neutron matter. In fact, if the system is contracted to sizes somewhat smaller than $\hbar/m_\rho c$, the potential of the repulsion forces of this N neutron system is estimated to be

$$V \sim N\left(\frac{g^2}{\hbar c}\right) m_n c^2,$$

where m_n is the neutron mass.

Since $\frac{g^2}{\hbar c} \geqslant 1$ then the energy of the mass localized at distances larger than $\alpha \frac{\hbar}{m_n c}$, $\alpha < 1$, turns out to be larger than the total rest mass of neutrons forming this system*. Further localization of the system of N neutrons (i.e. the "packing" of them in a still more narrow region) increases only the external energy of the meson field. The total mass of the system localized in the field when the machine operates is always larger

* At this stage of construction (dimensions $\frac{\hbar}{m_n c} \sim 10^{-14}$ cm) the gravitational mass defect is still negligible.

than Nm_n and never vanishes. From this point of view, the machine destroying baryons, about which writes Zel'dovich is impossible.

The Zel'dovich machine is impossible just because of the "hair" the source of which is the conserved baryon charge*. When discussing the operation of the machine we have disregarded the fact that the huge meson fields which are due to contraction will lead to the production of neutron-antineutron pairs. The antineutrons will be attracted by this system decreasing its baryon charge while the neutrons will be repulsed from the system. When the system will be completely neutralized in the baryon charge due to further contraction, the system may, in principle, become closed with zero total mass. In this case the system will have no baryon hair (meson field), but this fact is in agreement with the law of conservation of baryon number. Unfortunately the machine does not provide the desired energy yield from matter: due to the operation of the machine the same number of neutrons will be in the surrounding space. Roughly speaking, all the neutrons will be "squeezed out" from the system. Here it should be noticed that due to the vector meson repulsive forces the star collapse appears to be unable of developing infinitely.

The role of the short-range nuclear forces in the development of the gravitational collapse was repeatedly discussed. But usually one considered the situation when the sizes of the collapsing system (R) are much larger than the range of action

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*We imply here the systems in the initial state, which are in no way microscopic, however far from the critical masses of celestial bodies. The gravitational radius $\hbar/m_n c$ is assigned to the mass

$$M_0 \sim \frac{\hbar}{m_n c}\frac{c^2}{2\varkappa} \sim 10^{14} gr = 10^8 ton.$$

of nuclear forces ($\frac{\hbar}{m_n c}$). In this case $\left(R \gg \frac{\hbar}{m_n c}\right)$ it is possible to introduce the notion of pressure since the nuclear energy enters additively when summing small parts of the system. The general thermodynamic consideration shows that at this stage of collapse $\left(R \gg \frac{\hbar}{m_n c}\right)$ the nuclear forces do not cease the gravitational contraction. For $R < \frac{\hbar}{m_n c}$ the phenomenon goes nut the framework of the thermodynamical consideration and further stage of the collapse should be considered dynamically as in the case of the presence of electrostatic forces. As Novikov has remarked/33/, electrostatic forces, being long-range ones, are capable of stopping the gravitational collapse. When the system is localized in the region $R < \frac{\hbar}{m_n c}$ the fields become actually long-range ones, and there arises the complete analogy with the electrostatic forces in the treatment of the possibility of stopping the collapse. Naturally the matter density becomes here huge. In fact, the critical mass at which the collapse of a star can occur is $M \sim M_\odot \sim 10^{33}$ gr.

In the sphere of radius $\frac{\hbar}{m_n c}$ the density of such a mass is $\mu \sim 10^{33} \left(\frac{\hbar}{m_n c}\right)^{-3} \sim 10^{74} \frac{gr}{cm^3}$.

It is interesting that this density, at which the collapse is expected to stop, is about 20 orders of magnitude smaller than the so-called critical (quantum) density $\mu_{crit} \sim 10^{93}$ gr/cm^3 which in certain hypotheses is associated with the considerations* about a possible stopping of the star collapse**.

*These considerations consist in that, at such densities, nonquantum mechanical approaches are already invalid, and there remains, in principle, a hope that in this case other laws should impede further development of the collapse.

**The density $\mu \sim 10^{93}$ gr/cm^3 is reached for masses larger than $M_\odot$ by a factor of 10^{20}, i.e. for M $\sim 10^{53}$ gram. May be by chance this value coincides approximately with the mass of our Universe.

Apparently the problem of infinite development of the collapse up to the point limit does not simply exist. However in a comoving coordinate system there arises the problem of the star expansion after the collpse has stopped. The classic consideration shows that this expansion may not be an expansion into the same space[33]*.

We have discussed the impossibility of formation of closed and semi-closed systems in the framework of classical physics. It is, however, quite possible that in quantum theory there can occur situations when certain analogues of these systems may be realized. We imply the occurence of, e.g. rare quantum fluctuations in few-nucleon systems. Here we may refer to our ignorance of the laws in this domain of physical phenomena. But the formation of closed systems due to fluctuations would be a violation of the law of baryon conservation. The spontaneous formation of microscopic semi-closed systems or microscopic black holes does not contradict any laws of conservation. However when these systems have small sizes only extremely small (unit) electrical and baryon charges, i.e. systems of the type of fridmons[2], are more favorable energetically. These small systems with large charges are unstable due to pair production and vacuum polarization in strong fields near point sources.

However, in modern theory of elementary particles there are situations in which the discussion of possible formation of microscopic semi-closed systems may turn out to be important. We

*With such contractions it is necessary to take into account the space-time picture of pair production, mixing of charged particles and the motion of pair components toward periphery. It is quite possible that this fact can change noticeably the whole situation.

imply here the intermediate states in modern perturbation theory as applied to elementary particles. The states of semi-closed systems or the states of black holes would seem to belong to the complete set of the states which can spontaneously occur in these cases. Moreover, energetically these states are the lowest ones which, as we shall see below, is important. Thus, if in the intermediate state the particle emits a quantum of mass then, according to the Heisenberg relation, this mass is localized in the domain*

$$\ell \sim \frac{\hbar}{mc}, \text{ then } m \sim \frac{\hbar}{\ell c}. \qquad (67)$$

The complete set of intermediate states includes states with arbitrary large energies and, consequently, arbitratily large masses. In modern theory of elementary particles one legalized historically the violation of the logic namely to introduce into consideration states with arbitrary large masses and, at the same time, to disregard completely their gravitational effects. When in the intermediate state we have the mass of the order $m \sim \sqrt{\frac{\hbar c}{\varkappa}}$, then the gravitational radius of this mass is

$$r_{gr} = \frac{2\varkappa m}{c^2} = \frac{2\sqrt{\hbar c}\sqrt{\varkappa}}{c^2}. \qquad (68)$$

On the other hand, for this mass, the dimension of the domain where the mass is localized (according to (67)) $\ell \sim \frac{\hbar}{mc} \sim \frac{\sqrt{\hbar c}\sqrt{\varkappa}}{c^2}$ coincides with the gravitational radius of the object in this state. With further increase of the energy of the intermediate state the gravitational radius should also increase. But, on the other hand, the domain of localization

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*When a quantum of energy $E = \hbar\nu$ is emitted, following the Fermi language /35/, the particle "borrows" the energy $E = mc^2$. According to the uncertainty relation, the time of "borrowing" cannot be longer than $\hbar/mc^2$. During this time the emitted quantum cannot go away from the particle at a distance larger than $\sim \hbar/mc$.

should then decrease according to the Heisenberg relation, and for $m > \frac{\sqrt{\hbar c}}{\sqrt{\varkappa}}$ should become smaller than the gravitational radius. If such a situation occured in the range of application of classical physics we would say that we were dealing with a system the mass of which is under the Schwarzschild gravitational sphere. In other words, we would imply a system in a collapsing state. This might be either the state of the black hole or rather a state of the system with semi-closed metric if the bare mass of the intermediate state strongly decreases due to gravitational defect. At present we do not know whether our understanding of the metric remains valid in this state. But we know that with increasing energy of the intermediate state, according to the Heisenberg relation, the domain of mass localization decreases. Hence, due to large mass concentration, the gravitational mass defect should increase which then decrease the total mass of the intermediate state. It appears that the gravitational radius of the system may not exceed the dimensions allowed by the Heisenberg relation, if we take into account the gravitational mass defect. In this way the contradiction under discussion may be resolved.

If we can admit the estimates (40) for the total mass of the intermediate state of this kind, then it would be obtained from the relation $M_{tot} = m_p + \frac{\hbar}{r_0 c} - \varkappa \frac{M_{tot}}{2 r_0 c^2}$, where m_p is the particle mass, $\hbar / r_0 c$ is the mass of an emitted quantum in the intermediate state, or

$$M_{tot} = -\frac{r_0 c^2}{\varkappa} + \sqrt{\frac{r_0 c^4}{\varkappa^2} + \frac{\hbar c}{\varkappa} + \frac{2 r_0 c^2 m_p}{\varkappa}}$$

At $r_0 \to 0$

$$M_{tot} \to \sqrt{\frac{\hbar c}{\varkappa}}.$$

This value is the maximum possible one for the total mass (energy) of the intermediate states.

It is obvious that the adequate quantum description of collapsing systems can introduce essential corrections apparently to their space-time description. It is doubtful whether the energy picture of these states will change appreciably, or more exactly, whether this affect noticeably the gravitational defect of masses localized in a small domain.
If these considerations are really found to be essential for elementary particle theory this will precisely be that rare case when the discussion of the properties of collapsing cosmic bodies initiates the discussion of the fundamental problems of elementary particle theory.

In conclusion I takes it may pleasant duty to thank my colleagues R.Asanov, V.Beresin and V.Frolov for numerous discussions as a result of which some problems have became more clear and,on the contrary, others have lost their clearness.

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