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INVESTIGATION OF REACTIONS $\gamma\gamma \to K\bar{K}$ AND $\gamma + \gamma \to \gamma + \gamma$ BY THE METHOD OF DISPERSION RELATIONS

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Исаев П.С., Хлесков В.И. **E2 - 6666**

Исследование процессов $\gamma\gamma \to K\bar{K}$ и $\gamma\gamma \to \gamma\gamma$ методом дисперсионных соотношений

Дисперсионным методом при низких энергиях исследован процесс $\gamma\gamma \to K\bar{K}$ с двухпионным промежуточным состоянием. Получены сечения s и d волн процесса и сечение взаимодействия встречных пучков $ee \to ee K\bar{K}$. Рассчитан вклад $K\bar{K}$ промежуточного состояния в s и d волны реакции $\gamma\gamma \to \gamma\gamma$.

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Isaev P.S., Khleskov V.I. **E2 - 6666**

Investigation of Reactions $\gamma\gamma \to K\bar{K}$ and $\gamma\gamma \to \gamma\gamma$ by the Method of Dispersion Relations*

The process $\gamma + \gamma \to K + \bar{K}$ has been investigated by the method of dispersion relations with the account of the two-pion intermediate state. The cross sections of the s -and d -waves of the process and of the interaction of the colliding beams $ee \to ee K K$ have been obtained. The contribution of the intermediate $K\bar{K}$ -state to the reaction $\gamma\gamma \to \gamma\gamma$ has been calculated.

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The interaction processes of light with light accompanied by a production of hadrons have caused a significant interest during the last time /1, 2, 3/. This is connected with an essential role played by such a process in the interaction of colliding beams /4,5/, their experimental investigation being a subject of a few coming years.

In the preceding paper /6/ we investigated the reaction $\gamma\gamma \to \pi\pi$ by the dispersion method. This allowed us to calculate later the s -and d -waves of the light by light scattering and to conclude that the contribution of the two-pion intermediate state to the process $\gamma\gamma \to \gamma\gamma$ is significant /7/ .

In this paper the known amplitudes of the process $\gamma\gamma \to \pi\pi$ are used for studying the reaction $\gamma\gamma \to KK$ at low energies ($E \lesssim$ 1.5 - 1.6 GeV) on the basis of dispersion relations and the two-body unitarity condition. The study of the process $\gamma\gamma \to KK$ is of an independent interest and permits to estimate the contribution of the intermediate $K\bar{K}$ -state to the reaction $\gamma+\gamma \to \gamma+\gamma$ as well. The physical threshold of the process $\gamma+\gamma \to K+\bar{K}$ starts from two kaon masses: $2M_K \simeq$ 990 MeV. Aside from $K\bar{K}$, the following hadron intermediate states (the one particle sates being excluded): 2π, 3π, 4π, etc. contribute to the unitarity condition. For cross sections of the processes $\gamma\gamma \to 3\pi$, $\gamma\gamma \to 4\pi$ the following values have been predicted in the works /8,9/:

$$\sigma_{\gamma\gamma \to 4\pi} \sim 2 \cdot 10^{-33}\ cm^2 \qquad \text{for} \qquad t = (6\mu_\pi)^2$$

$$\sigma_{\gamma\gamma \to 3\pi^\circ} \ll \sigma_{\gamma\gamma \to \pi^+\pi^-\pi^\circ} \sim 10^{-35}\ cm^2 \qquad \text{for} \qquad t = (4\mu\ )^2$$

At the corresponding energies the magnitudes of the $\sigma_{\gamma\gamma\to\pi\pi}$ /8,6/ are at least, for two orders larger. In the reaction $\gamma\gamma\to K\bar{K}$, also on account of $\sigma(K\bar{K}\to S^*\to K\bar{K})/\sigma(\pi\pi\to S^*\to K\bar{K})\simeq 0{,}1$, one can confine oneself with an accuracy of 10 to 20% with the two-pion intermediate state only.

The lef-hand cuts in the complex t -plane in the dispersion equations are substituted approximately by the contribution of the K -meson Born terms and the K^*_0 and $K^*_\pm$ resonances. To normalize the amplitudes to the Tompson limit one subtraction in the dispersion equation has been carried out at the unphysical point $t = 0$.

In this work the cross sections of the s -and d -waves of the process $\gamma+\gamma\to KK\ (T=0)$ and of the interaction of colliding beams $ee\to eeK\bar{K}\ (T=0)$ have been obtained. The contributions of the $K\bar{K}(T=0)$ intermediate state to the s and d waves of the process of the light by light scattering are calculated from the obtained partical waves of the reaction $\gamma\gamma\to K\bar{K}(T=0)$. The graph of the comparison of the contributions of different intermediate states to the process $\gamma+\gamma\to\gamma+\gamma$ is presented.

I. Dispersion Equations for Partial Waves

Let $k=(k_0,\bar{k})$ and $k'=(k'_0,\bar{k}')$ be the 4-momenta of the initial photons; ϵ^μ and ϵ'^ν are their polarization vectors and $P=(P,\bar{P})$, $P'=(P',\bar{P}')$ are the 4-momenta of the final K mesons (see Fig.I.). The C -parity of the initial state is positive and the relative orbital moment l is even. For the process $\gamma\gamma\to K\bar{K}$ taking account of two-pion intermedium states only, one finds that there is only one isotopic amplitude with $T = 0$.

We write down the dispersion equations for the invariant amplitudes $T^{(T=0)}_{\mu\nu}(t,\cos\phi_+)$ in the center of mass system in the following way

$$<KK(T=0)|S|2\gamma> = \frac{i}{(2\pi)^4}\,\frac{1}{4k_0P_0}\,\delta^{(4)}(k+k'-P-P')\,T^{(T=0)}_{\mu\nu}(t,\cos\phi_+)\,\epsilon^{\mu}\epsilon'^{\nu} \tag{I}$$

here $t=4k_0^2=4P_0^2$, ϕ_+ is the scattering angle.

Note, that

$$T^{(T=0)}_{\mu\nu}(t, \cos\phi_+) = \frac{1}{\sqrt{2}} T^{K^+K^-}_{\mu\nu}(t, \cos\phi_+) - \frac{1}{\sqrt{2}} T^{K^0\bar{K}^0}_{\mu\nu}(t, \cos\phi_+) . \quad (2)$$

From eq. (2) it follows that in the Tompson limit $t \to 0,\ M_K^2 \to 0$ the amplitude $T^{(T=0)}_{(\mu\nu)} \to \frac{2e^2}{\sqrt{2}}$ because of $T^{K^+K^-}_{\mu\nu} \to 2e^2$ and $T^{K^0K^0}_{\mu\nu} \to 0.$

The dispersion equations for the partial s -and d -waves normalized to the Tompson limit can be written in this case as follows (see /6/):

$$\mathrm{Re}\, T^{(T=0)}_{\mu\nu}(t)_s = \frac{t}{\pi} P \int_{4\mu_\pi^2}^{\infty} \frac{\mathrm{Jm}\, T^{(T=0)}_{\mu\nu}(t')_s}{t'(t'-t)} dt' + K^{(T=0)}_{\mu\nu}\left(t, -\frac{1}{\sqrt{3}}\right) -$$

$$- K^{(T=0)}_{\mu\nu}\left(t=0, M_K^2=0, \frac{1}{\sqrt{3}}\right) + B_{\mu\nu}\left(t, \frac{1}{\sqrt{3}}\right) + \frac{2e^2}{\sqrt{2}} .$$

$$\mathrm{Re}\, T^{(T=0)}_{\mu\nu}(t)_d = \frac{t}{\pi} P \int_{4\mu_\pi^2}^{\infty} \frac{\mathrm{Jm}\, T^{(T=0)}_{\mu\nu}(t')_d}{t'(t'-t)} dt' + K^{(T=0)}_{\mu\nu}\left(t, \sqrt{\frac{7}{15}}\right) -$$

$$- K^{(T=0)}_{\mu\nu}\left(t, \frac{1}{\sqrt{3}}\right) + K_{\mu\nu}^{(T=0)}\left(t=0, M_K^2=0, \frac{1}{\sqrt{3}}\right) - K^{(T=0)}_{\mu\nu}\left(t=0,\right.$$

$$\left. M_K^2 = 0, \sqrt{\frac{7}{15}}\right) + B^{(T=0)}_{\mu\nu}\left(t, \sqrt{\frac{7}{15}}\right) - B^{(T=0)}_{\mu\nu}\left(t, \frac{1}{\sqrt{3}}\right).$$

Here the partial waves are singled out by the method of the fixing of definite scattering angles /6/ . $B_{\mu\nu}(t, \cos\phi_+)$, $K_{\mu\nu}(t, \cos\phi_+)$ are the contributions of the K meson Born terms and of the K_0^* , $K_\pm^*$ resonances.

The electromagnetic decays of the K^* mesons are determined by the effective Hamiltonian of the interaction

$$H_{K_0^* \to K_0 \gamma} = g_{K_0^* K_0 \gamma} \frac{\partial K^*_{0\,\mu}(x)}{\partial x^\nu} F_{\sigma\delta}(x) K_0(x) \epsilon^{\mu\nu\sigma\delta} ,$$

$$H_{K_\pm^* \to K_\pm \gamma} = g_{K^*_\pm K_\pm \gamma} \frac{\partial K^*_{\pm\,\mu}(x)}{\partial x^\nu} F_{\sigma\delta}(x) K_\pm(x) \epsilon^{\mu\nu\sigma\delta} ,$$

where $K^*_{0,\pm}(x)$, $K_{0,\pm}(x)$ are the operators of the $K^*_{0,\pm}$, $K_{0,\pm}$-meson fields and $F_{\sigma\delta}(x)$ is tensor of the electromagnetic field;

$\epsilon^{\mu\nu\sigma\delta}$ is an antisymmetrical tensor. The constants $g_{K_0^* K_0 \gamma}$ and $g_{K_\pm^* K_\pm \gamma}$ are connected with $g_{\omega\pi\gamma}$ in a known way /11/ :

$$g_{K_\pm^* K_\pm \gamma} = \frac{1}{3} g_{\omega\pi\gamma} , \quad g_{K_0^* K_0 \gamma} = -\frac{2}{3} g_{\omega\pi\gamma} . \tag{4}$$

Making use of eq. (4), the contributions of the Born and resonance terms can be evaluated according to the formulas analogous to those from the work /6/ , if the replacements $M^2_\omega \to M^2_{K^*}$, $\mu^2_\pi \to M^2_K$ are carried out and the relation (2) is taken into account.

The imaginary part of the partial amplitudes is determined by the condition of the two-body unitarity with the two-pion intermediate state

$$Jm\, T^{(T=0)}_{\mu\nu}(t)_\ell = \frac{1}{32\pi} \sqrt{1 - \frac{4\mu^2_\pi}{t}}\, T^{(T=0)}_{\mu\nu}(t)_\ell \, \Pi^*_\ell(t) , \tag{5}$$

where $T^{(T=0)}_{\mu\nu}(t)_\ell$ are the partial waves of the process $\gamma\gamma \to \pi\pi(T=0)$ /6/, $\Pi_\ell(t)$ are the partial amplitudes of the reaction $\pi\pi \to KK$ $(T=0)$.

The s -wave of the process $\pi\pi \to KK$ $(T=0)$ was considered in the work /12/. We took from that work the resonance solution which agrees well with the experimental data (see Fig.2.). The d -wave of the reaction $\pi\pi \to K\bar{K}(T=0)$ was approximated by the f -meson with the total width $\Gamma_f \cong$ 150 MeV and the mass $M_f \cong$ 1260MeV, in the maximum of the resonance the cross section being $\sigma^{d}_{\pi\pi\to K\bar{K}(T=0)}(M_f^2)\simeq 1\,mb$.

For calculations according to the formulas (3) and (5) the amplitudes of the process $\pi\pi \to K\bar{K}(T=0)$ were analytically continued to the region $4\mu_\pi^2 < t < 4M_K^2$.

2. The Light by Light Scattering Through the $K\bar{K}(T=0)$ Intermediate State

From the evaluated partial amplitudes of the process $\gamma\gamma \to K\bar{K}(T=0)$ one can obtain the cross sections of the s and d waves of the light by light scattering through the state $K\bar{K}(T=0)$. The condition of the two-body unitarity for the partial amplitudes $T_{\mu\nu j\delta}(t)_\ell$ of the light by light scattering (μ, ν and j, δ are the indices of the polarization of the initial and final photons) has the following form

$$Jm\,T_{\mu\nu j\delta}(t)_\ell = \frac{1}{32\pi}\sqrt{1-\frac{4M_K^2}{t}}\;T^{(T=0)}_{\mu\nu}(t)^*_\ell\;T^{(T=0)}_{j\delta}(t)_\ell\,, \tag{6}$$

where $T^{(T=0)}_{\mu\nu}(t)$ are the described above partial waves of the process $\gamma+\gamma\to K\bar{K}(T=0)$.

The real part of the amplitude $T_{\mu\nu j\delta}(t)_\ell$ can be restored from the imaginary part on the basis of the dispersion integral with one subtraction

$$Re\,T_{\mu\nu j\delta}(t)_\ell = \frac{t}{\pi}\,P\int_{4m_K^2}^{\infty}\frac{Jm\,T_{\mu\nu j\delta}(t')_\ell}{t'(t'-t)}\,dt' \tag{7}$$

neglecting the narrow resonances in the direct channel and the crossing cuts (analogically to /7/).

3. Cross Sections of the Processes

For the nonpolarized initial photons with the density matrices $e^{\mu\mu'} = \frac{1}{2}\delta^{\mu\mu'}$, $e^{\nu\nu'} = \frac{1}{2}\delta^{\nu\nu'}$ the partial cross sections of the process $\gamma\gamma \to K\bar{K}(T=0)$ can be written as follows

$$\sigma_\ell = \frac{(2\ell+1)}{16\pi}\,\frac{1}{t}\,T^{(T=0)}_{\mu\nu}(t)^*_\ell\;T^{(T=0)}_{\mu'\nu'}(t)_\ell\;e^{\mu\mu'}e^{\nu\nu'}$$

$\ell = s, d.$

The amplitudes $T^{(T=0)}_{\mu\nu}(t)_\ell$ satisfy the equations (3) and (5). For the s -wave of the process $\gamma\gamma \to K\bar{K}(T=0)$ there exist two solutions which correspond to the two s -waves of the reaction $\gamma\gamma\to\pi\pi(T=0)$ /6/ : down and down-up. The curves of the partial cross sections of the process $\gamma\gamma \to K\bar{K}(T=0)$ are presented in Fig.3. One can notice the anomalous increase of the cross sections of the s -wave at the threshold of the reaction $\gamma+\gamma\to K\bar{K}(T=0)$. This is due to the process $\pi\pi \to K\bar{K}(T=0)$ for the down-up solution it is more distinct. The contribution of the d -wave to the total cross section at the threshold is small, however, when going to the region of the f -meson, its cross section strongly increases exceeding the cross section of the s wave in the maximum of the resonance.

The cross sections of the reaction $\gamma\gamma \to K\bar{K}(T=0)$ in the region from the threshold to the values, where the twobody unitarity becomes a rough approximation (i.e. to $\sqrt{t} \approx 1{,}4-1{,}6\, Gev$), are of the order of the cross section of $\gamma+\gamma \to \pi\pi$ given in the work /6/ .

In Figure 4 the cross section of the interaction of the colliding beams $ee\to ee K\bar{K}(T=0)$ calculated in the approximation of the method of the equivalent photons /1/ , is shown. It is at least, for two orders smaller, than the cross section of $ee \to ee\,\pi\pi$ /6/.

The partial s -and d -cross sections of the reaction $\gamma\gamma \to \gamma\gamma$, obtained from eq. (6) and eq. (7) are shown in Fig.5a and Fig.5b. The magnitudes of these cross sections are small in comparison with the cross sections of the light by light scattering through the two-pion state /7/ and could lead in the region of the production of the $K\bar{K}$ -pair to the negligible fluctuations of the total cross section of the $\gamma\gamma \to \gamma\gamma$. Note also, that due to the positive parity the contribution of the $KK(T=0)$ intermediate state, to the process $\gamma+\gamma\to\gamma+\gamma$ obtained in this work, as well as the contribution of the two-pion intermediate state described in the work /7/ , do not interfere with the pole contributions of the pseudoscalar mesons. In Figure 6 the qualitative comparison of the cross section of the light by light scattering through the $K\bar{K}$ and the $\pi\pi$ /7/ intermediate states with the cross section of the resonant light by light scattering /13/ and with the cross section of the photon scattering within the framework of quantum electrodynmics /14/ , is presented. The cross section of the light by light scattering due to the vacuum polarization by the pairs of the $\mu\bar{\mu}$ mesons must be smaller (in the ratio of the square mass $(\mu/m_\ell)^2$, in 4 orders) that the cross section of the $\gamma\gamma \to \gamma\gamma$ with the electron-positron polarization of the vacuum and should be compatible only with the contribution of the $K\bar{K}$ -state to the process $\gamma\gamma\to\gamma\gamma$.

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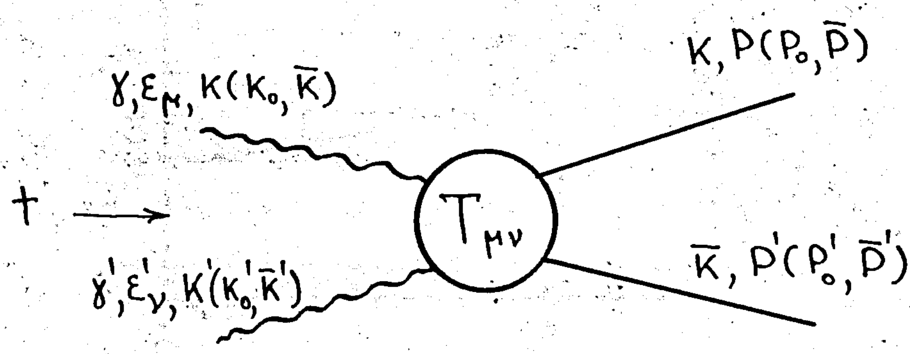

Fig.1

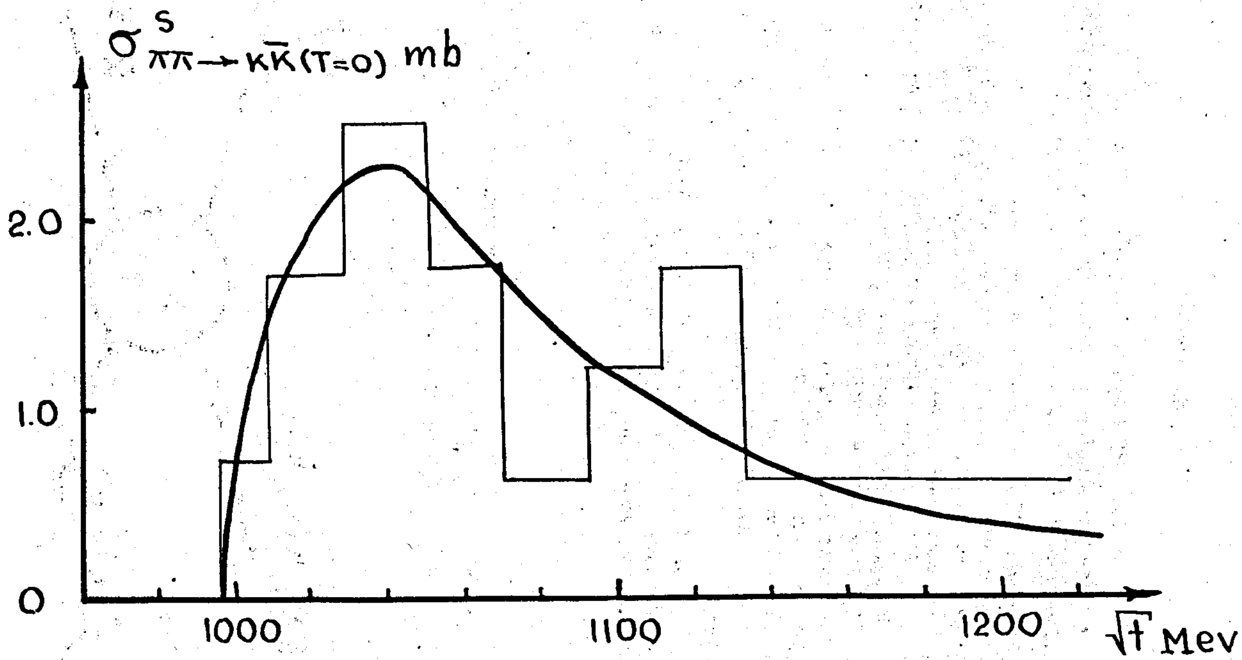

Fig.2

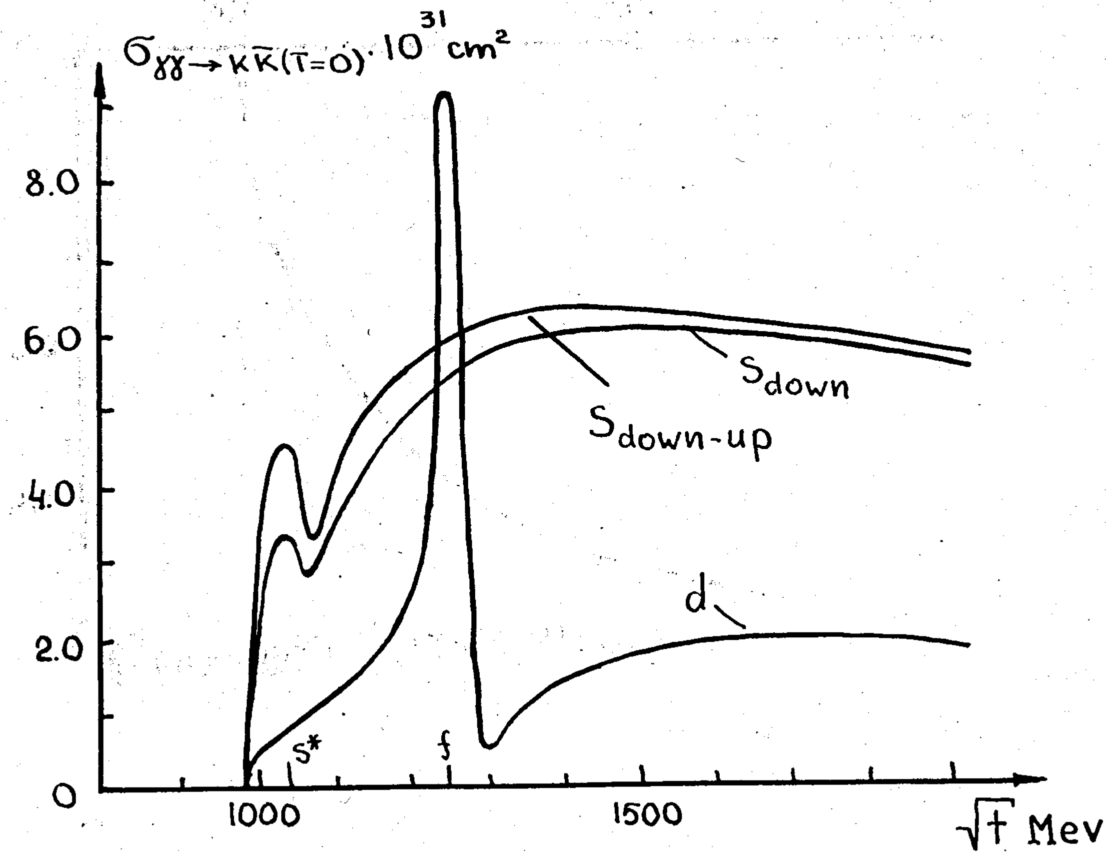
$\sigma_{\gamma\gamma \to K\bar{K}(T=0)} \cdot 10^{31}\,cm^2$
8.0
6.0
4.0
2.0
0
1000
1500
$\sqrt{t}$ Mev
S_{down}
$S_{down-up}$
d
S*
f

Fig.3

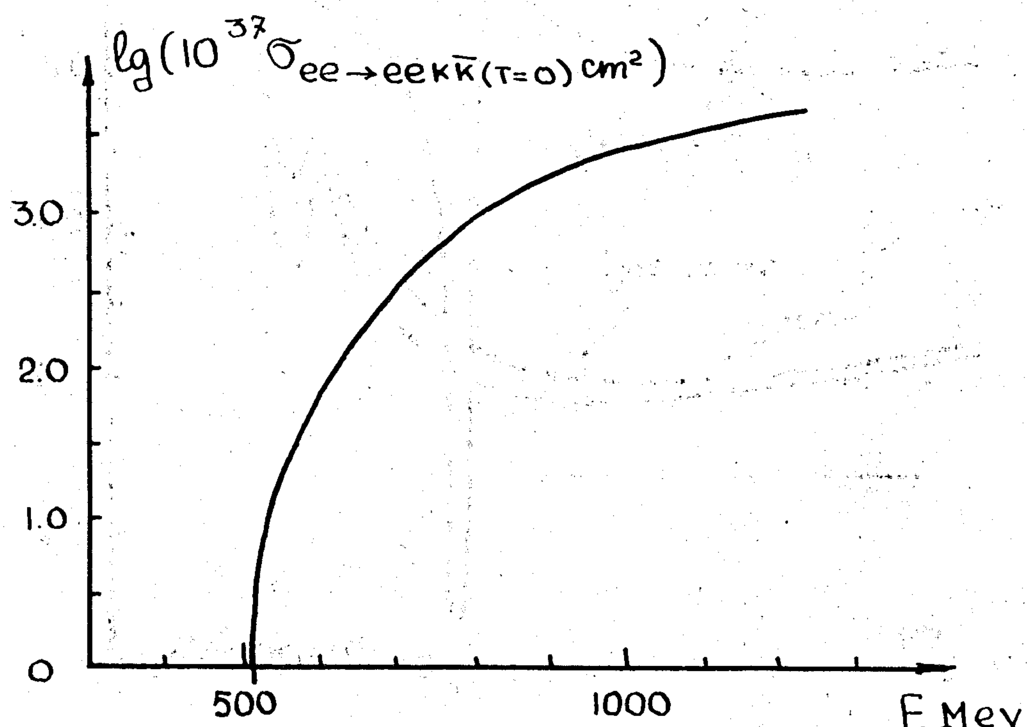

Fig.4

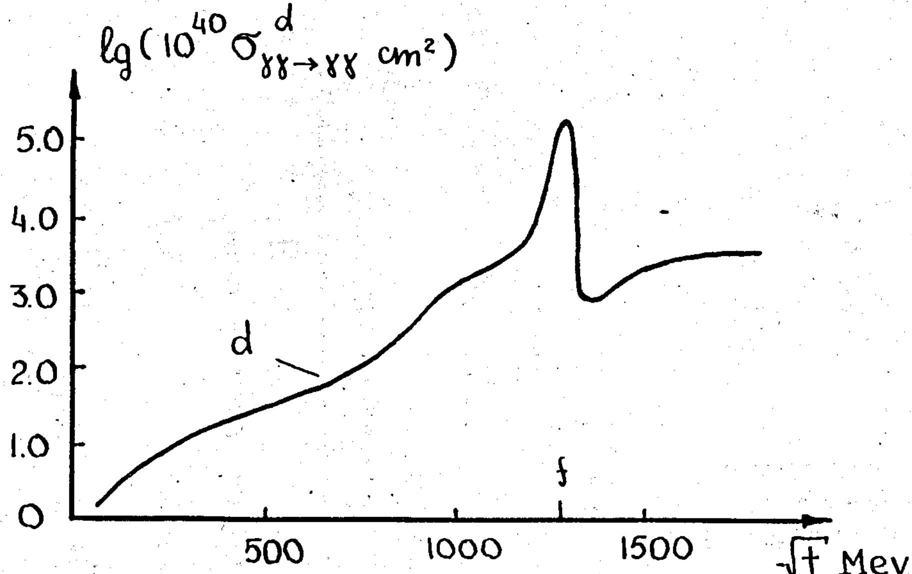
lg(10^40 σ^d_γγ→γγ cm²)
5.0
4.0
3.0
2.0
1.0
0
d
f
500
1000
1500
√t Mev

Fig.5a

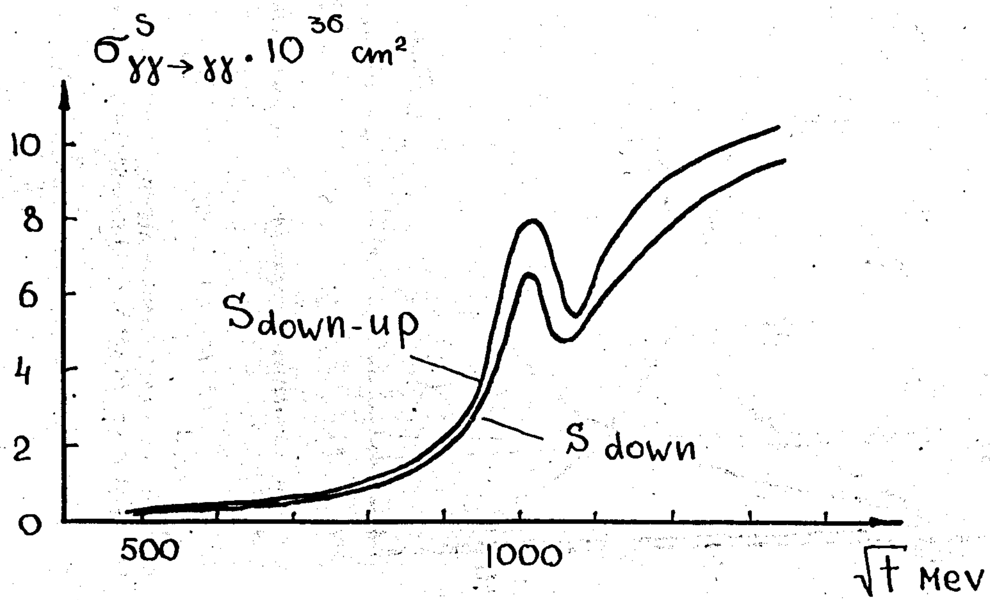
σ^S_γγ→γγ · 10^36 cm²
10
8
6
4
2
0
S down-up
S down
500
1000
√t Mev

Fig.5b

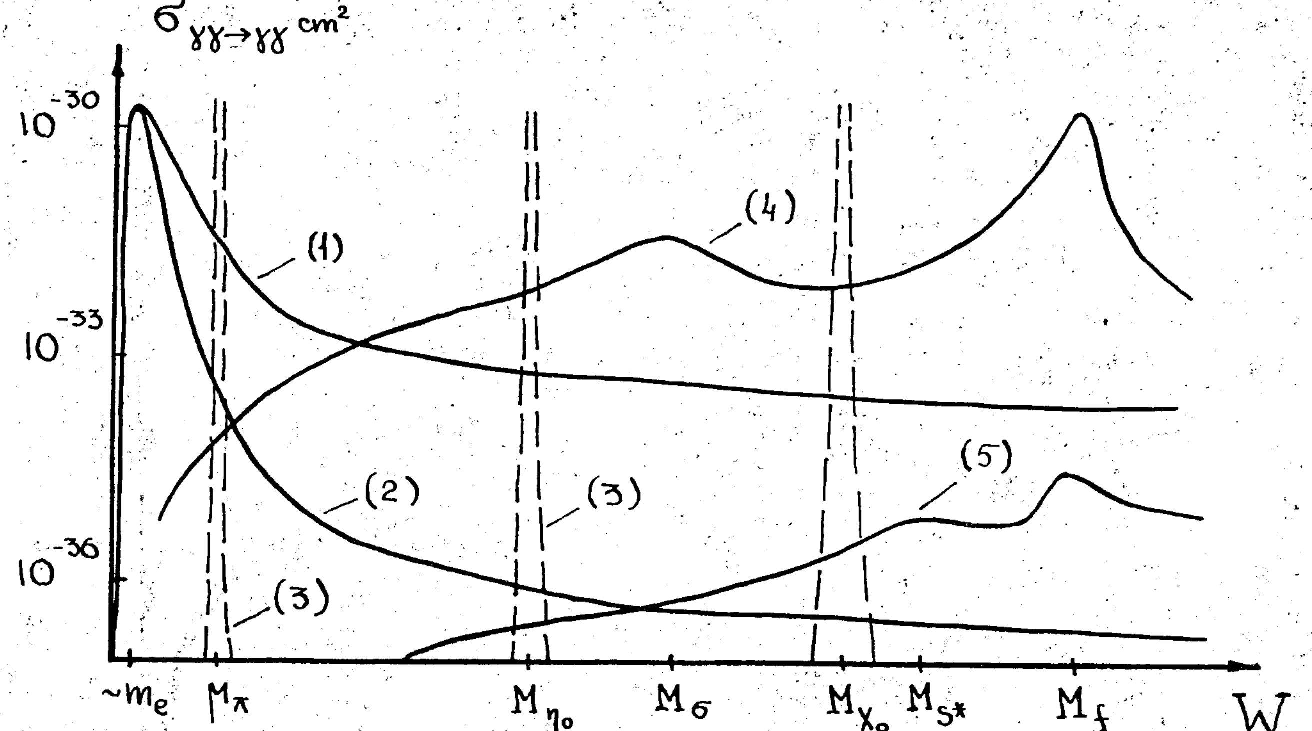

Fig.6 The qualitative comparison of the contributions from different intermediate states to the process

(1) $-4\pi \frac{d\sigma}{d\Omega}(W, \theta = 0)$, (2) $-1\pi \frac{d\sigma}{d\Omega}(W, \theta = \frac{\pi}{2})$ /14/;

(3) - resonant light by light scattering /13/;
(4) - scattering of light by light through $\pi\pi$ intermediate state $(\sigma^{s}_{down-up} + \sigma^{d})$ /7/;

(5) - scattering through KK intermediate state $(\sigma^{s}_{down-up} + \sigma^{d})$.