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A DISPERSION CALCULATION OF THE REAL PART OF THE FORWARD $\pi\,^4$He ELASTIC SCATTERING AMPLITUDE

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I. Introduction

In recent paper of I.V.Falomkin et al.[1] the authors were discussing the inconsistency of the dispersion relation prediction[2] for the real part of the forward πHe^4 scattering amplitude with new experimental data. In this work we analyse in detail the calculation of the real part of the forward amplitude from FDR and show that it is possible to obtain the qualitative agreement with the new experimental data when we shift the resonance peak in the total cross section to the lower energies and to higher values.

The precise form of the FDR is given in section 2. We shall not derive that relation from an analytical assumption (no formal proof of FDR exists so far for our case) because it is discussed in review paper of Ericson and Locher[2]. Subtraction constant $Re f(1)$ and the contribution from the unphysical region are evaluated in section 3. In sections 4 and 5 the contributions from the asymptotical and physical regions, respectively, are calculated. Finally, conclusions are given in section 6.

2. Dispersion Relation

Owing to the fact that the spin and isospin of He^4 is zero, there is only one amplitude completely describing the

πHe^4 scattering. The once-subtracted FDR for the real part of this amplitude in the laboratory system takes the form/2/ (units $\hbar = c = \mu = 1$ are used; μ is the pion mass).

$$\mathrm{Re}\, f(\omega) = \mathrm{Re}\, f(1) + \frac{2(\omega^2-1)}{\pi} P\left\{ \int_{\omega_{TN}}^{1} + \int_{1}^{17.14} + \int_{17.14}^{\infty} \right\} \frac{\omega' \, \mathrm{Im}\, f(\omega')}{(\omega'^2-1)(\omega'^2-\omega^2)} d\omega' \quad (1)$$

where ω is the incident pion laboratory total energy and $\omega_{TN} \approx 0.143\ [\mu]$ is the threshold of the inelastic process $\pi He^4 \longrightarrow TN$ (T means threenucleon) starting below the elastic threshold $\omega = 1[\mu]$. There are present no poles, since the πHe^4 system has isospin unity and no bound fournucleon state of this isospin exists. The integrals in (1), which we denote by $\mathcal{J}_1$, $\mathcal{J}_2$, $\mathcal{J}_3$, are contributions to $\mathrm{Re}\, f(\omega)$ from the unphysical, physical and asymptotical regions respectively. Such a decomposition of the integrals is appropriate since the main source of our information about the behaviour of $\mathrm{Re}\, f(\omega)$ is in the total cross section (through optical theorem) and this is known only for $1 < \omega \leq 17.14\ [\mu]$ There is no experimental information about σ_{TOT} for $\omega > 17.14[\mu]$ and no information on the $\mathrm{Im}\, f(\omega)$ can be obtained from direct measurements in unphysical region. Therefore $\mathrm{Im}\, f(\omega)$ must be calculated from some models in these energy intervals.

To find the $\mathrm{Re}\, f(\omega)$ means to evaluate the subtraction constant $\mathrm{Re}\, f(1)$ and the contributions from the integrals $\mathcal{J}_1$, $\mathcal{J}_2$ and $\mathcal{J}_3$.

3. The Unphysical Region Contribution

Before the evaluation of the unphysical region contribution to the $\mathrm{Re}\, f(\omega)$ we say a few words about the subtraction constant. This is taken as a value of $\mathrm{Re}\, f(\omega)$ at the elastic threshold and is equal to the real part of the complex S-wave scattering length. To show it we are in need of the forward scattering amplitude (FSA) expanded in terms of partial wave amplitudes.

$$f(\omega) = \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(\omega), \ (\text{because } P_\ell(\vartheta=0)=1) \quad (2)$$

where

$$f_\ell(\omega) = \frac{1}{k} e^{i\delta_\ell} \sin \delta_\ell \quad (3)$$

k is c.m. momentum and δ_ℓ are complex πHe^4 phase shifts. Defining the πHe^4 complex scattering lengths by

$$k^{2\ell+1} \cot g\, \delta_\ell = \frac{1}{A_\ell} + B_\ell k^2 \approx \frac{1}{A_\ell}\ ; \quad A_\ell = a_\ell + i b_\ell \quad (4)$$

and taking into account the threshold behaviour of the partial wave amplitudes

$$f_\ell(\omega) \approx A_\ell k^{2\ell} \quad (5)$$

the FSA in (2) may be written in the following form

$$f(\omega) \approx A_0 + 3A_1 k^2 + 5A_2 k^4 + \dots \quad (6)$$

The limit $k \to 0$ in relation (6) gives $f(1) = A_0$ and from here

$$\mathrm{Re}\, f(1) = a_0 \quad (7)$$

Numerical values of a_0 and b_0 , a_1 , b_1 (these will be useful for us later) are calculated from the phase-shifts of Nordberg and Kinsey/3/

$$\begin{aligned} a_0 &= -0.133 \pm 0.003 \quad [\mu^{-1}] \\ b_0 &= +0.081 \pm 0.006 \quad [\mu^{-1}] \\ a_1 &= +0.265 \pm 0.003 \quad [\mu^{-3}] \\ b_1 &= +0.022 \pm 0.022 \quad [\mu^{-3}] \end{aligned} \tag{8}$$

Now we evaluate the unphysical region contribution to the $\mathcal{R}e\, f(\omega)$ represented by the integral

$$\mathcal{J}_1 = \frac{2(\omega^2-1)}{\pi} \int_{\omega_{TH}}^{1} \frac{\omega'\ \mathcal{J}m\, f(\omega')}{(\omega'^2-1)(\omega'^2-\omega^2)}\, d\omega' \tag{9}$$

where $\mathcal{J}m\, f(\omega')$ is found by means of the analytical continuation of the zero-effective-range amplitude

$$f(\omega) \approx \sum_{\ell=0}^{\infty} (2\ell+1) \frac{A_\ell k^{2\ell}}{1 - i A_\ell k^{2\ell+1}} \tag{10}$$

obtained from (2) writing (3) in the form

$$f_\ell(\omega) = \frac{1}{k \cot g\, \delta_\ell - ik} \tag{11}$$

and substituting in the last formula $k \cot g\, \delta_\ell$ from the relation (4).

Further we keep only first two terms in the sum of the relation (10). The reason of this is following. There are dominating only S - and P -waves in the physical region at low energies and we hope higher partial waves remain negligible

also after the analytical continuation of the formula (10) into the unphysical region. Then (10) takes the form

$$f(\omega) \approx \frac{A_0}{1 - i A_0 k} + 3\, \frac{A_1 k^2}{1 - i A_1 k^3} \tag{12}$$

where k as a function of the pion laboratory total energy ω is

$$k = K_1 \left(\frac{\omega^2 - 1}{\omega + K_2} \right)^{1/2} \tag{13}$$

and constants K_1 , K_2 written in terms of the mass of helium m_α have the form

$$K_1 = \left(\frac{m_\alpha}{2} \right)^{1/2}, \quad K_2 = \frac{m_\alpha^2 + 1}{2 m_\alpha} \tag{14}$$

Having in mind that k in (12) for $\omega < 1$ is complex

$$k = i K_1 \left(\frac{1 - \omega^2}{K_2 + \omega} \right)^{1/2} \tag{15}$$

for the imaginary part of the FSA in unphysical region we obtain

$$\mathcal{J}m\, f(\omega) \approx \frac{b_0}{1 + 2 a_0 K_1 \left(\frac{1-\omega^2}{K_2+\omega}\right)^{1/2} + (a_0^2 + b_0^2) K_1^2 \left(\frac{1-\omega^2}{K_2+\omega}\right)} - \frac{3 b_1 K_1^2 \left(\frac{1-\omega^2}{K_2+\omega}\right)}{1 - 2 a_1 K_1^3 \left(\frac{1-\omega^2}{K_2+\omega}\right)^{3/2} + (a_1^2 + b_1^2) K_1^6 \left(\frac{1-\omega^2}{K_2+\omega}\right)^3} \tag{16}$$

This explicit form of the $\mathcal{J}m\, f(\omega)$ (with the values of the scattering lengths (8)) was used in the dispersion integral (9) to calculate the unphysical region contribution to the $\mathcal{R}e\, f(\omega)$ which is graphically shown in fig. 1.

4. The Contribution from Asymptotical Region

We mentioned in section 2. that we have no information about σ_{TOT} for $\omega > 17.14\ [\mu]$. To obtain the explicit form of σ_{TOT} in the asymptotical region we proceed from two assumptions:

$$\begin{aligned} &1. \quad \text{for} \quad \omega \to \infty \qquad \sigma_{\pi He^4} \geq \sigma_{\pi N} \\ &2. \quad \text{for} \quad \omega > 17.14\,[\mu] \quad \sigma_{\pi He^4}(\omega) \leq \sigma_{\pi He^4}(17.14) \equiv 5.24\,[\mu^{-2}] \end{aligned} \tag{17}$$

Having in mind asymptotical behaviours of σ_{TOT} of other processes (as e.g. πN, KN and NN) we write $\sigma_{\pi He^4}$ in the form

$$\sigma_{TOT} \approx C_1 + \frac{C_2}{\omega} \tag{18}$$

where constants $C_1\,[\mu^{-2}]$ and $C_2\,[\mu^{-1}]$ are determined from assumptions (17). Then we have two extreme forms for σ_{TOT} in the asymptotical region

$$\begin{aligned} \sigma_{MAX} &= 5.24\ [\mu^{-2}] \\ \sigma_{MIN} &= 1.2 + \frac{69.25}{\omega}\ [\mu^{-2}] \end{aligned} \tag{19}$$

which when we put through optical theorem

$$\mathcal{I}m\, f(\omega) = \frac{k}{4\pi}\, \sigma_{TOT}(\omega) \tag{20}$$

into the integral

$$J_3 = \frac{2(\omega^2-1)}{\pi} \int_{17.14}^{\infty} \frac{\omega'\, \mathcal{I}m\, f(\omega')}{(\omega'^2-1)(\omega'^2-\omega^2)}\, d\omega' \tag{21}$$

we obtain the asymptotical region contributions (see fig. 1) J_{3MAX} and J_{3MIN} respectively.

5. The Physical Region Contribution and $\mathcal{R}e\, f(\omega)$

In this region $\mathcal{I}m\, f(\omega)$ is known from the experimental values of the total cross section, which are shown in fig. 2. It is not so simple to find one smooth curve sufficiently good interpolating the whole experimentally known region of σ_{TOT}. To avoid this difficulty we decompose the interval $1 < \omega \leq 17.14\ [\mu]$ into three parts each of which is interpolated by means of the functions (see fig. 2).

$$\begin{aligned} \sigma_1 &= \frac{4.6\,\omega}{(\omega-2.16)^2+0.7} && \text{for} \quad 1 < \omega < 3.2914\,[\mu] \\ \sigma_2 &= 1.03\,\omega^2 - 9.89\,\omega + 29.04 && \text{for} \quad 3.2914 < \omega < 4.6789\,[\mu] \\ \sigma_3 &= -0.046\,\omega^2 + 1.001\,\omega + 1.638 && \text{for} \quad 4.6789 < \omega < 17.14\,[\mu] \end{aligned} \tag{22}$$

respectively. Now using the optical theorem (20) (with k given by (13)) and (22) we can evaluate the physical region contribution

$$J_2 = \frac{2(\omega^2-1)}{\pi}\, \mathcal{P} \left\{ \int_1^{3.2914} \mathcal{I}m_1 f(\omega') + \int_{3.2914}^{4.6789} \mathcal{I}m_2 f(\omega') + \int_{4.6789}^{17.14} \mathcal{I}m_3 f(\omega') \right\} \frac{\omega'}{(\omega'^2-1)(\omega'^2-\omega^2)}\, d\omega' \tag{23}$$

represented by the curve J_2 in fig. 1.

Then the sum of all contributions gives

$$\mathcal{R}e\, f(\omega) = a_0 + J_1(\omega) + J_2(\omega) + J_3(\omega) \tag{24}$$

graphically shown in fig.3 where it is also compared with the new experimental results /4/. One can see immediately that the real part does not agree well with the experimental data. To know the cause let us look on the independent contributions from the integrals in (23) shown in fig.4. We see only that the first integral is (comparably also with the unphysical and asymptotical region contributions) mainly responsible for the shape of the $\mathcal{R}e\, f(\omega)$ at low energies. In other words, to obtain the better behaviour of $\mathcal{R}e\, f(\omega)$ we have to change the form of the interpolating function or the experimental σ_{TOT} at the resonant region. If we shift the maximum of the interpolating curve to the lower energies and to the higher values (see curves σ_4 and σ_5 in fig.2) we obtain the qualitatively good agreement of the $\mathcal{R}e\, f(\omega)$ with the new experimental data, as is shown in fig.5.

6. Conclusions

In this section we summarize what has been learned from the analysis of the πHe^4 FDR. As is possible to see in fig.1, the main contribution to the $\mathcal{R}e f(\omega)$ at low energies is from the physical region where the total charge independent cross section is used as an experimental input in FDR. The unphysical region contribution is for $\omega > 1.46\ [\mu]$ roughly speaking cancelled out with the subtraction constant. The contribution from the asymptotical region is appearing to be remarkable only for $\omega > 3\ [\mu]$ where, as yet, there is no experimental information about the $\mathcal{R}e\, f(\omega)$.

We believe that it would be desirable to repeat the analysis when more experimental information about the total cross section in resonant region will be available.

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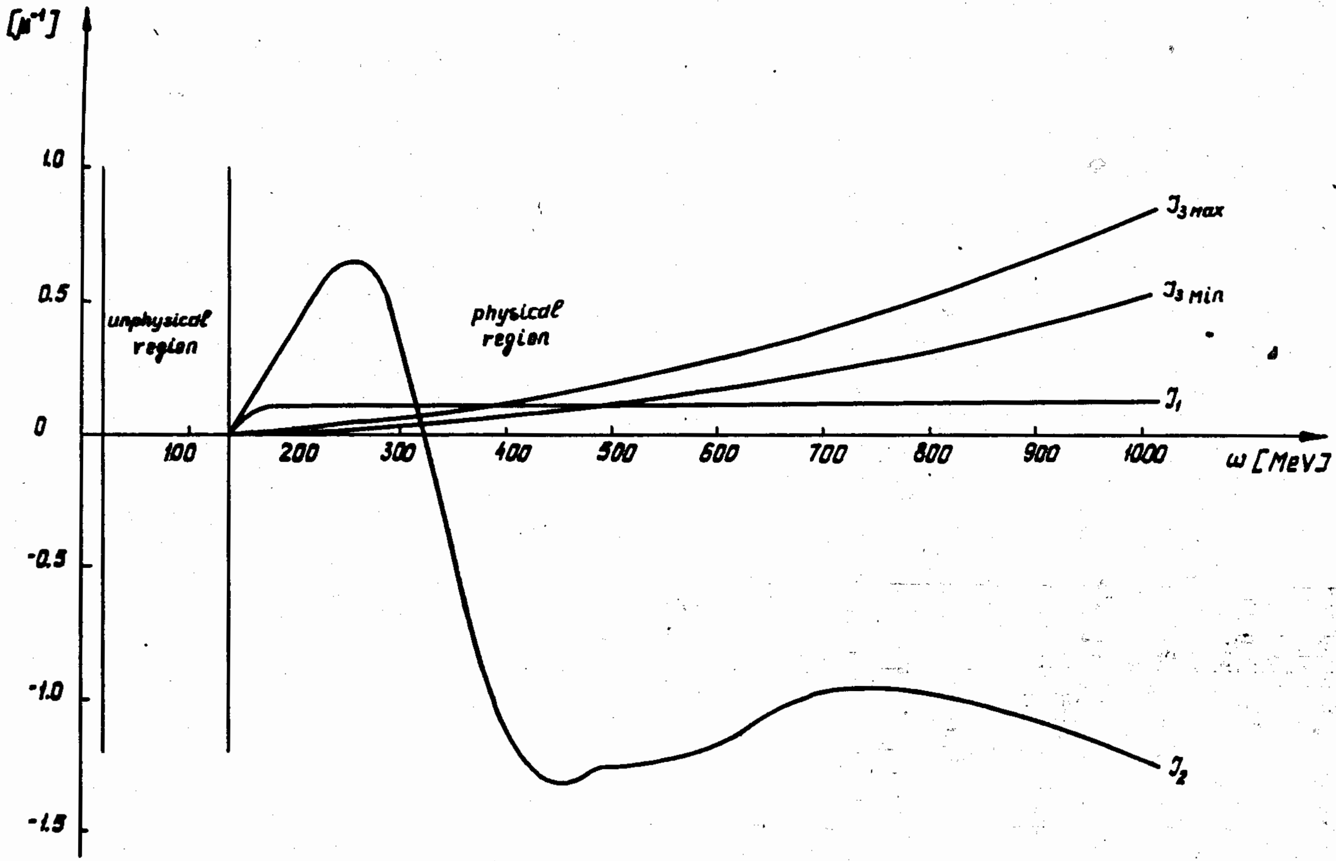

[μ⁻¹]
1.0
0.5
0
-0.5
-1.0
-1.5
unphysical region
physical region
100
200
300
400
500
600
700
800
900
1000
ω [MeV]
$J_{3\,max}$
$J_{3\,min}$
J_1
J_2

Fig. 1

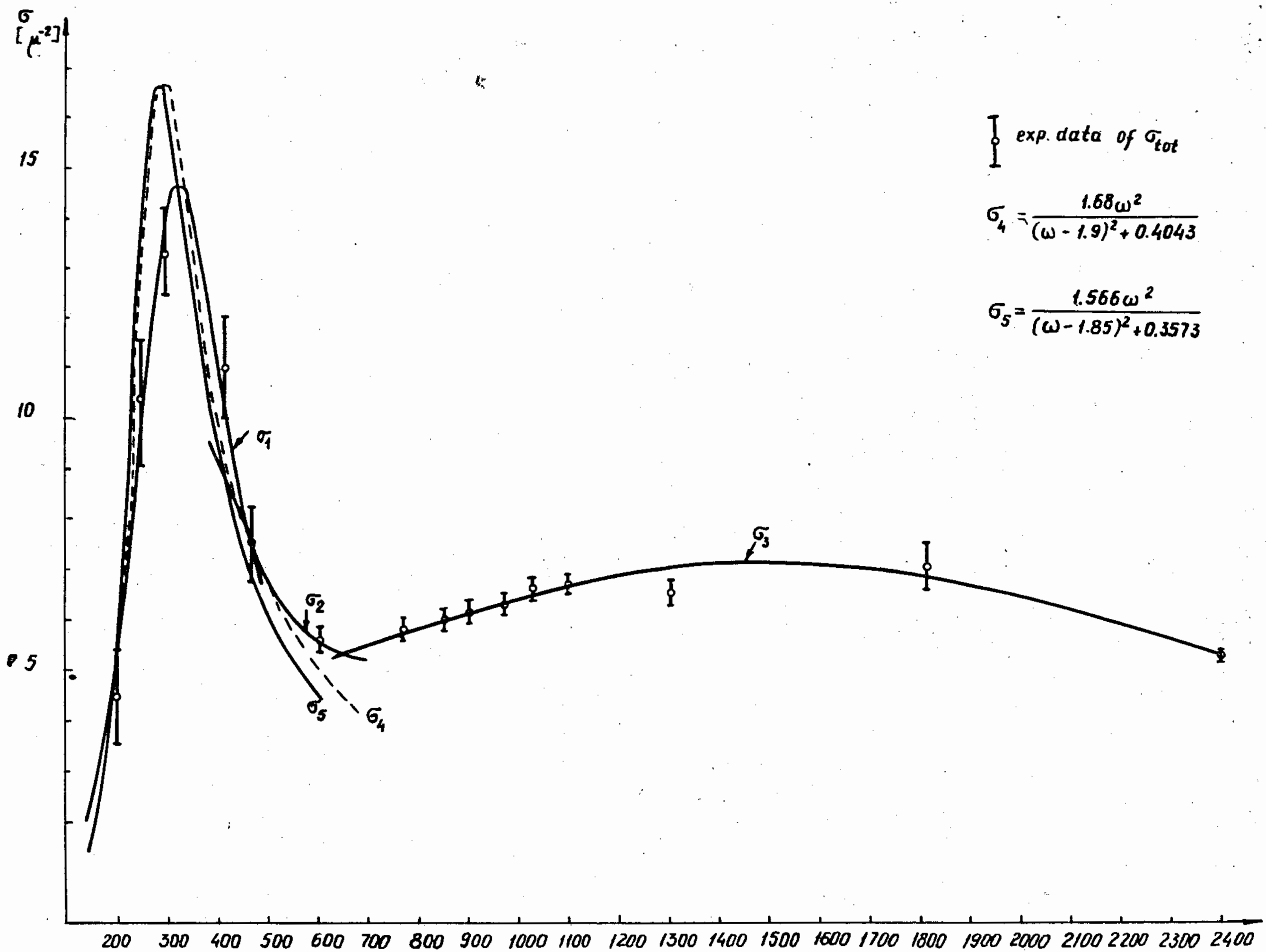
σ [μ^-2]
15
10
5
exp. data of σ_tot
σ_4 = 1.68ω² / ((ω - 1.9)² + 0.4043)
σ_5 = 1.566ω² / ((ω - 1.85)² + 0.3573)
σ_1
σ_2
σ_3
σ_4
σ_5
200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000 2100 2200 2300 2400
ω [MeV]

Fig. 2

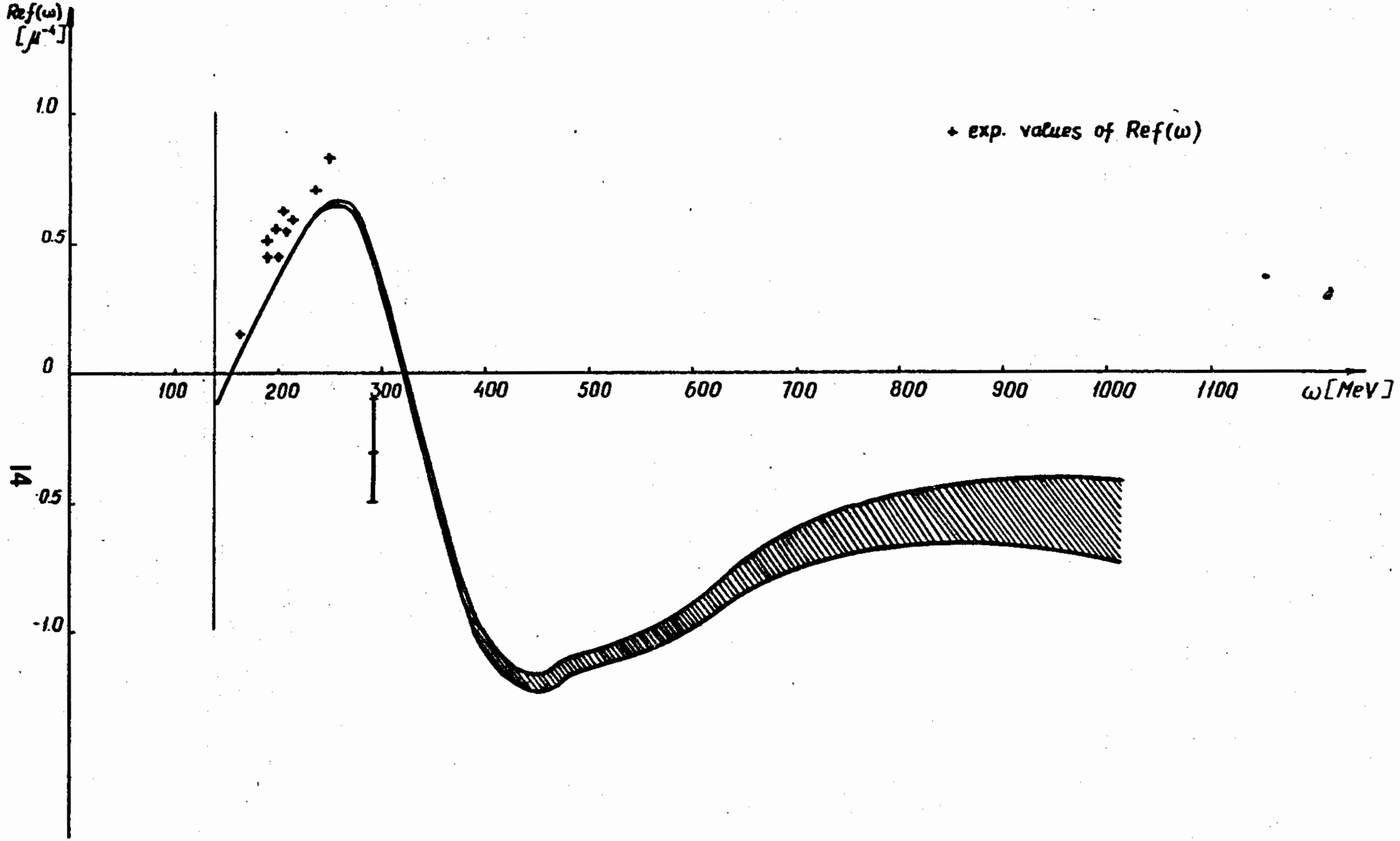
Ref(ω) [μ⁻¹]
1.0
0.5
0
-0.5
-1.0
100
200
300
400
500
600
700
800
900
1000
1100
ω [MeV]
+ exp. values of Ref(ω)

Fig. 3

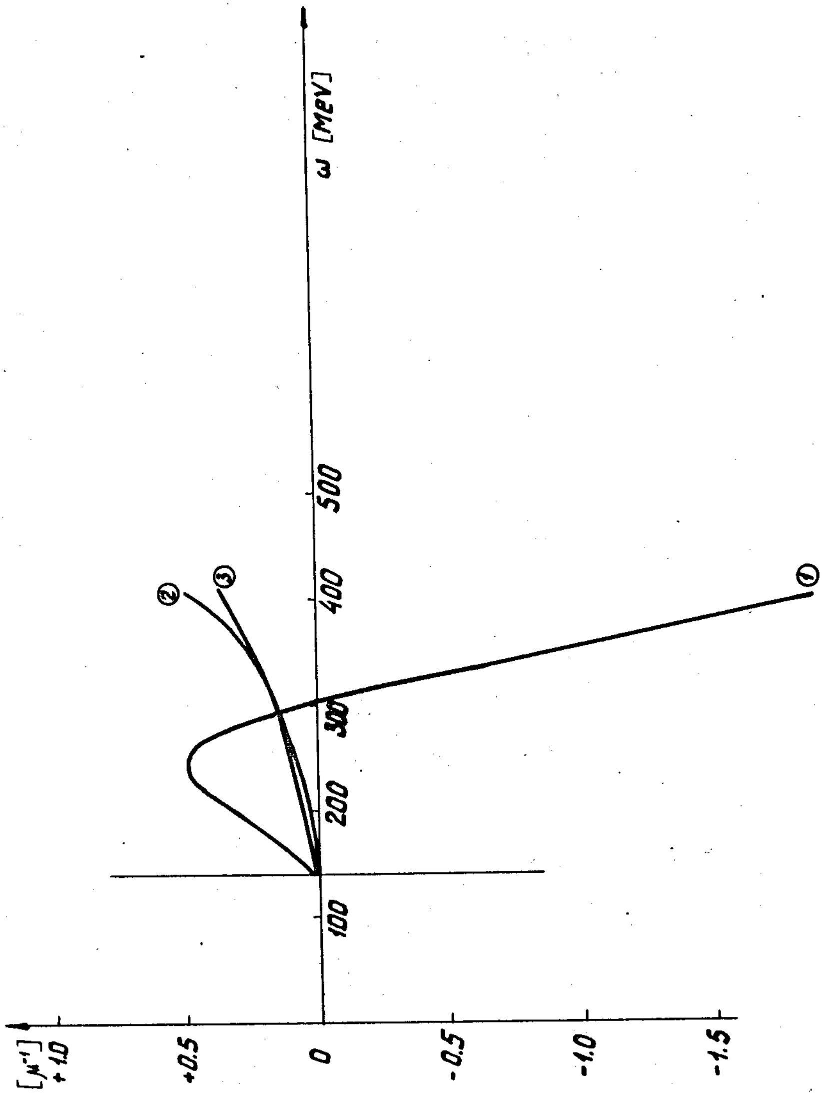

Fig. 4

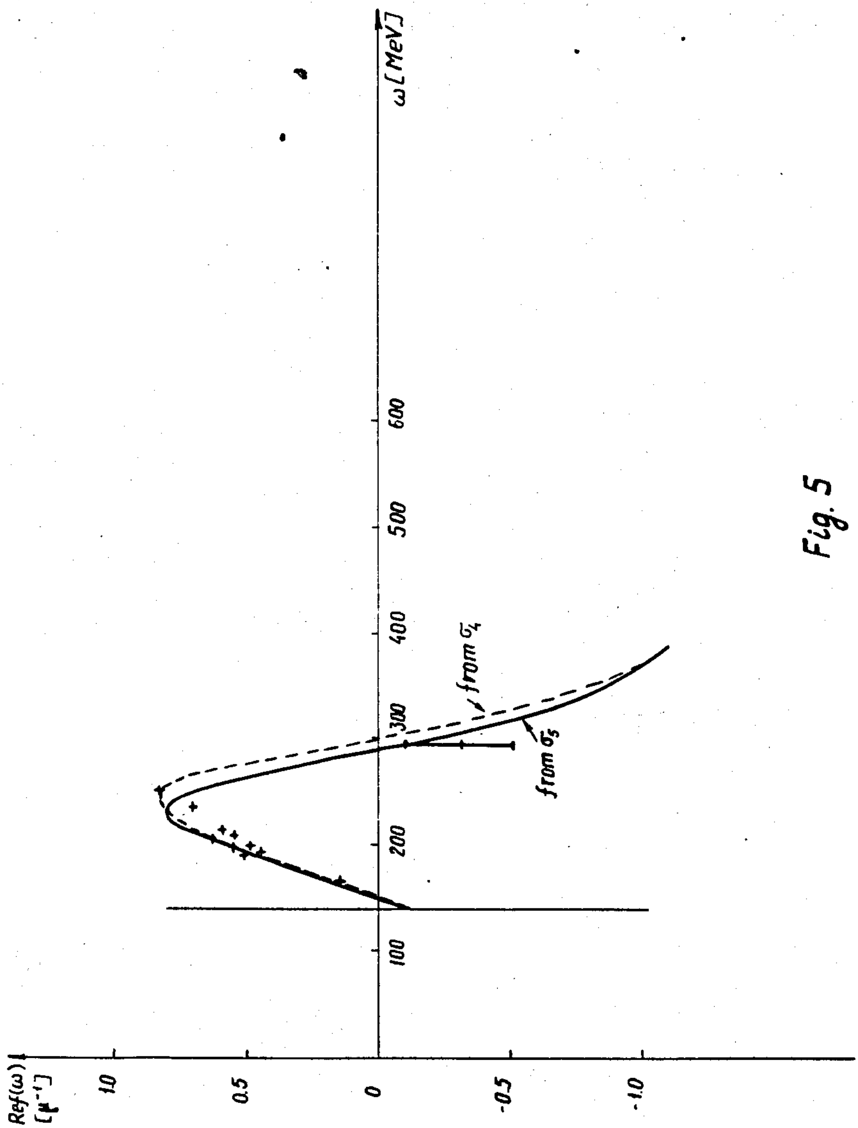

Fig. 5