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π N -SCATTERING AT HIGH ENERGIES IN THE QUASIPOTENTIAL APPROACH

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1. Introduction

Recently the investigation of the high energy hadron collisions has become a rather independent branch of strong interaction physics. A large amount of experimental data has been obtained in cosmic rays and by accelerators. This raises a very timely problem of their classification and theoretical explanation. Theoretical investigations in these lines may be divided conventionally into two classes: 1) Consequences of the general statements of quantum field theory (see, e.g., review papers [1] and references cited therein), 2) Search for models for the description of high energy processes (see, e.g., papers [2-8] and references cited therein).

In the present paper we concentrate our attention on the description of πN-scattering processes within the framework of the quasipotential approach. Quasipotential approach in quantum field theory suggested by Logunov and Tavkhelidze [9] has recently been successfully developed along a number of lines [10-15]. In particular, it turned out to be very convenient in the study of elastic and quasielastic processes at high energy. One of the important points of this approach is the problem of smoothness of the local quasipotential. The idea of smothness has been put forward in paper [16] and further discussed in detail in Refs. [6, 17, 18].

Analysis of the experimental data at high energies has been performed from various points of view but the authors confined themselves to the description of a restrictednumber of phenomena (description of the data at a given energy, description of some characteristics of the process,etc.). The most complete analysis of the data on the basis of Regge phenomenology has been done in Refs. [19, 2o]. Below an analysis of the experimental data on πN-scattering which became available by the end of 1971 is performed on the basis of the quasipotential approach.

A rather complete review of high energy models can be found in Ref. [21]. Experimental data on πN-scattering at high energies have been discussed in detail in Ref. [22].

2. Solution of the quasipotential equation for the two-particle system with spins 0 and $1/2$. The choice of the local quasipotential.

The quasipotential equation for the wave function of two interacting particles with spins 0 and $1/2$

is of the form [23, 24] :

$$\left[E\gamma_0 + \left(1 + \omega(\vec{p})/W(\vec{p})\right)(\vec{\gamma}\vec{p} - M)\right]\psi(\vec{p}) = \frac{-1}{\omega(\vec{p})}\int V(E;\vec{p},\vec{k})\,\psi(\vec{k})\,d\vec{k}\,. \quad (2.1)$$

Here E is the total energy of the two-particle system, $\omega(\vec{p}) = \sqrt{\mu^2 + \vec{p}^{\,2}}$, $W(\vec{p}) = \sqrt{M^2 + \vec{p}^{\,2}}$, $s = E^2 = \left(\sqrt{\mu^2+\vec{p}^{\,2}} + \sqrt{M^2+\vec{p}^{\,2}}\right)^2$, $\vec{p}$ -is the relative momentum in the c.m. frame, μ and M are the masses of the scalar and the spinor particles respectively, $(\gamma_0, \vec{\gamma})$ are the Dirac matrices, $V(E;\vec{p},\vec{k})$ is the quasipotential.

It is convenient to solve Eq.(2.1) in the Foldy-Wouthuysen representation in the configuration space. In the case of the local quasipotential this equation looks as follows:

$$\left[E\gamma_0 - \omega(-i\vec{\nabla}) - W(-i\vec{\nabla})\right]\psi(\vec{r}) = \frac{-1}{\omega(-i\vec{\nabla})}\,V(E;\vec{r})\,\psi(\vec{r})\,. \quad (2.2)$$

The Fourier transform of the quasipotential is 4x4 matrix:

$$V(E;\vec{r}) = \begin{pmatrix} V_{11}, & V_{12} \\ V_{21}, & V_{22} \end{pmatrix}. \quad (2.3)$$

Where V_{ij} are 2x2 matrices.

It was shown in Ref. [25] that in the kinematical region $|t/s| << 1$, which we consider, V_{11} term of the quasipotential contributes mainly. We represent it in the form (in what follows we omit the indices of V_{11} and this will not cause any misunderstanding)

$$V(\varepsilon;\vec{r}) = V^{(+)}(\varepsilon;\vec{r}) + \frac{1}{2ip} V^{(-)}(\varepsilon;\vec{r})(\vec{\sigma}\vec{L}), \qquad \vec{L} = -i[\vec{r}\times\vec{\nabla}] \tag{2.4}$$

$V^{(+)}$ and $V^{(-)}$ are assumed to be smooth functions of r^2 increasing as p when $p \to \infty$. (This leads to the total cross section to be constant.)

The scattering amplitude obtained from Eq. (2.1) with the quasipotential (2.4) may be written in the form

$$T(\vec{p},\vec{k}) = \chi^{+}_{1/2,m_z'}(\vec{k})\Big[T_0(\vec{p},\vec{k}) + \frac{1}{2ip}T_1(\vec{p},\vec{k}) + \dots\Big]\chi_{1/2,m_z}(\vec{p}), \tag{2.5}$$

where

$$T_K = T_K^{(+)} + i\sigma_y T_K^{(-)}. \tag{2.6}$$

The amplitudes $T^{(+)}$ and $T^{(-)}$ describe the spin nonflip and spin-flip processes respectively. Here we write down the expressions for $T_0^{(+)}$ and $T_0^{(-)}$ only.

$$T_0^{(+)}(E;\vec{\Delta}^2) = -ip\int_0^\infty \rho d\rho J_0(\rho\Delta_\perp)\left[e^{\chi^{(+)}}\cos\chi^{(-)} - 1\right] \qquad (2.7a)$$

$$T_0^{(-)}(E;\vec{\Delta}^2) = p\int_0^\infty \rho d\rho J_1(\rho\Delta_\perp)\, e^{\chi^{(+)}}\sin\chi^{(-)} \qquad (2.7b)$$

$$\vec{\Delta}^2 = (\vec{p} - \vec{k})^2 = -t\ .$$

The expressions for $T_1^{(+)}$, $T_1^{(-)}$ may be found in Ref. [24] .

The functions $\chi^{(+)}(E;\rho)$ and $\chi^{(-)}(E;\rho)$ are connected with the quasipotential by the following relations:

$$\chi^{(+)}(E;\rho) = -\frac{1}{2ip}\int_{-\infty}^{\infty} V^{(+)}(E;\vec{r})\,dz\,, \qquad (2.8a)$$

$$\chi^{(-)}(E;\rho) = -\frac{1}{2ip}\cdot\frac{\rho}{2}\int_{-\infty}^{\infty} V^{(-)}(E;\vec{r})\,dz\,. \qquad (2.8b)$$

Experimentally observable quantities, total cross section, differential cross section, polarization and polarization rotation parameter can be expressed in terms of $T^{(+)}$ and $T^{(-)}$:

$$\sigma_{tot} = \frac{4\pi}{p}\,\mathrm{Im}\,T^{(+)}(E;\vec{\Delta}^2=0) \qquad (2.9a)$$

$$d\sigma/d|t| = (\pi/p^2)\left[|T^{(+)}|^2 + |T^{(-)}|^2\right] \qquad (2.9b)$$

$$P = 2\,\mathrm{Im}\left(T^{(+)}T^{(-)*}\right)\Big/\left(|T^{(+)}|^2 + |T^{(-)}|^2\right) \qquad (2.9c)$$

$$R = \frac{\left(|T^{(+)}|^2 - |T^{(-)}|^2\right)\cos\theta - 2\,\mathrm{Re}\left(T^{(+)}T^{(-)*}\right)\sin\theta}{|T^{(+)}|^2 + |T^{(-)}|^2} \qquad (2.9d)$$

θ is the scattering angle in the c.m. frame.

In order to connect our considerations with the physical processes in the πN-system, we shall study the isotopic structure of the quasipotential. Let us consider the quasipotentials (and hence amplitudes,etc.) with definite isospin in s or t-channel. Notice that the quasipotentials and all the functions connected with it will be labelled by an additional index $I = 0, 1$ (the total isotopic spin in t -channel), or $I = 1/2, 3/2$ (the total isotopic spin in s -channel) e.g.:

$$V^{(\pm)} \to V^{(I, \pm)}. \tag{2.10}$$

Thus in the isotopic space in the case of t-channel consideration, we have:

$$V^{(+)} = V^{(0,+)} P^{(0)} + V^{(1,+)} P^{(1)}, \tag{2.11a}$$

$$V^{(-)} = V^{(0,-)} P^{(0)} + V^{(1,-)} P^{(1)}. \tag{2.11b}$$

Where $P^{(0)}$ and $P^{(1)}$ are the projection operators onto the states with definite isospin.

Keeping in Eq.(2.7) only the terms corresponding to single exchange with $I = 1$ we obtain the following expressions for the scattering amplitude with definite isospin in t -channel:

$$T^{(0,+)} = -ip \int_0^\infty \rho d\rho \, J_0(\rho \Delta_\perp) \left[e^{\chi^{(0,+)}} \cos \chi^{(0,-)} - 1 \right], \tag{2.12a}$$

$$T^{(0,-)} = p \int_0^\infty \rho d\rho \, J_1(\rho \Delta_\perp) \, e^{\chi^{(0,+)}} \sin \chi^{(0,-)}, \tag{2.12b}$$

$$T^{(1,+)} = -ip \int_0^\infty \rho d\rho \, J_0(\rho \Delta_\perp) \, e^{\chi^{(0,+)}} \left[\chi^{(1,+)} \cos \chi^{(0,-)} - \chi^{(1,-)} \sin \chi^{(0,-)} \right], \tag{2.12c}$$

$$T^{(1,-)} = p \int_0^\infty \rho d\rho \, J_1(\rho \Delta_\perp) \, e^{\chi^{(0,+)}} \left[\chi^{(1,-)} \sin \chi^{(0,-)} + \chi^{(1,-)} \cos \chi^{(0,-)} \right], \tag{2.12d}$$

where

$$\chi^{(0,+)}(E;\rho) = \frac{-1}{2ip} \int_{-\infty}^{\infty} V^{(0,+)}(E;\vec{r})\, dz, \tag{2.13a}$$

$$\chi^{(0,-)}(E;\rho) = \frac{-1}{2ip} \frac{\rho}{2} \int_{-\infty}^{\infty} V^{(0,-)}(E;\vec{r})\, dz, \tag{2.13b}$$

$$\chi^{(1,+)}(E;\rho) = \frac{-1}{2ip} \int_{-\infty}^{\infty} V^{(1,+)}(E;\vec{r})\, dz, \tag{2.13c}$$

$$\chi^{(1,-)}(E;\rho) = \frac{-1}{2ip} \frac{\rho}{2} \int_{-\infty}^{\infty} V^{(1,-)}(E;\vec{r})\, dz. \tag{2.13d}$$

We may define the scattering amplitude with definite isospin in S -channel too:

$$T^{(I,+)} = -ip\int_0^\infty \rho\, d\rho\, J_0(\rho\Delta_\perp)\left[e^{\chi^{(I,+)}}\cos\chi^{(I,-)} - 1\right], \tag{2.14a}$$

$$T^{(I,-)} = p\int_0^\infty \rho\, d\rho\, J_1(\rho\Delta_\perp)\, e^{\chi^{(I,+)}}\sin\chi^{(I,-)}, \tag{2.14b}$$

Here $I = 1/2$ or $3/2$.

The phases χ are connected with quasipotentials as follows:

$$\chi^{(I,+)}(\varepsilon;\rho) = \frac{-1}{2ip}\int_{-\infty}^{\infty} V^{(I,+)}(\varepsilon;\vec{r})\,dz , \tag{2.15a}$$

$$\chi^{(I,-)}(\varepsilon;\rho) = \frac{-1}{2ip}\,\frac{\rho}{2}\int_{-\infty}^{\infty} V^{(I,-)}(\varepsilon;\vec{r})\,dz . \tag{2.15b}$$

The amplitudes $T^{(I,\pm)}$ are related to the amplitudes of the physical processes in a normal way.

Let us choose the quasipotentials as smooth functions of r^2 . For the case with the definite isospin in t -channel:

$$V^{(0,+)} = 2ip\, g_{1,0}^{(0,+)}\left[a_1^{(0,+)}\right]^{-3/2} e^{-r^2/4a_1^{(0,+)}} + \left[g_{0,0}^{(0,+)} + g_{0,2}^{(0,+)} r^2\right] e^{-r^2/4a_0^{(0,+)}} , \tag{2.16a}$$

$$V^{(0,-)} = 2ip\left[a_1^{(0,-)}\right]^{-3/2}\left[g_{1,0}^{(0,-)} + g_{1,2}^{(0,-)} r^2\right] e^{-r^2/4a_1^{(0,-)}} + \left[g_{0,0}^{(0,-)} + g_{0,2}^{(0,-)} r^2\right] e^{-r^2/4a_0^{(0,-)}} , \tag{2.16b}$$

$$V^{(1,+)} = \left[g_{0,0}^{(1,+)} + g_{0,2}^{(1,+)} r^2\right] e^{-r^2/4a_0^{(1,+)}} , \tag{2.16c}$$

$$V^{(1,-)} = \left[g_{0,0}^{(1,-)} + g_{0,2}^{(1,-)} r^2\right] e^{-r^2/4a_0^{(1,-)}} . \tag{2.16d}$$

In the case of the S -channel consideration, we choose the quasipotentials in the form:

$$V^{(I,+)} = 2ip\, g_{1,0}\, a_1^{-3/2}\, e^{-r^2/4a_1} + \left[g_{0,0}^{(I,+)} + g_{0,2}^{(I,+)} r^2\right] e^{-r^2/4a_0^{(I,+)}} \qquad (2.17a)$$

$$V^{(I,-)} = \left[g_{0,0}^{(I,-)} + g_{0,2}^{(I,-)} r^2\right] e^{-r^2/4a_0^{(I,-)}}, \quad {}^{+)} \qquad (2.17b)$$

It is easy to see that the classification of the lower indices is the following: the first index of the quantities $g_{i,j}^{(I,\pm)}$ denotes the p degree and the second the r degree in the corresponding terms. The lower index of the $a_i^{(I,\pm)}$ paramteres denotes the p degree in the corresponding term.

The parameters $a_i^{(I,\pm)}$ and $g_{i,j}^{(I,\pm)}$ are in general complex energy dependent quantities. This will be discussed in detail in the next section.

+) The main terms of the quasipotentials $V^{(1/2,+)}$ and $V^{(3/2,+)}$ are chosen to be equal so as not to violate the known asymptotic theorem:

$$\frac{T^{(3/2)} - T^{(1/2)}}{T^{(3/2)} + T^{(1/2)}} \to 0 \quad \text{when } p \to \infty .$$

3. Comparison with Experiment

The above considerations have been applied to the description of experimental data on πN-scattering. The parameters entering the definition of the quasi-potential have been found by the minimization of the χ^2 functional making use of a standard program, which has been suggested in the JINR for the solution of the problems of such a kind.

The functional χ^2 is written in the following form:

$$\chi^2 = \sum_{k,i} \frac{\left[F_i^k - M_k F_i^k(x_n) \right]^2}{(\sigma_i^k)^2} , \qquad (3.1)$$

where F_i^k is the quantity which has been measured at the i -th point of the k -th experiment, $F_i^k(x_n)$ the value of the quantity F_i^k calculated by means of the parameters x_n, which we are searching for, σ_i^k experimental error of the quantity F_i^k, M_k the norm of the k -th experiment. The parameter M_k allows one to take into account systematic error of the k - th experiment+).

+) All the experimental data, except for differential cross sections, enter our calculations with unit norm.

The relation between the quantities $F_i^{\kappa}(x_n)$ and the scattering amplitude is given by formulas (2.9). $T's$ are here the amplitudes of physical processes in $\pi\mathcal{N}$ -system. The amplitudes possessing a definite isospin in the s or t - channel are connected with them by means of the following formulas

$$T(\pi^+p\to\pi^+p)= T^{(0)}-\frac{1}{2}T^{(1)}= T^{(3/2)}, \tag{3.2}$$

$$T(\pi^-p\to\pi^-p)= T^{(0)}+\frac{1}{2}T^{(1)}=\frac{1}{3}T^{(3/2)}+\frac{2}{3}T^{(1/2)}, \tag{3.3}$$

$$T(\pi^-p\to\pi^0 n)=\frac{-1}{\sqrt{2}}T^{(1)}= \frac{\sqrt{2}}{3}\left(T^{(3/2)}-T^{(1/2)}\right). \tag{3.4}$$

$T's$ are the amplitudes in the eikonal approximation (see formulas (2.7)). According to our choice of the amplitudes with definite isospin ($I=0, 1$ in the t -channel, $I=1/2, 3/2$ in the s -channel) the amplitudes $T_0^{(I,\pm)}$ are given either by (2.12) or by (2.14).

Numerical integration of the eikonal amplitude is difficult to be performed because of technical reasons+).

That is why we expand the integrands in (2.12) and (2.14) in power series. We assume that the phases with $I = 1/2$ and $I = 3/2$

$$\chi^{(I,+)} = -h e^{-\rho^2/4a_1} + \frac{i}{p}\left(d_1^{(I,+)} + f_1^{(I,+)}\rho^2\right) e^{-\rho^2/4a_0^{(I,+)}} = \tag{3.5}$$

$$= \chi_0^{(I,+)} + \frac{1}{p}\chi_1^{(I,+)},$$

$$\chi^{(I,-)} = -\frac{\rho}{2ip}\left(d_1^{(I,-)} + f_1\rho^2\right) e^{-\rho^2/4a_0^{(I,-)}}, \tag{3.6}$$

satisfy the following conditions

$$\left|\frac{1}{p}\chi_1^{(I,+)}\right| \ll 1, \quad \left|\chi^{(I,-)}\right| \ll 1. \tag{3.7}$$

+) When we are looking for the minimum of the χ^2 functional it is necessary to know the functions and their partial derivatives with respect to the parameters x_n. Therefore the number of integrals which have to be calculated for any experimental point is equal to $4(N+1)$, where N is a number of parameters.

Here
$$h = \frac{2\sqrt{\pi}\, g_1}{a_1},$$

$$d_1^{(I,\pm)} = \sqrt{\pi a_0^{(I,\pm)}} \left(g_{0,0}^{(I,\pm)} + 2a_0^{(I,\pm)} g_{0,2}^{(I,\pm)} \right), \tag{3.8}$$

$$f_1^{(I,\pm)} = \sqrt{\pi a_0^{(I,\pm)}}\; g_{0,2}^{(I,\pm)} .$$

Confining ourselves to considering only the terms of the first order of smallness we obtain from (2.14), e.g., for $T_0^{(I,+)}$:

$$\begin{aligned} T_0^{(I,+)} = &- 2ip\, a_1 \sum_{n=1}^{\infty} \frac{(-h)^n}{n \cdot n!} e^{a_1 t/n} + \\ &+ 2d_1^{(I,+)} \sum_{n=0}^{\infty} \frac{(-h)^n}{n!} B_n^{(I,+)} e^{B_n^{(I,+)} t} + \\ &+ 8f_1^{(I,+)} \sum_{n=0}^{\infty} \frac{(-h)^n}{n!} \left[B_n^{(I,+)} \right]^2 \left(1 + B_n^{(I,+)} t \right) e^{B_n^{(I,+)} t}, \end{aligned} \tag{3.9}$$

where $$B_n^{(I,+)} = \frac{a_1 a_0^{(I,+)}}{a_1 + a_0^{(I,+)} n} .$$

In the case of t -channel isospin decomposition, the phases

$$\chi^{(0,\pm)} = \chi_0^{(0,\pm)} + \frac{1}{p} \chi_1^{(0,\pm)} \tag{3.1o}$$

and $\chi^{(1,\pm)}$ are assumed to satisfy the following conditions:

$$\left|\frac{1}{p}\chi_1^{(0,\pm)}\right| \ll 1, \quad \left|\chi^{(1,\pm)}\right| \ll 1 \,. \tag{3.11}$$

By a complete analogy with the s -channel isospin decomposition it is easy to perform the integration in (2.12) and obtain the amplitudes $T_0^{(0,\pm)}$ and $T_0^{(1,\pm)}$ which are expressed in terms of elementary functions.

The parameters of the quasipotential are in general some complex functions of energy. The energy dependence of the quasipotential is in general unknown. However there are some general ideas on the energy dependence of the leading term of the quasipotential. In the case of scattering of spinless particles it has been shown [25] that for the quasipotential of Gaussian type the parameter a (where $R=2\sqrt{a}$ is the interaction radius) can not increase faster than $\ln s$. Therefore we assume the following s -dependence of all the parameters a :

$$a^{(I,\pm)} = \alpha^{(I,\pm)} + \beta^{(I,\pm)}\left[\ln\frac{s}{s_0} - \frac{i\pi}{2}\right]. \tag{3.12}$$

Some of the parameters $\beta^{(I,\pm)}$ can be equal to zero. In order to have the constancy of the total cross sections at asymptotic energies the energy dependence of the parameter $g_{1,0}$ in the leading term is chosen in the form

$$g_{1,0} = \gamma + \delta \left[\ln \frac{s}{s_0} - \frac{i\pi}{2} \right]^{-1}. \tag{3.13}$$

Other parameters entering the definition of the quasipotential are assumed to be energy independent complex numbers. These considerations allow to introduce 30 real parameters$^{+)}$.

We have performed an analysis of all the existing experimental data (total and differential cross sections, polarizations, etc.) on πN -scattering for $0{,}01 \leqslant |t| \leqslant 1.0 \, (GeV/c)^2$ and $P_L \gtrsim 10 \, GeV/c$ (see Table 1). Only the points which deviate from the expected values more than by 3 standard errors have been excluded from the consideration.

$^{+)}$Note that some parameters turn out to be unimportant and have been taken to be equal to zero.

The result of the analysis are presented in Table 2. Satisfactory description of the data $(C.L. \approx 2\%)$ has been obtained in the case of the S -channel isospin consideration and renormalization of the differential cross section data. In the case of t -channel isospin decomposition the description is rather poor.

The obtained values of the parameters are listed in Tables 3 and 4. The calculated curves for the experimentally measurable quantities together with some experimental data are given in Figs.1 - 13. They are drawn for the case of S -channel isospin decomposition.

When these results have been obtained data on the measurement of R at $P_L = 16\ GeV/c$ have been reported [32]. If they are included in the fit the calculated value of χ^2 increasis. That is why an attempt have been made to find a new solution of the problem.

In order to make the fit better in the second term of the quasipotential (2.17a) and in (2.17b) factors $(P/\sqrt{s_0})^{1-\xi}$ $(\xi \lesssim 1)$ have been introduced. Paramters which correspond to new solution are listed in Table 4. In Figs. 14 - 23 some predictions of our considerations are shown.

4. Conclusions

Analysis we have performed in this paper shows that theoretical claculations give statistically satisfactory fit of πN -scattering data in the region $0{,}01 \leq |t| \leq 1.0 (GeV/c)^2$ and $P_L \gtrsim 10$ GeV/c .
One of the important points of our consideration is the choice of the local quasipotential which gives an adequate picture of high energy scattering.

A problem of taking into account of corrections of the relative order $1/p$ as compared to the main contributions is of great interest. Consideration of the backward scattering phenomena will also clear up some aspects of the scheme developed here. These problems require special investigations and we will probably return to them elsewere.

We should like to stress however that the approach considered in this paper doesn't claim to the completness. It is a theoretical model and its result depends on the choosen form of the local quasipotential.

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Table I
Fitted Experimental Data

Reaction	Quantity	P_L GeV/c	Number of points	Number of excluded points	Norm	References	
1	2	3	4	5	6	7	
$\pi^- p$	$\frac{d\sigma}{d\|t\|}$	9.84	9		1.009	Foley	(1968) 26
		9.89	12		1.030	-"-	
		10.8	15	I	1.102	-"-	(1963)
		11.89	11		1.005	-"-	(1968)
		12.4	20		1.039	Harting	(1965)
		13.	13		1.109	Foley	(1963)
		14.16	12	I	1.018	-"-	(1968)
		14.84	8		1.163	-"-	(1965)
		15.	13		1.093	-"-	(1963)
		15.99	14		0.985	-"-	(1968)
		16.	17	I	0.978	-"-	
		17.	12		1.069	-"-	(1963)
		18.19	14		1.009	-"-	(1968)
		18.4	15		1.138	Harting	(1965)
		18.9	6		1.103	Foley	(1963)
		19.75	7		1.244	-"-	(1965)
		23.18	7		1.222	-"-	
		24.22	19		0.967	-"-	(1968)
		25.34	8		1.192	-"-	(1965)
		20.15	17		0.976	-"-	(1968)
		20.38	18		0.962	-"-	
		22.13	19		0.971	-"-	
		26.23	20		0.950	-"-	

Table I. (Cont'd)

1	2	3	4	5	6	7
π^+p	$\frac{d\sigma}{d\|t\|}$	9.86	9		1.129	Foley (1968) 26
		10.02	12		1.078	-"-
		10.8	15		1.088	-"- (1963)
		11.95	15		1.030	-"- (1968)
		12.4	19		0.919	Harting (1965)
		12.8	14		1.097	Foley (1963)
		14.	12		1.023	-"- (1968)
		14.8	13		1.088	-"- (1963)
		16.02	18		1.012	-"- (1968)
		16.7	13		1.027	-"- (1963)
		17.96	17		0.973	-"- (1968)
		20.19	18	I	0.944	-"-
$\pi^-p\to\pi^0n$	$\frac{d\sigma}{d\|t\|}$	9.8	14		1.069	Stirling (1965)
		10.	7		1.021	Wahling (1968)
		13.3	14	I	1.090	Stirling (1965)
		13.3	11	1	1.090	Sonderegger (1966)
		18.2	14	1	0.900	Stirling (1965)
		18.2	11	2	0.900	Sonderegger (1966)
π^-p	σ_{tot}	10–28.68	27			26
		21–65	19			Prokoshkin 27
π^+p	σ_{tot}	9.84–22.1	30			26
		15–60	10			Prokoshkin 28

Table I. (Cont'd)

1	2	3	4	5	6	7
$\pi^- p$	P	10	15			Borghini (1967) 26
		12	13			-"- 26
$\pi^- p$	P	10	21			-"- (1971) 29
		14	19			-"- 29
$\pi^+ p$	P	10	7	I		Borghini (1967) 26
		12	5			-"- 26
		14	7			Bell (1968) 26
		10	15			Borghini (1971) 29
		14	19			-"-
		17.5	8			-"-
		8	6			
$\pi^- p \rightarrow \pi^0 n$	P	11.2	7			Bonami (1970) 30
$\pi^- p$	α	9.84-26.23	11			Dumbrais 31
$\pi^+ p$	α	9.86-20.19	7			-"-
$\pi^- p$	R	16	8			de Lesquen 32

Table 2

Isospin Decomposition	Interval	χ^2	$\overline{\chi^2}$	C.L. %
1/2, 3/2	$0.01 \leq \|t\| \leq 1.0 (GeV/c)^2$ $p_L \gtrsim 10$ GeV/c	806	734	2
0,1	$0,01 \leq \|t\| \leq 1,0 (GeV/c)^2$ $p_L \gtrsim 10$	930	734	1
1/2, 3/2	without norms $0,01 \leq \|t\| \leq 1,0 (GeV/c)^2$ $p_L \gtrsim 10$ GeV/c	1114	734	1
1/2, 3/2	$0,01 \leq \|t\| \leq 2,5 (GeV/c)^2$ $p_L \gtrsim 6$	2000	1130	1

Table 3

Parameters of the quasipotential for the case of s-channel isospin decomposition (I=1/2, 3/2). Data on R at $p_L=16$ GeV/c are not included.

$$a_1 = \left[(1.2261 \pm 0.1086) + (0.4159 \pm 0.0316)\left(\ln\frac{s}{s_0} - \frac{i\pi}{2}\right)\right]\left(GeV/c\right)^{-2}$$

$$g_1 = \left[(0.9808 \pm 0.0253) - (1.5922 \pm 0.1473)\left(\ln\frac{s}{s_0} - \frac{i\pi}{2}\right)^{-1}\right]\left(GeV/c\right)^{-2}$$

$$a_0^{(3/2,+)} = (11.875 \pm 0.3587)\left(GeV/c\right)^{-2}$$

$$a_0^{(3/2,-)} = (16.1760 \pm 0.3851)\left(GeV/c\right)^{-2}$$

$$g_{0,0}^{(3/2,+)} = \left[(0.0762 \pm 0.0037) + i(0.0274 \pm 0.0011)\right]\left(GeV/c\right)^{2}$$

$$g_{0,2}^{(3/2,+)} = (-0.0015 \pm 0.00006)\left(GeV/c\right)^{4}$$

$$g_{0,0}^{(3/2,-)} = \left[(-0.0015 \pm 0.0009) - i(0.0033 \pm 0.0005)\right]\left(GeV/c\right)^{2}$$

$$g_{0,2}^{(3/2,-)} = \left[(-0.00005 \pm 0.00001) + i(0.00006 \pm 0.000006)\right]\left(GeV/c\right)^{4}$$

$$a_0^{(1/2,+)} = \left[(9.7045 \pm 0.2396) + (0.5102 \pm 0.1285)\left(\ln\frac{s}{s_0} - \frac{i\pi}{2}\right)\right]\left(GeV/c\right)^{-2}$$

$$a_0^{(1/2,-)} = \left[(7.7140 \pm 0.1699) + (0.9639 \pm 0.0903)\left(\ln\frac{s}{s_0} - \frac{i\pi}{2}\right)\right]\left(GeV/c\right)^{-2}$$

$$g_{0,0}^{(1/2,+)} = \left[(0.0461 \pm 0.0037) + i(0.0627 \pm 0.0021)\right]\left(GeV/c\right)^{2}$$

$$g_{0,2}^{(1/2,+)} = \left[(-0.0011 \pm 0.00006) - i(0.0004 \pm 0.00003)\right]\left(GeV/c\right)^{4}$$

$$g_{0,0}^{(1/2,-)} = \left[(-0.0184 \pm 0.0014) + i(0.0094 \pm 0.0014)\right]\left(GeV/c\right)^{2}$$

$$g_{0,2}^{(1/2,-)} = \left[(-0.0003 \pm 0.00003) + i(0.00005 \pm 0.00001)\right]\left(GeV/c\right)^{4}$$

$$s_0 = 1\left(GeV/c\right)^{2}$$

Table 4

Parameters of the quasipotential for the case of s-channel isospin decomposition (I=1/2, 3/2). Data on R at p_L = 16 GeV/c are included.

$$a_1 = \left[1,1670 \pm 0,0865 + (0,4726 \pm 0,0222)\left(\ln\frac{s}{s_0} - \frac{i\pi}{2}\right)\right](GeV/c)^{-2}$$

$$g_1 = \left[0,6961 \pm 0,0199 + (0,0602 \pm 0,0120)\left(\ln\frac{s}{s_0} - \frac{i\pi}{2}\right)\right](GeV/c)^{-2}$$

$$a_0^{(3/2,+)} = (8,4589 \pm 0,2523)(GeV/c)^{-2}$$

$$a_0^{(3/2,-)} = (11,684 \pm 0,4136)(GeV/c)^{-2}$$

$$g_{0,0}^{(3/2,+)} = \left[0,0998 \pm 0,0053 + i(0,0236 \pm 0,0013)\right](GeV/c)^{2}$$

$$g_{0,2}^{(3/2,+)} = (-0,0022 \pm 0,0001)(GeV/c)^{4}$$

$$g_{0,0}^{(3/2,-)} = \left[-0,0034 \pm 0,0004 - i(0,0043 \pm 0,0003)\right](GeV/c)^{2}$$

$$g_{0,2}^{(3/2,-)} = \left[(-2,0 \pm 0,5).10^{-5} + i(4,0 \pm 0,3).10^{-5}\right](GeV/c)^{4}$$

$$a_0^{(1/2,+)} = \left[6,6452 \pm 0,1460 + (0,2667 \pm 0,0409)\left(\ln\frac{s}{s_0} - \frac{i\pi}{2}\right)\right](GeV/c)^{-2}$$

$$a_0^{(1/2,-)} = (4,7169 \pm 0,1538)(GeV/c)^{-2}$$

$$g^{(1/2,+)} = \left[0,1632 \pm 0,0047 + i(0,0132 \pm 0,0024)\right](GeV/c)^{2}$$

$$g_{0,2}^{(1/2,+)} = \left[-(4,4 \pm 0,1).10^{-3} + i(6,0 \pm 0,6).10^{-4}\right](GeV/c)^{4}$$

$$g_{0,0}^{(1/2,-)} = \left[-(2,5 \pm 0,2).10^{-3} + i(5,9 \pm 1,1).10^{3}\right](GeV/c)^{2}$$

$$g_{0,2}^{(1/2,-)} = \left[(5,0 \pm 0,7).10^{-4} + i(4,0 \pm 0,8).10^{-4}\right](GeV/c)^{4}$$

$$\xi = 0,7492 \pm 0,0906$$

$$s_0 = 1\,(GeV/c)^{2}$$

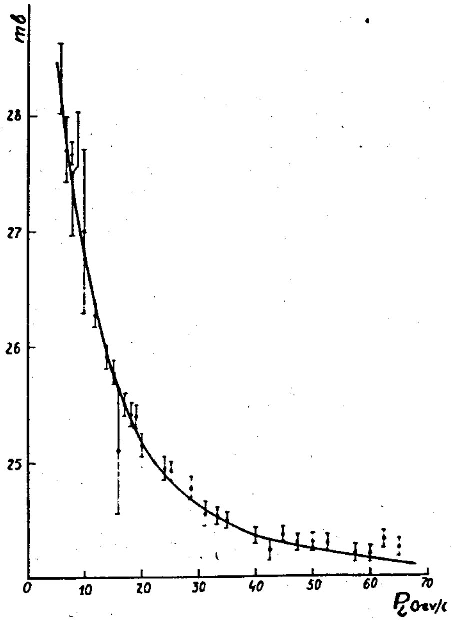

Fig.1. Total cross section for $\pi^- p$ -interaction

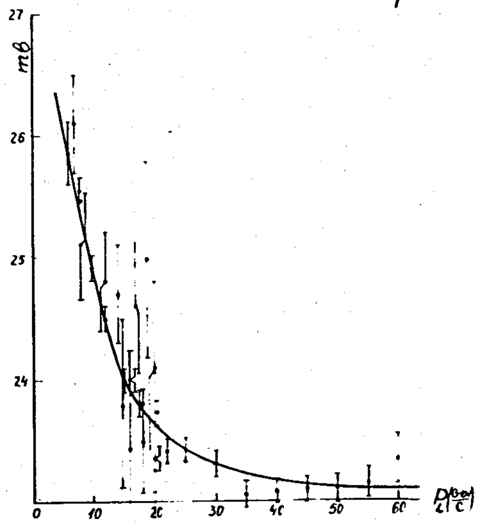

Fig.2. Total cross section for $\pi^+ p$ -interaction

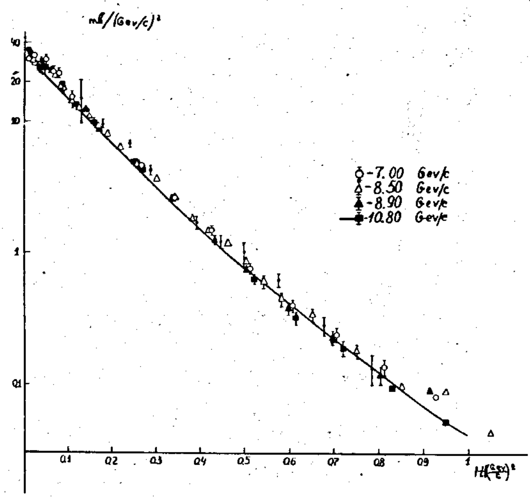

Fig.3. Differential cross section for $\pi^- p$-scattering at P_L =10,8 GeV/c

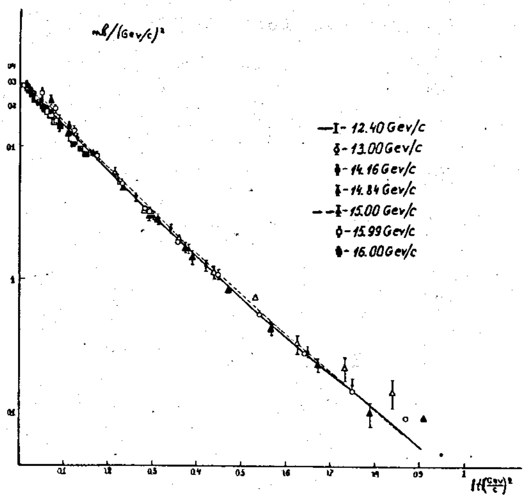

Fig.4. Differential cross section for πp-scattering at P_L = 12,4 and 15,0 GeV/c

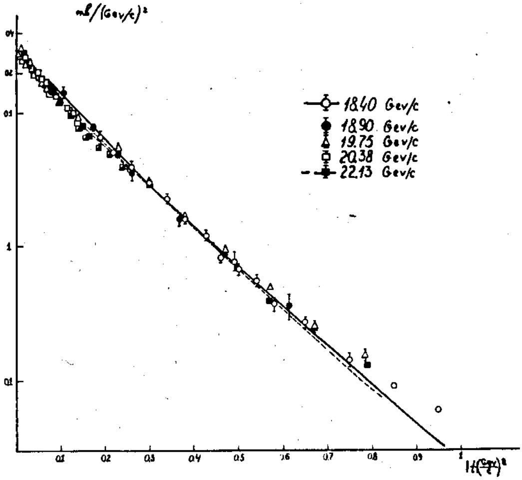

Fig.5. Differential cross section for π^-p-scattering at P_L = 18,4 and 22,13 GeV/c

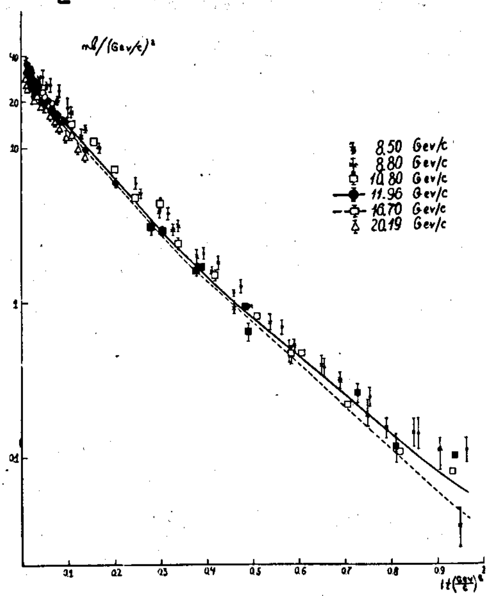

Fig.6. Differential cross section for π^+p-scattering at P_L =11,96 and 16,7 GeV/c

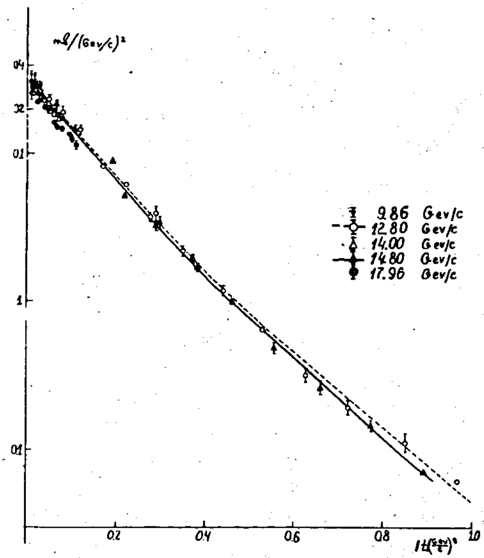

Fig.7. Differential cross section for π^+p -scattering at p_L = 12,8 and 14,8 GeV/c

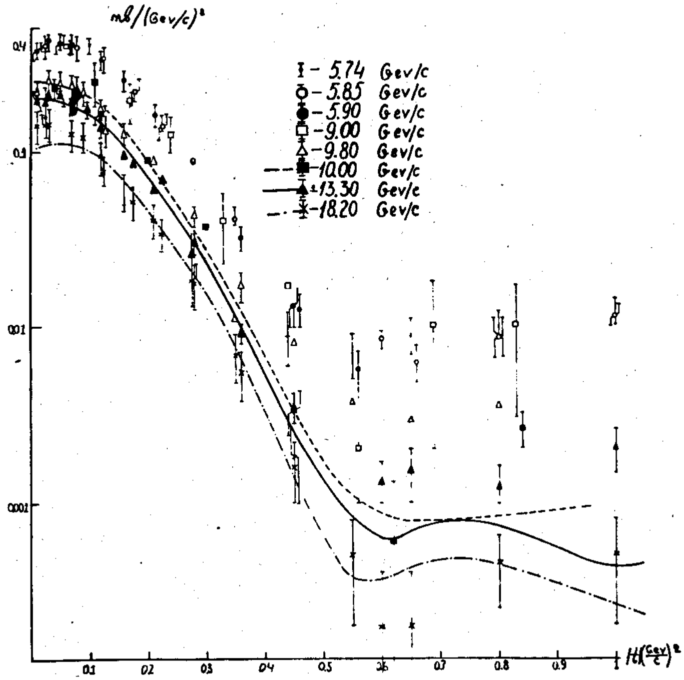

Fig.8. Differential cross section for the reaction $\pi^- p \to \pi^\circ n$ at P_L =10,0; 13,3 and 18,2 GeV/c

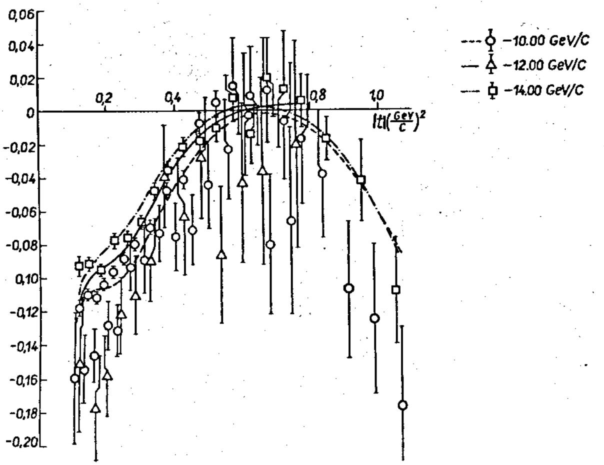

Fig.9. **Polarization in π^-p -scattering at P_L =10,0; 12,0 and 14,0 GeV/c**

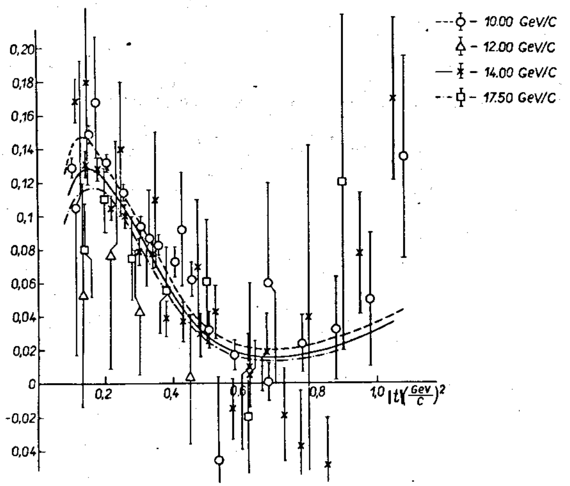

Fig.10. **Polarization in π^+p -scattering at P_L =10,0 14,0 and 17,5 GeV/c**

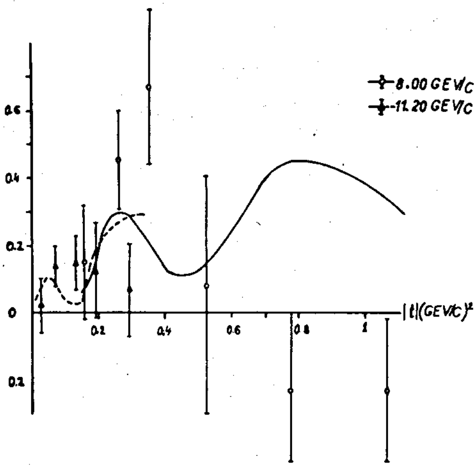

Fig.11. Polarization in $\pi^- p \to \pi^0 n$ reaction at p_L =8,0 and 11,2 GeV/c

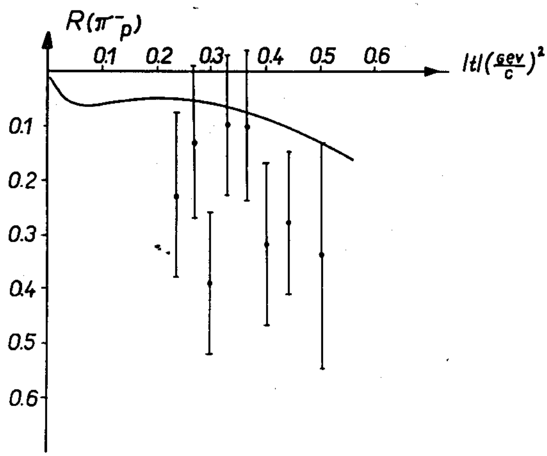

Fig.12. Polarization rotation parameter in $\pi^- p$ -scattering at p_L =16 GeV/c

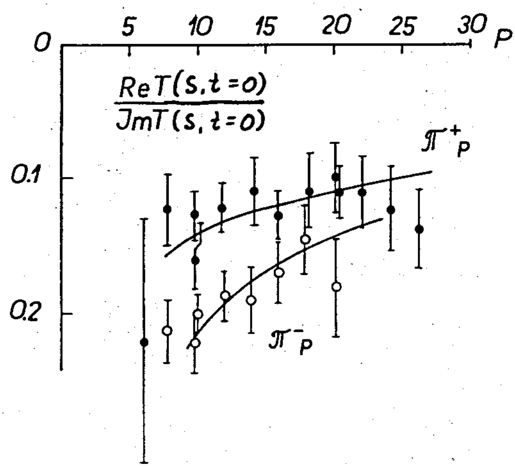

Fig.13. Ratio $\alpha = \frac{Re\,T(s,t=0)}{Im\,T(s,t=0)}$ for $\pi^- p$ and $\pi^+ p$ scattering

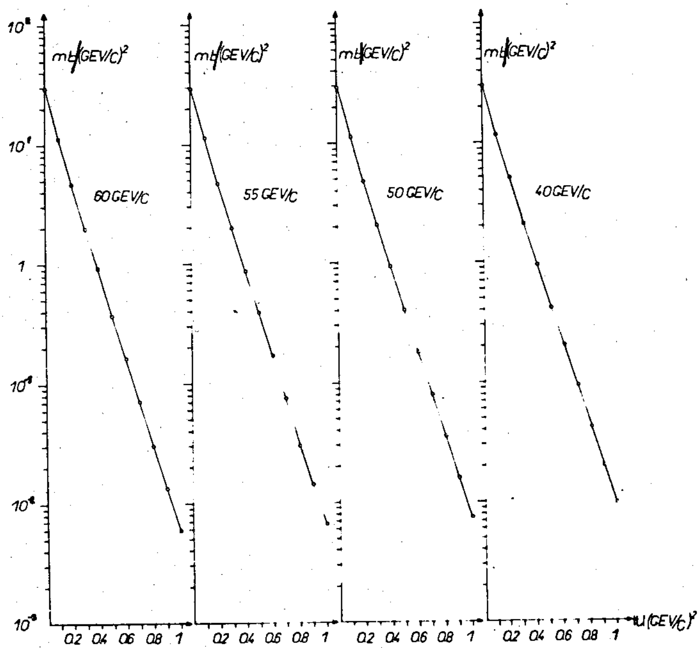

Fig.14. Predictions for differential cross section of $\pi^- p$ -scattering at P_L =40, 50, 55 and 60 GeV/c

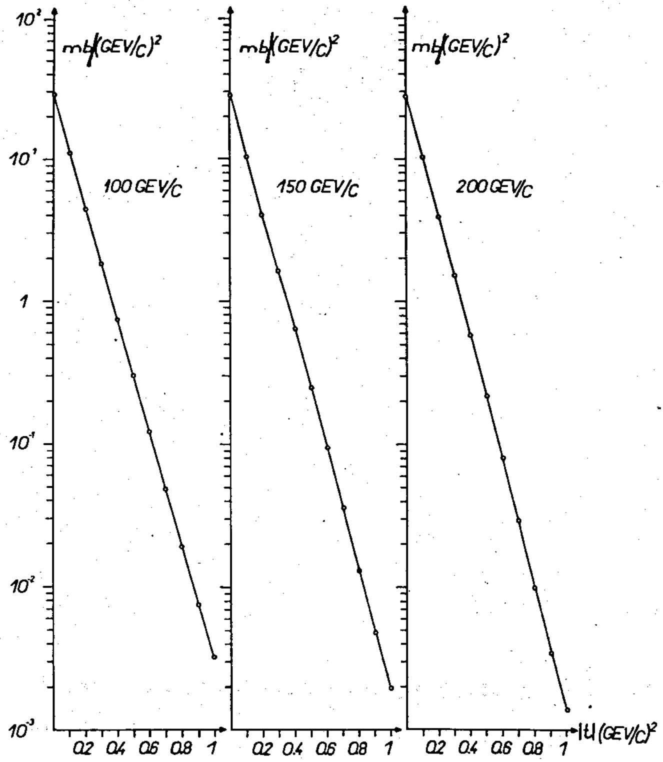

Fig.15. Predictions for differential cross section of $\pi^- p$-scattering at p_L =100, 150 and 200 GeV/c

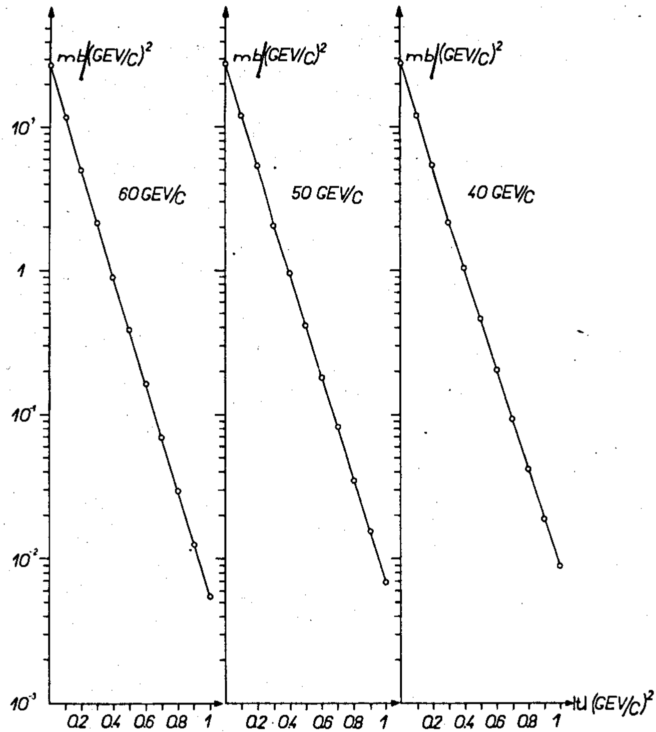

Fig.16. Predictions for differential cross section of π^+p -scattering at p_L =40, 50, and 60 GeV/c

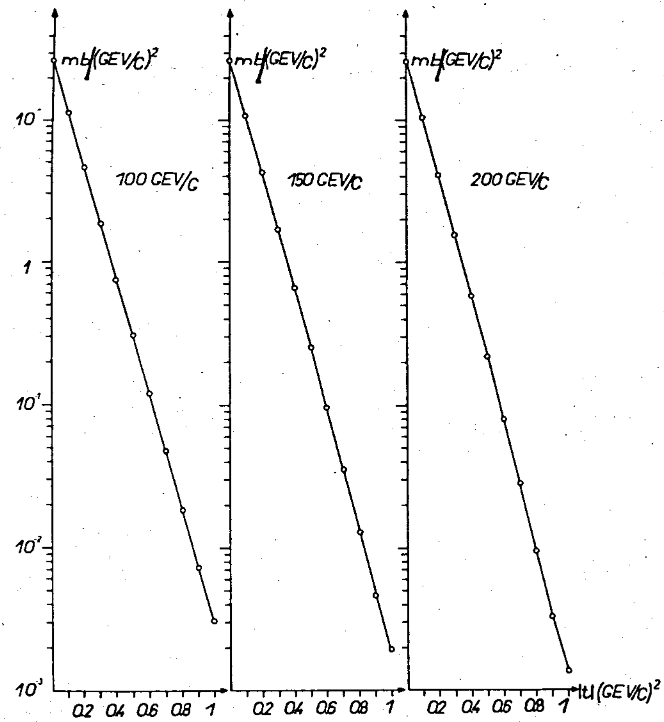

Fig.17. Predictions for differential cross section of π^+p-scattering at p_L =100, 150 and 200 GeV/c

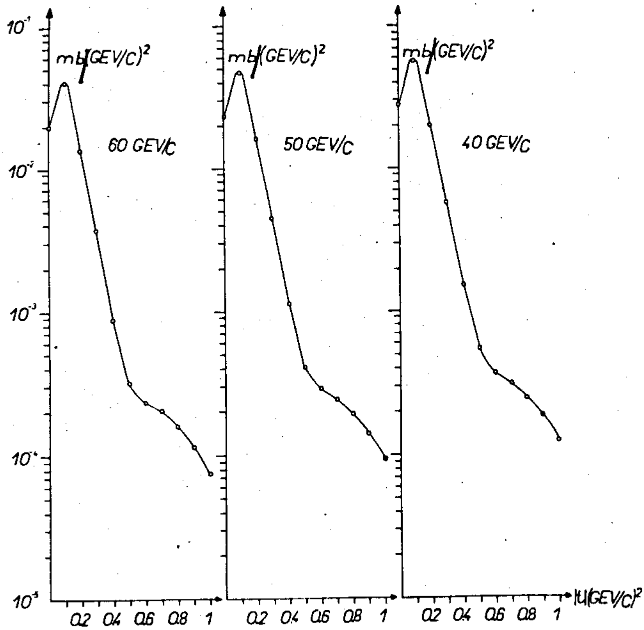

Fig.18. Predictions for differential cross section of $\pi^- p \to \pi^0 n$ reaction at p_L =40, 50 and 60 GeV/c

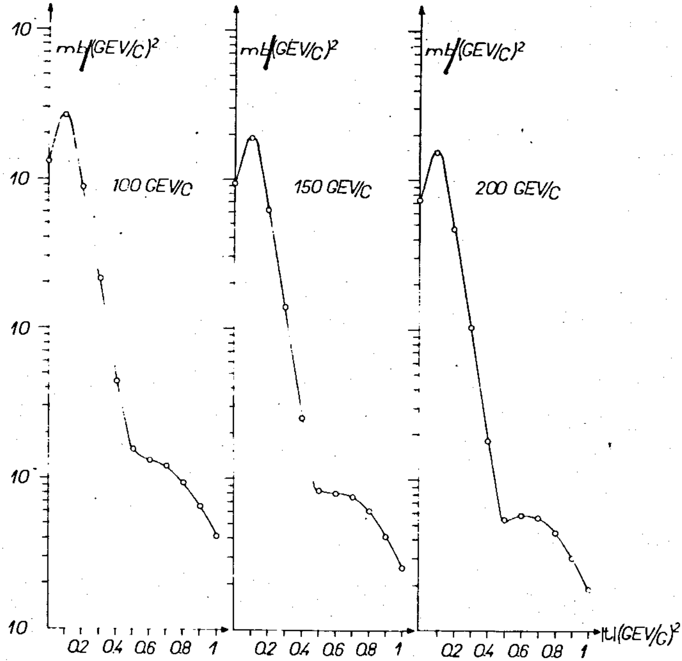

Fig.19. Predictions for differential cross section of $\pi^- p \to \pi^0 n$ reaction at P_L =100, 150 and 200 GeV/c

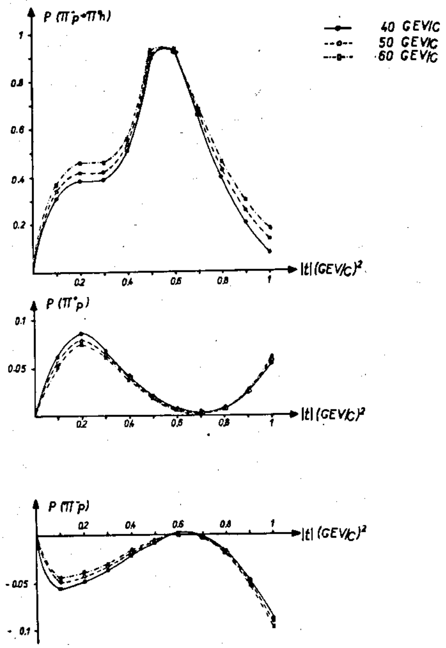

Fig.20. Predictions for polarization in $\pi^- p$, $\pi^+ p$ elastic scattering and $\pi^- p \to \pi^0 n$ reaction at p_L = 40, 50 and 60 GeV/c

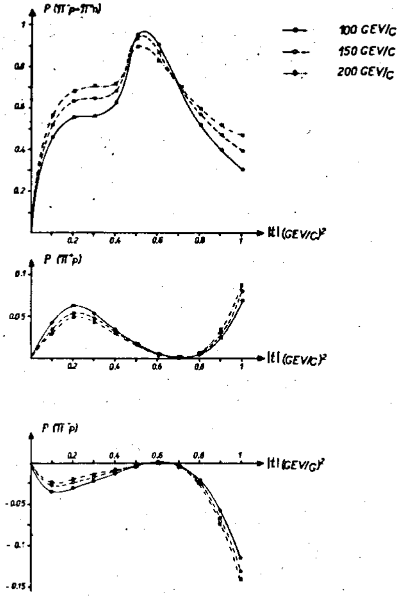

Fig.21. Predictions for polarization in $\pi^- p$, $\pi^+ p$ elastic scattering and $\pi^- p \to \pi^0 n$ reaction at p_L =100, 150 and 200 GeV/c

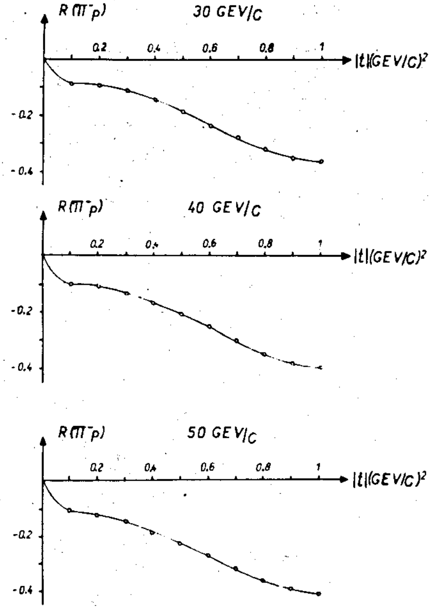

Fig.22. Prediotions for polarization rotation parameter in $\pi^- p$ - scattering at P_L =30, 40 and 50 GeV/o

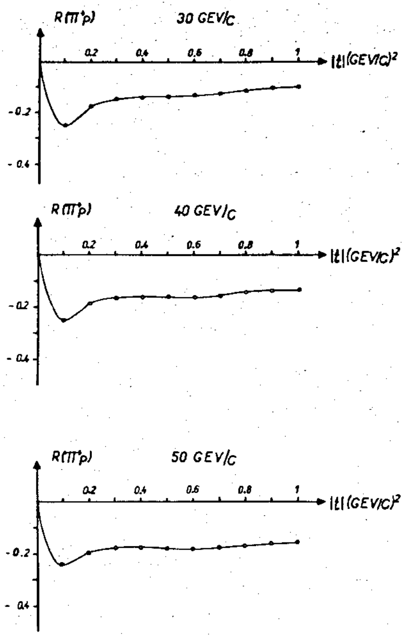

Fig.23. Predictions for polarization rotation parameter in π^+p -scattering at p_L =30, 40 and 50 GeV/c