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ON SUM RULES FOR THE PHOTON-LEPTON INTERACTION CROSS-SECTIONS

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ON SUM RULES FOR THE PHOTON-LEPTON INTERACTION CROSS-SECTIONS

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О правилах сумм для сечений взаимодействия фотонов с лептонами

Работа посвящена получению и проверке по теории возмущений релятивистского правила сумм для флюктуации дипольного момента свободного лептона и лептона, находящегося в кулоновском поле. Рассмотрено применение правила сумм для магнитных моментов для оценки влияния гипотетических "тяжелых" лептонов на магнитные моменты электрона и мюона.

Сообщение Объединенного института ядерных исследований
Дубна, 1972

Gerasimov S.B., Moulin J. **E2 - 6722**

On Sum Rules for the Photon-Lepton Interaction Cross-Sections

The relativistic dipole moment fluctuation sum rules for a free lepton and for a lepton bound in a Coulomb field are derived and checked in the lowest order of perturbation theory. The dispersion sum rule for the anomalous magnetic moment is applied to estimate the influence of a possible existence of heavy "excited" leptons on the electron and muon magnetic moments.

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I. Introduction

In the present work we examine some applications of sum rules (s.r.) in the framework of quantum electrodynamics of leptons.

S.r. connect the integrals of the total photon-lepton interaction cross sections and static electromagnetic properties of leptons.

As is known /1/, the charge radius and leptonic magnetic moments have been calculated by perturbation theory and verified experimentally up to terms of the order of α^2 and α^3, respectively $(\alpha = e^2/4\pi = 1/137,\ \hbar = c = 1)$. The electromagnetic properties of the simplest hydrogen-like atoms are known too and testify to the exceptionally good agreement of quantum electrodynamics with experiment /1/. The very high degree of reliability and elaboration of the theory of leptonic electromagnetic interactions permits us to use the known results of the ordinary approach for testing results of other methods and approaches less characteristic for quantum electrodynamics - for example, "parton" models and techniques of "infinite" momentum /2/ or dispersion theory methods /3/. The aim of the present paper is, firstly, to discuss the relativistic s.r. for the dipole moment fluctuation of a free lepton and of a bound lepton in the field of a Coulomb center.

Using the techniques of a "parton" model in an infinite momentum frame, we obtained a relativistic generalization of an old nonrelativistic s.r. for electric dipole photoabsorption known in the theory of atomic /4/ and nuclear photoeffect /5/.

We also examine the application of the dispersion s.r. for anomalous magnetic moments /6-8/ to leptons and estimate the influence of a possible existence of "heavy" leptons /9/ on electron and muon magnetic moments.

2. Sum Rule for Dipole Moment Fluctuation of a Lepton. Derivation and Verification by Perturbation Theory

The techniques of dipole moment akgebra in an infinite momentum frame were first applied for derivation of the Cabibbo-Radicati s.r. /10/ . Our way for obtaining operator relations between currents and their moments lies out of current algebra and depends on the model assumptions about the particle structure and the structure of current operator in the framework of composite models or "parton" models.

An analogous approach was previously used in the work /11/ where s.r. for electro- and photoproduction on proton was obtained in the framework of a non-relativistic quark model.

However, the uncertainties existing in the experimental interpretation and verification of the obtained relations in /11/ hamper the verification of the initial model assumption themselves. It is therefore very desirable to verify the parton approach results in the framework of field theory models, where it is possible to carry out all necessary calculations and comparisons. It is one of the reasons why we carried out the following calculation.

Let us consider the state vector expansion of a "physical" ("dressed") electron in terms of his constituent states (in the sense of a parton model /2/) of the point Dirac electrons and virtual photons:

$$|e^-\rangle_{phys} = c_0\,|e^-\rangle + c_1\,|e^-, \gamma\rangle + c_2\,|e^-, 2\gamma\rangle + c_3\,|e^-, e^+, e^-\rangle + \ldots \quad (1)$$

$$\sum_n |c_n|^2 = 1 .$$

We shall take into account in (1) the states containing only one electron and an arbitrary number of photons, neglecting for the moment the presence of electron-positron pairs (we shall return to the discussion of that question later). Then, proceeding from the definitions of dipole moment operator and the mean-square radius of the charge distribution we get the following operator relation:

$$\vec{D}^2(\vec{x}, t) = e^2 \int \vec{x}^{\,2} \rho(\vec{x}, t)\, d^3x . \quad (2)$$

Sandwiching Eq. (2) by one-electron states and using standard infinite-momentum techniques /10/ we obtain the s.r.

$$4\pi^2 a\left(\frac{1}{3}\langle r_1^2\rangle - \frac{1}{4m^2}\kappa^2\right) = \int_0^\infty \frac{d\omega}{\omega}\sigma_{tot}(\omega), \tag{3}$$

where $\langle r_1^2\rangle = -6F_1'(0)$ is the charge radius, $\kappa = F_2(0)$ - the anomalous magnetic moment, $F_1(q^2)$ and $F_2(q^2)$ being the Dirac and Pauli form-factors and $\sigma_{tot}(\omega)$ is the total cross section of interaction of photons with the considered leptons. It is clear, that the convergence condition of integral in (3) will be violated if we include the lepton pair production processes.

This is the reason why the s.r. (3) may have a strict sense only in the first order of perturbation theory. The cross section $\sigma_{tot}(\omega)$ coincides then with total cross section of scattering of photons by electrons.

The integral in (3) converges at the upper limit, but, because of the zero photon mass and of the non zero limit $\lim_{\omega\to 0}\sigma_{tot}(\omega) \neq 0$ we get a logarithmic divergence in the photon low frequency domain.

However it is just the same type of infrared divergence which appears in the calculation of $\langle r_1^2\rangle$ with the lower order Feyman diagram represented in Fig.I.

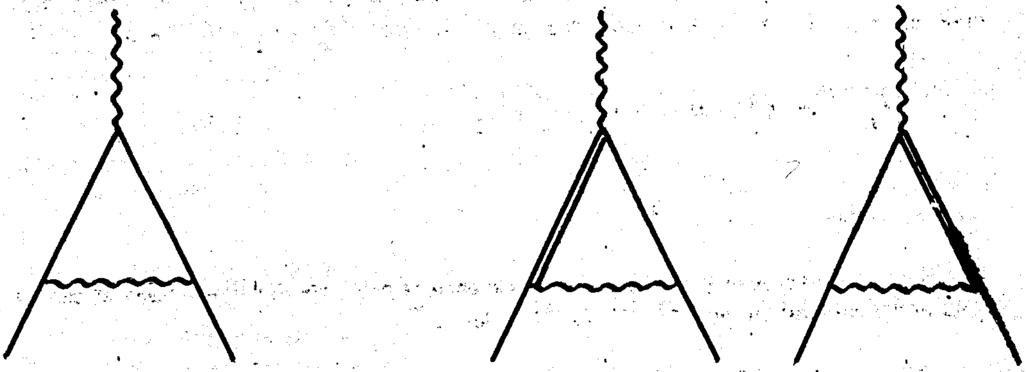

Fig.I. Fig.2.

The lowest order vertex diagrams. The solid line represents electron or muon, the wavy line corresponds to photon and the double line corresponds to the "excited" lepton.

A formal elimination of infrared singularities may be carried out by two equivalent methods: either by introducing a certain minimal frequency ω_{min} or, by giving the photon a finite rest mass.

We choose the first method and shall take a photonic minimal frequency ω_{min} as lower limit of all divergent integrals. The calculation of the electronic radius $<r_1^2>^{(1)}$ corresponding to Fig.I. gives:

$$<r_1^2>^{(1)} = \frac{2\alpha}{\pi m^2}\left(\ln\frac{m}{2\omega_{min}} + \frac{11}{24}\right) \tag{4}$$

Neglecting $\kappa^2 \approx \alpha^2$ and carrying out an elementary calculation of the integral in (3) using the classical Klein-Nishina-Tamm formula for $\sigma_{tot}(\omega)$ we conclude that s.r. (3), and consequently the spin-averaged matrix element of the operator relation (2), are true in lowest order of perturbation theory.

Now, we discuss briefly the role of pair configuration in the expansion (I).

Let us carry out a "numerical experiment", making an ad hoc assumption that pair states are unimportant for the determination of the electronic charge radius of order α^2. Then, instead of the operator relation (2) we should obtain a relation between matrix elements of (2), for states without pair configurations.

Formally s.r. (3) conserves its form but now the pair production diffractive processes which would lead to a divergent integral do not contribute to $\sigma_{tot}(\omega)$.

Up to order α^2, the radiative corrections of order α to the Compton effect and the double Compton effect will contribute to the integral.

We performed a numerical calculation of the integral, using the results of Mork's work /12/. It gives, however a value around 5 times less than the one of the left-hand side of Eq. (3), if we put there the following values for $<r_1^2>^{(2)}$ /1/ and $\kappa^{(1)}$:

$$<r_1^2>^{(2)} = -6F_1'(0) = 2,82\cdot\left(\frac{\alpha}{\pi}\right)^2\frac{1}{m^2}$$

$$\kappa^{(1)} = \frac{\alpha}{2\pi}. \tag{5}$$

The observed discrepancy shows that the presence of pair configurations plays an essential role in the calculation of $<r_1^2>^{(2)}$ to the order α^2.

The question of a possible corresponding generalization of s.r. (3) which would correctly take into account terms of the order of α^2, remains therefore open.

3. Sum Rule for Interaction of Photons with Atomic Electrons

Let us consider an electron in a bound state in the Coulomb field of a heavy spinless particle of mass $M(m/M \ll 1)$.

If we take the matrix element of the operator relation (2) in the system of "zero impulse", i.e. in the rest frame of the atom as a whole, then we get the well-known sum rule for electric dipole photoabsorption /4,5/

$$\frac{4\pi^2\alpha}{3}\langle r^2\rangle = \int_0^\infty \frac{d\omega}{\omega}\sigma_{E1}(\omega), \tag{6}$$

where $\sigma_{E1}(\omega)$ is the electric dipole photoeffect cross section, calculated in the static approximation neglecting the retardation effects,

$$\langle r^2\rangle = \int r^2 |\Psi(\vec{r})|^2 d^3 r . \tag{7}$$

and $\Psi(r)$ is the non relativistic wave function of the atomic electron.

We have verified the relation (2) in the lowest order of perturbation theory for the derivation of the relativistic s.r. (3) for a free electron.

A relativistic analogue of the s.r. (6) may be obtained assuming the validity of the equality (2) in an infinite momentum frame.

Taking into account that the considered system has a spin $J = 1/2$ and that "anomalous" atomic magnetic moment coincides (due to $m/M \ll 1$) with total magnetic moment μ, we obtain the relativistic sum rule:

$$4\pi^2\alpha\left(\frac{1}{3}\langle r_1^2\rangle - \frac{1}{4m^2}\mu^2\right) = \int_0^\infty \frac{d\omega}{\omega}\sigma_{tot}(\omega), \tag{8}$$

which differs from Eq. (6) in two respects:

1) The total photoelectric absorption cross section which enters the integral (8) takes into account all retardation effects and all multipole transitions.

2) The sum rule (8) includes the radiative corrections of order α to the charge radius and the atomic magnetic moment, in the left-hand side of Eq. (8), and the radiative corrections to the atomic photodisintegration cross section and the Compton cross section (both elastic and inelastic, when an atomic electron undergoes a transition from the ground state to the excited atomic states or to the continuum) on a bound electron, in the right-hand side of Eq. (8). We may expand both sides of Eq. (8) in powers of the interaction

constant with the radiation field and get relations between separated coefficients of expansion (8) for each degree of α. For example:

$$4\pi^2 \alpha\left(\frac{1}{3}\langle r_1^2\rangle^{(1)} - 2\mu^{(0)}\mu^{(1)}\frac{1}{4m^2}\right) = \int_0^\infty \frac{d\omega}{\omega}\,\sigma_{tot}^{(1)}(\omega), \tag{9}$$

where

$$\langle r_1^2\rangle = \langle r_1^2\rangle^{(0)} + \langle r_1^2\rangle^{(1)} + \ldots,$$

$$\langle r_1^2\rangle^{(n)} \approx 0(\alpha^n), \quad etc.$$

The sum rule for quantities with index "0" has the same form as formula (8). It does not include radiative corrections in terms of the radiation field but contains all terms in powers of αZ (interaction constant of electron with the Coulomb center). The relations (8) and (9) present interest for verification and comparison with results of the relativistic theory of radiative processes in a Coulomb field.

4. Sum Rule for Leptonic Magnetic Moments and Hypothetical "Heavy" Leptons

The dispersion sum rule for magnetic moments has been obtained assuming the validity of a low energy theorem and an unsubtracted dispersion relation for the spin-dependent amplitude of the forward Compton scattering /6–8/. For spin $1/2$ particles the sum rule reads /7/ :

$$2\pi^2 \alpha \frac{1}{m^2}\kappa^2 = \int_0^\infty \frac{d\omega}{\omega}\left(\sigma_p(\omega) - \sigma_a(\omega)\right), \tag{10}$$

where κ is the "anomalous" magnetic moment of a particle with mass m, charge eQ and total magnetic moment μ : $\mu = \frac{e}{2m}(Q + \kappa)$, and the index "p" ("a") corresponds to the parallel (antiparallel) spin orientation of the colliding particles.

The sum rule (10) has explicitly been verified for weakly-bound two-particle systems /13–15/. For leptons themselves only the "trivial" cancellation of terms of the order of α^2 in the right-hand side of Eq. (10) was checked earlier /7/. This cancellation is necessary for the sum rule (10) to hold, because, the first non-zero term in the left-hand side of Eq. (10) is of the order of α^3. We shall assume further, that Eq. (10) is valid in all orders in α, and also taking into account all possible leptonic interactions. Our aim is to make use of Eq. (10) for evaluating the influence of a possible existence of "excited" leptons on the electron and muon magnetic moments.

One should note, that up to now no "heavy" leptons l^* were discovered experimentally /16/, which have the following decay mode: $l^* \to l + \gamma$. Following Low /9/, the decay vertex $l^* \to l + \gamma$ is conventionally described by the effective Lagrangian

$$L_{int} = e\left(\frac{\lambda}{m^*}\right) \bar{\Psi}_l \, \sigma_{\mu\nu} \, \Psi_{l^*} + h.c. \qquad (11)$$

i.e. it is assumed that the "excited" lepton l^* with mass m has spin $1/2$. One can obtain the upper limit for the coupling constant

$$\left|\frac{\lambda}{m^*}\right| < 5\cdot 10^{-2}\ Gev^{-1} \qquad (12)$$

from the upper limit of experimental production cross-section of l^* /16/.

It is interesting to discuss the effect of the existence of "heavy" leptons on magnetic moments of electrons and muons. Using the Lagrangian of Eq. (11) one can try to evaluate the (divergent) Feynmann diagrams of Fig.2. This approach was adopted and investigated in Refs. /17, 18/. Our purpose is to point out that the application of the sum rule (10) enables us to get a number of qualitatively new consequences. It appears logically admissible to consider the contributions of "heavy" leptons and those of the ordinary higher order radiative corrections independently, in the sense, that the sum rule (10) would hold when the radiative corrections are "swiched off". Then positiveness of the integral in Eq. (10) when one assumes a resonance "saturation" scheme requires the existence of "excited" leptons with spin $J \geq 3/2$.

Indeed, the intermediate resonance states with spin $1/2$ contribute to $\sigma_a(\omega)$ only. If there were no higher spin excited leptons the integral in Eq. (10) would be negative, which is impossible. Hence, the existence of a spectrum of "excited" leptons, including the higher spin states, appears to be necessary for the sum rule (10) to be valid under the aforementioned conditions. It follows from Eq. (10) that the contribution of each excited lepton l^* to the square of the anomalous magnetic moment is proportional to the square of the coupling constant of the radiative transition $l^* \to l + \gamma$. This gives a new type of dynamical relation between static lepton properties and the "heavy" lepton coupling constants, which lies out of the scope of the perturbative calculations /17,18/ of the Feynman diagrams in Fig.2. To get a numerical estimate for the "excited" lepton coupling constants we made a simple calculation of the contribution of the spin-parity $J^P = 3/2^+$ "excited" state to the integral entering the sum rule (10). In the narrow-width approximation and assuming $m^* \gg m$ we obtain

$$\Delta\sigma_{-1}(J^P = 3/2^+) = 2\pi^2 \alpha \left(\frac{\lambda}{m^*}\right)^2, \qquad (13)$$

where λ/m^* is the coupling constant in the interaction Lagrangian

$$L_{int} = e\left(\frac{\lambda}{m^*}\right) \bar{\Psi} \gamma_\mu \gamma_5 \Psi_\nu F_{\mu\nu} + h.c. \qquad (14)$$

and Ψ_ν is the field operator of the spin $3/2$ particle.

A comparison of the experimental values of the leptonic anomalous magnetic moments with theory led to the following bounds /1/ on the possible corrections to the conventional theory:

$$\begin{aligned} -7{,}1 \cdot 10^{-9} &\lesssim \Delta\kappa_e \lesssim 11{,}7 \cdot 10^{-9}, \\ -325 \cdot 10^{-9} &\lesssim \Delta\kappa_\mu \lesssim 887 \cdot 10^{-9}. \end{aligned} \qquad (15)$$

Putting Eq. (13) into Eq. (10) and barring "accidental" cancellation between positive (from higher spin states) and negative (from spin $1/2$ "excited" leptons) contributions to the right-hand side of Eq. (10) we find, using (15):

$$\begin{aligned} \left|\frac{\lambda}{m^*}\right|_e &\lesssim 2{,}3 \cdot 10^{-5}\, Gev^{-1}, \\ \left|\frac{\lambda}{m^*}\right|_\mu &\lesssim 0{,}84 \cdot 10^{-5}\, Gev^{-1}. \end{aligned} \qquad (16)$$

Note, that these estimates are of the same order of magnitude as a simple dimensional estimate of the magnitude of the weak-radiative coupling constant:

$$e\left(\frac{\lambda}{m^*}\right) \approx e \cdot G \cdot m_{eff} \approx e \cdot 10^{-5} \cdot \left(\frac{m^*}{m_N}\right) Gev^{-1} , \tag{17}$$

where $G \approx 10^{-5} \cdot m_N^{-2}$ is the universal Fermi constant, m_N -the nucleon mass and m_{eff} is some characteristic mass connected with the effective interaction region (we rather arbitrary put it equal to the "excited" lepton mass m^*).

The smallness of the coupling constants given by Eq. (16) suggests that instead of looking for "excited" leptons in processes based on the existence of the radiative decay channel $l^* \to l \pm \gamma$ (like in Ref. /16/) a more easy and perspective way might be to search for $l^*\bar{l}^*$ - pairs in reactions like $e^+e^- \to l^*\bar{l}^*$ or $\gamma A \to l^*\bar{l}^* A$, where A is a nucleus or nucleon.

Conclusion

We summarize the main results of this work:

1) A relativistic dipole moment fluctuation sum rule has been derived and checked for free leptons in the lowest order of perturbation theory.

2) A generalization of the dipole sum rule for the atomic photoabsorption is proposed, which includes total photoelectric cross-section with the lowest order radiative correction and the total photon scattering cross-section on the atomic electron.

3) The dispersion sum rule for the magnetic moments is applied to leptons for estimating the effects of hypothetical "heavy" leptons on the electron and muon magnetic moments. Assuming the validity of a resonance "saturation" of the sum rule we conclude that the existence of the "excited" leptons with spin $J \geq 3/2$ is necessary. We obtain an estimate on the coupling constants of radiative transitions $l^* \to l + \gamma$, which in the order of magnitude, turn out to have the strength of the weak-electromagnetic coupling constants. This is a much more restrictive bound than the ones obtained from the analysis of the other experimental data /16/ .

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