ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2 - 6653

D.I.Blokhintsev

GEOMETRY AND PHYSICS OF THE MICROWORLD

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1972

E2 - 6653

D.I.Blokhintsev

GEOMETRY AND PHYSICS OF THE MICROWORLD

Submitted to "УФН"

Блохинцев Д.И. **E2 - 6653**

Геометрия и физика микромира

Настоящая работа является очерком проблем, возникающих при переносе геометрических понятий из классической физики в мир элементарных частиц.

Препринт Объединенного института ядерных исследований.
Дубна, 1972

Blokhintsev D.I. **E2 - 6653**

Geometry and Physics of the Microworld

The present paper is an essay of problem arising as a results of the transfer of geometric notions from classical physics to the world of elementary particles.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1972

It is important for the problems to be considered below that the metric (1) is indefinite. Hence, it follows that the notion of nearness of two events $\mathcal{P}$ and $\mathcal{P}'$ in the $\mathcal{R}_4(x)$ space is not an invariant notion and may be formulated only as applied to the given frame of reference Σ.

These are the most important features of geometry, rather chronogeometry [7], which is the content of special relativity.

It is also possible to choose another method of arithmetization of events and thereby obtain another geometry.

The basic advantage of the method suggested by the Einstein theory of relativity consists in that it is just this method of arithmetization that reveals invariance of fundamental laws of physics. A law which in the frame Σ is expressed by the relation

$$F(A, B, x \ldots) = 0 \tag{3}$$

in the frame $\bar{\Sigma}$ is expressed by the realtion

$$F(\bar{A}, \bar{B}, \bar{x} \ldots) = 0, \tag{3'}$$

where $\bar{A}, \bar{B}, \bar{x}, \ldots$ are scalars, spinors, vectors or tensors.

Therefore not any means of arithmetization is valid. The means of arithmetization must be, first of all, physically realizable (at least in an ideal experiment) and, secondly, maximum universal. The latter means that it must be based on the widest range of phenomena [x].

Here we can stop the description of the method of arithmetization of events commonly accepted in classic relativistic physics.

In what follows possible restrictions on this method are considered, which are seen to be useful for understanding more complicated situation in the worlds of elementary particles.

2. Possible Restrictions on the Arithmetization Accepted

In classic physics, to the concept of point-like event $\mathcal{P}(x)$ there well corresponds the concept of material point - an object of finite mass $(m_0 \neq 0)$ and infinitely small dimensions $(a \to 0)$.

In virtue of the fact that the space $\mathcal{R}_4(x)$ is assumed to be continuous, at each of its points one can construct a space of tangent vectors of infinitesimal displacements and a covariant momentum space $\mathcal{R}_4(p)$. The metric of this space is also indefinite and has the form

$$dp^2 = dp_0^2 - d\vec{p}^{\,2}, \tag{4}$$

---

[x] Thus, for example, the assumption based on the velocity of sound U, rather than on the velocity of light C, would intoduce very private particularities of sound phenomena to the consideration of all physical phenomena. In a similar way, measurement of lengths by means of a spring dynamometer would introduce special properties of the spring (see refs.[4,6]) .

where $d\vec{\rho}^{\,2} = d\rho_1^2 + d\rho_2^2 + d\rho_3^2$. This form is unambiguously defined by the metric of the space $R_4(x)$. Thus, the structures of the spaces $R_4(x)$ and $R_4(\rho)$ are not independent.

The motion of a material point (or of a system of material points) can be formulated, in the most general way, in terms of the Finsler geometry. In this geometry, the Lagrange variational principle looks like the condition of motion of a material point along a geodesical line. The element of length ds of this line is ✓

$$ds = L(x, dx), \tag{5}$$

where L is a homogeneous function of the first-power displacements dx [7].

Within the framework of special relativity, there are no logical contradictions between the Einstein way of arithmetization and the mechanics of material points. Therefore, in special relativity, material points may be thought of as objects which realize physically a point event $\mathcal{J}(x)$. ✓ pace

Restrictions come from the side of gravitation. If the gravitational radius a_g of a material point

$$a_g = \frac{8\pi k m_0}{c^2} \tag{6}$$

(here $k = 6{,}7 \cdot 10^{-8}\,\frac{cm^3}{g \cdot sec^2}$ is the Newton gravitational constant, m_0 the rest mass of the matrial point) is larger than its dimension a then "inside" the material point, the metric relations change so strongly that the division of the manifold

$\mathcal{R}_\gamma(x)$ into the space-like and the time-like parts loses its sense. Therefore the space inside the sphere of radius $a \simeq a_g$ is nonlocal in the sense that the usual ordering of events inside it becomes impossible.

This situation is due to the state of matter. For $a = a_g$ there arises a certain critical density of matter

$$\rho_g = \frac{3}{4\pi} \frac{c^6}{(8\pi k)^3} \frac{1}{m_0^2} . \tag{7}$$

It is seen from (6) that it is advisable to have as objects marking points in space-time material points with least mass $(m_0 \to 0)$. However, in so doing, the matter density $\rho_g \longrightarrow \infty$ tends to infinity and can go beyond the limits we are aware of from elementary particle physics.

Then a curious question arises: whether or not the method of arithmetization of events adopted in the theory of relativity can lose its validity before the condition $\rho = \rho_g$ is reached. In fact, if for a certain density of matter $\rho_\kappa < \rho_g$ not a single light signal and even not a single neutrino signal, can propagate in a medium, due to extremely strong extinction, ordering of events in that medium by means of light or neutrino waves becomes impossible. Under these conditions a sound signal may turn out to be a more suitable tool for ordering events. The velocity of such a signal U may be higher than the light velocity in vacuum C. Nevertheless there occurs no contradiction with causality since the ordering is performed by the U signal rather than the C signal (for more detail see ref.[6]).

Restrictions of another kind come from the side of stochastic gravitational fields. Fields induced by turbulent motion of matter lead inevitably to that the metric tensor $g_{\mu\nu}(x)$ becomes a stochastic quantity $\hat{g}_{\mu\nu}(x)$. The interval

$$d\hat{s}^2 = \hat{g}_{\mu\nu}(x)\, dx_\mu\, dx_\nu \tag{8}$$

becomes also a stochastic quantity. If the fluctuations $\hat{h}_{\mu\nu}(x)$ of the metric tensor $\hat{g}_{\mu\nu}(x)$ are not large compared to the average values of $\langle \hat{g}_{\mu\nu} \rangle = \bar{g}_{\mu\nu}$ then it is advisable to write this tensor in the form

$$\hat{g}_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \hat{h}_{\mu\nu}(x). \tag{9}$$

In this case the ordering of events can be based on the metric defined by the principle part of the metric tensor $\bar{g}_{\mu\nu}(x)$ (comp. refs.[6,8]) . If the fluctuations are not small then the ordering in $\mathcal{R}_\gamma(x)$ becomes essentially stochastic.

Spaces with stochastic metric were considered in refs.[9,10] from the axiomatic point of view. However , axiomatic of these spaces is restricted by a positive definite metric. An extension of axiomatic to stochastic spaces of the Minkovsky type is still an open problem. Some relevant problems are discussed in refs.[6,11].

The main question to be answered rather by physicists than by mathematicians is related to the indication of a physical method of ordering. Are we approaching here the limits of applicability of the concept of ordering?

Problems associated with the metric at large fluctuations and extremely large densities appear to become of foremost value when analysing earlier stages of "Big Bang".

We know that nowadays matter is governed by definite laws and there exist definite symmetries. But we have no reason to assert that these forms of the existenoe of matter are prescribed for ever. It is quite possible that the vacuum and the world of elementary particles we are aware of are nothing less than one of possible ways of evolution of the Universe chosen as a result of competition of various possibilities.

However, at the present stage of our knowledge we have not much information for being able of disoussing in more detail this facet of the problem.

3. Point-Like Events in the Microworld

Now we pass to the world of elementary particles. Modern quantum field theory which describes the behaviour of elementary particles is based on the oondition of looality

$$[\varphi(x), \varphi(y)] = \mathcal{D}(x-y), \tag{10}$$

$$\mathcal{D}(x-y) = 0, \quad \text{for } (x-y)^2 < 0. \tag{10'}$$

Here $\varphi(x)$ is the field operator at a point (x), $\varphi(y)$ the operator of the same field at a point (y); $[\hat{A}, \hat{B}]$ the commutator of the operators $\hat{A}$ and $\hat{B}$ x.

x) or the antioommutator if the field is a spinor field.

We write down explicitly the condition for the scalar field $\varphi(x)$.

The condition (1^0) is the causality principle and means the independence of the fields if the points (x) and (y) are spaced by a space-like interval $(x-y)^2 < 0$. In other words, an arbitrary variation of the field at the point (x) cannot affect the field at the point (y) since a signal of velocity $V \leq C$ cannot in this case reach the point (y) and the inverse.

In the condition of locality (1^0) the coordinates of the points (x) and (y) are assumed to be determined indefinitely exactly. This assumption is equivalent to supposing the existence of point-like events $\mathfrak{J}(x), \mathfrak{J}(y), ---$
We have to analyse how much this assumption is consistent in the framework of the same local theory

Elementary particles, analogues of material points in classic physics, are natural candidates for playing the role of representatives of point-like events. However, this analogy is not far-reaching due to a number of particular features dictated by the laws of quantum physics.

First of all, all particles with rest mass $m_0 = 0$ should be excluded from the analogy since they are nonlocalizable in the space $R_4(x)$. They can be localized only in a tangent space $R_4(p)$. However, there is a trouble with particles of rest mass $m_0 \neq 0$, too. Bosons of rest mass $m_0 \neq 0$ cannot be localized in the space $R_4(x)$ more exactly than in the limits $\Delta x \simeq \hbar / m_0 c$.

In fact, the density $\rho(\vec{x},t)$ of the meson field $\varphi(\vec{x},t)$ obeying the conservation law is equal to

$$\rho(0,\vec{x}) = \varphi^*(0,\vec{x})\Omega\,\varphi(0,\vec{x}), \tag{11}$$

where $\Omega = \sqrt{m_0^2 - \nabla^2}$ is the frequency operator and $\vec{\nabla}$ is the gradient operator. It is positive definite only in the domain $|\vec{\nabla}| \ll m_0$ i.e. in a nonrelativistic domain. In this case, the quantity

$$\rho(0,\vec{x}) \simeq |\varphi(0,\vec{x})|^2 > 0 \tag{12}$$

may be thought of as the density of probability for a boson to be located at the point $\vec{x}$ at the time moment $t=0$. However, for $|\vec{\nabla}| \ll m_0$ the density $\rho(0,\vec{x})$ is distributed in the space, in the domain $|\Delta\vec{x}| \gg \hbar/m_0c$.

For spinor particles obeying the Dirac equation there exists a positive definite probability density

$$\rho(0,\vec{x}) = \bar{\psi}(0,\vec{x})\psi(0,\vec{x}) \geq 0, \tag{13}$$

where $\psi(0,x)$ is the wave function for a single-particle state. There exists a belief that for the single-particle state $\overline{\Delta x}^2 > (\hbar/m_0c)^2$. In reality the Dirac particles obey the usual uncertainty relation $\overline{\Delta x}^2 > \frac{1}{4}\,\hbar^2/\overline{\Delta p}^2$ (see refs.[6,12]). Therefore one might think that the Dirac particles can be localized with any accuracy. However, we should bear in mind that the creation of a wave packet of dimensions $\Delta x \lesssim \hbar/m_0 c$ by means of the external field, even if the latter is adiabatically switched on, will result in the production of pairs of particles x . Therefore, a possible localization of spinor particles turns out to be illusory, too.

We see that in the microworld there are no objects which would serve as a model of the point-like event $\mathcal{P}(x)$ since elementary particles cannot be localized more exactly than

$$\Delta x > \hbar/m_0 c. \tag{14}$$

In classic physics any material point can be considered not only as the realization of a point-like event but also as a body of reference ("Bezugkörper") which fixes the frame of reference. In the world of elementary particles this is impossible to do.

If we choose as a body of reference an elementary particle of rest mass m_0 then in the Lorentz transformation (2) U will be the four-dimensional velocity of the particle $u = \frac{p}{m_0 c}$, (p is the particle momentum) and the space shift components a_1, a_2 and a_3 its coordinates at the moment t=0.

It follows from the uncertainty relation

x For example, in a compound nucleus which is produced when two nuclei with charges Z_1 and Z_2 come near each other, under the condition $Z_1 + Z_2 > 137$, there arises an electronic orbit of radius $a_0 \simeq \hbar/m_0 c$. However, in this case an adiabatic production of pairs e^+e^- non-single-particle phenomenon, takes place, ref.[13].

$$[p_i, a_k] = i\hbar\,\delta_{ik} \tag{15}$$

that the parameters of the transformation (2) become operators. Therefore, the coordinates $(\bar{x})$ reckoned from this body of reference become also operators. In particular, from (2) and (15) it is not difficult to calculate the commutator $\bar{x}, \bar{t}$

$$[\bar{x}, \bar{t}] = i\frac{\hbar}{m_0 c}(x - \hat{v}t), \tag{16}$$

where $\hat{v}$ is the operator of the three-dimensional velocity of the particle.

Thus, elementary particles of finite mass can be utilized neither as objects by means of which one marks points in the $R_4(x)$ space nor as bodies of reference,

On the other hand, experimental facts point out that the predictions of the local field theory based on the microscopic causality (10) are satisfied up to scales of the order of 10^{-15} cm x.

It should thus be supposed that there exist elementary particles with a mass much more heavier than the mass m_p of a nucleon for which $\Delta x \simeq \hbar/m_0 c = 2\cdot 10^{-14}$ cm.

Thus, it follows that <u>local theory assumes inexplicitly the existence of arbitrary heavy elementary particles ($m_0 \to \infty$).</u>

x See ref.[14]

Under this assumption the contradiction between the use of the notion of arbitrary exact coordinates in the space $R_4(x)$ and the absence of objects playing the role of point-like events would be eliminated.

The limitation of the particle masses from above by a certain limit $m_0 = M$ would imply a limitation of principle of applicability of the local theory for scales of the order of $\Delta x \sim \hbar/Mc$.

The requirements of an ideal experiment concerning the marking of space-time points are opposite in classic and quantum physics. In what follows we consider possible reasons for the existence of the upper limit of the elementary particle mass.

4. Gravitation in the Microworld

In just the same way as in macroscopic physics, limitations on the elementary particle mass can come to the microworld from the side of gravitation.

According to the basic idea of A.Einstein, the curvature of the space-time R and its metric $g_{\mu\nu}$ are defined by the motion of matter. The basic equation of the theory reads:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi k}{c^2} T_{\mu\nu}(x). \tag{17}$$

We remind, here: $R_{\mu\nu}$ is the curvature tensor, $T_{\mu\nu}$ the momentum energy tensor, k the gravitational constant.

A detailed description of the motion of matter is definitely associated with quantum phenomena. Consequently, the tensor $T_{\mu\nu}(x)$ should be considered as a stochastic quantity represented by the operator $\hat{T}_{\mu\nu}(x)$. The quantities in the l.h.s. of (17) become also operators. In other words, whenever the motion of matter is viewed within the accuracy up to quantum phenomena the gravitational field becomes a quantum field [x].

The question about the role of gravitational phenomena in quantum domain is of a quite different nature.

For example, the gravitational field generated by zero oscillations of a solid body is related to the field created by the mass of its atoms as $\frac{\hbar\omega_0}{m_0 c^2}$ where ω_0 is the Debay frequency and m_0 the mass of an atom (or a molecule) The magnitude of this ratio is $10^{-11}/A$ (A is the atomic weight of atoms).

We decompose the momentum- energy tensor $T_{\mu\nu}(x)$ into two parts:

$$T_{\mu\nu}(x) = \bar{T}_{\mu\nu}(x) + \hat{t}_{\mu\nu}(x), \qquad (18)$$

[x] There are also other opinions concerning the possible extension of the Einstein equation to quantum phenomena. The approach in question is the most natural development of the Einstein idea.

where $\overline{T}_{\mu\nu}(x)$ is defined by the average motion of matter and $\hat{t}_{\mu\nu}(x)$ by the fluctuations of this motion. We represent the metric tensor $g_{\mu\nu}(x)$ in the form (9). Then the Einstein equation (17) is

$$A_{\mu\nu}^{\rho\sigma}\hat{h}_{\rho\sigma}+B_{\mu\nu}^{\rho\sigma\alpha}\frac{\partial\hat{h}_{\rho\sigma}}{\partial x_\alpha}+C_{\mu\nu}^{\rho\sigma\alpha\beta}\frac{\partial^2\hat{h}_{\rho\sigma}}{\partial x_\alpha\partial x_\beta}=\frac{8\pi k}{c^2}\hat{t}_{\mu\nu}(x) \qquad (19)$$

where the tensors A, B and C depend only on the average tensor $\overline{g}_{\mu\nu}$ and its derivatives.

If the dimensions of the mass m defining the average metric are of the order of a the space curvature R is in the order of magnitude equal to

$$R=\frac{1}{\ell^2}\simeq\frac{1}{a^2}\frac{a_g}{a} \qquad (20)$$

as it immediately follows from the Einstein equation (17). Here a_g is the gravitational radius of the body (see eq. (6)) and ℓ - is the length characterizing the space curvature. If the characteristic fluctuation mass is Δm and its characteristic size is b then eq. (19) can be schematically written as

$$\frac{\alpha}{\ell^2}\hat{h}+\frac{\beta}{\ell\ell'}\hat{h}+\frac{\gamma}{\ell'^2}\hat{h}\simeq\frac{b_g}{b^3} \qquad (21)$$

where

$$b_g=\frac{8\pi k\Delta m}{c^2} \qquad (22)$$

is the gravitational radius of fluctuation, ℓ' - the length scale characterizing the stochastic field gradient $\hat{h}$; α, β, γ - are the numerical coefficients. Following the meaning of eq. (14) $\ell' \ll \ell$. Next, it follows from the linearity of the equation that the scale of length defining the gradient of the tensor $\hat{h}$ and the scale defining the gradient of the matter tensor should be compatible. Hence it follows $\ell' \simeq \ell$. Thus, on the basis of (21) we have

$$\hat{h} \simeq \frac{\ell_g}{\ell} . \tag{23}$$

This equality defines the order of magnitude of the fluctuations of the gravitational field by the magnitude of the matter fluctuations. Hence, it is not difficult to determine the fluctuations of the metric tensor which are caused by zero oscillations of a certain quantum field. For example, for the scalar field $\varphi(x)$ zero mass oscillations of a scale exceeding ℓ are

$$\Delta m(\ell) = \frac{\Delta E(\ell)}{c^2} = \frac{\ell^3}{c^2} \int_0^{1/\ell} \frac{\hbar \omega_k}{2} \, d^3k = \frac{\hbar}{c\ell} , \tag{24}$$

where $\frac{\hbar\omega_k}{2}$ is the zero energy of a k-th oscillation.

Inserting (24) in (23) we get

$$\hat{h} \simeq \frac{8\pi k}{c^2} \frac{\Delta m(\ell)}{\ell} = \Lambda_g^2/\ell^2 , \tag{25}$$

where

$$\Lambda_g = \sqrt{\frac{8\pi k \hbar}{c^3}} = 0{,}82 \cdot 10^{-32}\ cm \tag{26}$$

is the known length uniting the gravitational constant and the Plank constant. It is seen from (25) that this length defines the magnitude of the fluotuations of the metric tensor which are caused by the quantum fluctuations of a material field. These fluctuations are small throughout the whole range of frequencies for which

$$\ell \gg \Lambda_g . \tag{27}$$

Now we return to the collapsing particle considered in section 2 and take into account quantum effeots. A partiole of mass m has an effective dimension of the order of $a = \frac{\hbar}{mc}$. We assume that this partiole has reaohed its oritical mass so that its gravitational radius is $a_g = a$. Then it follows from the condition $\frac{8\pi k m}{c^2} = \frac{\hbar}{mc}$:

$$a = \Lambda_g, \quad m = M_g = \frac{\hbar}{\Lambda_g c} = 0{,}52 \cdot 10^{-5}\ g . \tag{28}$$

Thus, the length Λ_g defines the maximum mass of a partiole obeying the laws of quantum theory. Aocording to (7) the density of matter reaches its limit value.

This particle was called by Markov maximon[15,16].

From (27) and (28) quite different conclusions can be drawn concerning the role of gravitation in the world of elementary particles depending on to what frequencies $\Omega_0 = \frac{c}{a}$ the spectrum of vacuum fluctuations is really limited. According to contemporary theory, it is almost uniformly distributed over the frequencies and the high frequencies are expected to give an infinitely large contribution to gravitation. If later on, for some or other reason, it will turn out that the possible frequencies in the microworld are limited by an "elementary" length $a \gg \Lambda_g$, then gravitational effect will be nonessential.

In the opposite case they will play the fundamental role in the microworld, but they will be represented in their quantum interpretation[x].

Predictions and hopes based on classic calculations of gravitation will be "majorized" by quantum effects. At the same time the "classic" average metric will be no longer predominant and the situation like that mentioned in section 2 arises: the notion of interval between events as well as the idea itself about a possible ordering of events in $R_4(x)$ space becomes more than doubtful. Here we are approaching the edge of an abyss without being sure if it is not too early to peep into it.

---

[x] Over many years this line has been developed by Wheeler and co-workers[17].

In what follows we shall consider other possible restrictions on local theory. Among various competitors the "weak" interaction claims to this role.

5. A "Weak Maximon"

We distinguish three modes of interactions: strong, electromagnetic and weak interactions. Let us compare their behaviour at high energies using a criterion suggested in ref.[18] According to it, an interaction is assumed to be strong if in the course of the interaction the density of the kinetic energy $\mathcal{E}_K$ is much smaller than the absolute density of the energy of their interaction W:

$$\mathcal{E}_K \ll |W|. \tag{29}$$

We first consider a collision of a nucleon $(\mathcal{N})$ and a pion (π). In this case the density of the total energy is

$$H = \hbar c \bar{\psi} \partial \psi + Mc^2 \bar{\psi} \psi + \frac{1}{2}(\Box \varphi^2 + m^2 \varphi^2) + \\ + g \bar{\psi} \gamma_5 \tau \varphi \psi, \tag{30}$$

where ψ is the nucleon field, φ the meson field, M the nucleon mass, m the meson mass; $\partial = \gamma^\mu \frac{\partial}{\partial x^\mu}$, g is the interaction constant. Let ℓ be a length defining the magnitude of the gradient in the c.m.s. ($\ell \simeq \frac{\hbar}{p} = \lambda\!\!\!^{-}$, p is the particle momentum). Then the density of the kinetic energy $\mathcal{E}_N$ of the nucleon is of an order of magnitude

$$\mathcal{E}_N \simeq \frac{\hbar c}{\ell}\,\bar{\psi}\psi \tag{31}$$

(since $\partial \sim \frac{1}{\ell}$); the density of the meson kinetio energy is

$$\mathcal{E}_\pi \simeq \frac{\varphi^2}{\ell^2} \tag{32}$$

(sinoe $\Box^2 \sim \frac{1}{\ell^2}$). Henoe,

$$/W/ \simeq g\bar{\psi}\psi\varphi \simeq \frac{g}{\hbar c}\,\ell^2 \mathcal{E}_N \mathcal{E}_\pi^{1/2}. \tag{33}$$

Taking into consideration the faot that $\mathcal{E}_K = \mathcal{E}_N + \mathcal{E}_\pi$ from condition (29), we get

$$1 \ll \frac{g}{\hbar c}\,\frac{\mathcal{E}_N}{\mathcal{E}_N + \mathcal{E}_\pi}\,\mathcal{E}_\pi^{1/2}\,\ell^2. \tag{34}$$

Then

$$\mathcal{E}_\pi \simeq \frac{pc}{\ell^3} \simeq \frac{\hbar c}{\ell^4} \tag{35}$$

and

$$\frac{\mathcal{E}_N}{\mathcal{E}_N + \mathcal{E}_\pi} \simeq \frac{1}{2}\,. \tag{36}$$

As a result, we find

$$\frac{g^2}{\hbar c} \gg 1. \tag{37}$$

Thus, it follows that according to the criterion used, the strong interaction is strong under all the conditions since the inequality (37) is always fulfilled .

Now we apply the same criterion to the interaction of an electromagnetic field with charged spinor particle.

In so doing, we have

$$W = e\bar{\Psi}A\Psi, \tag{38}$$

where $A = \gamma^{\mu}A_{\mu}$, A_{μ} is a vector potential and e the particle charge. Following the same procedure we get

$$\frac{e^2}{\hbar c} \gg 1. \tag{39}$$

This inequality does not hold. Consequently, following our criterion, electromagnetic interactions do not belong to strong interactions [x] .

Now we consider the case of weak interaction we are interested in. In this case the density of the total energy is

$$H = \hbar c\bar{\Psi}\partial\Psi + Mc^2\bar{\Psi}\Psi + \hbar c\bar{\varphi}\partial\varphi + mc^2\bar{\varphi}\varphi + \\ + g_F\bar{\Psi}O_{\alpha}\Psi\cdot\bar{\varphi}O^{\alpha}\varphi, \tag{40}$$

where Ψ and φ are the fields of a nucleon and a lepton respectively, M and m are the masses of these particles, G_F is the Fermi constant, O_{α} the spinor operator.

[x] We note that we have not taken into account the direct interaction between a gamma quantum and vector mesons $(\rho, \omega \ldots)$

It is not difficult to see that in this case the interaction energy is of the order

$$|W| \simeq g_F \frac{\varepsilon_N \ell}{\hbar c} \frac{\varepsilon_e \ell}{\hbar c}, \tag{41}$$

where ε_N is the density of the nucleon kinetic energy and ε_e the density of the lepton kinetic energy. Taking into consideration the fact that $\varepsilon_N \simeq \varepsilon_e \simeq \frac{pc}{\ell^3} = \frac{\hbar c}{\ell^4}$ from condition (29), we obtain

$$\frac{\Lambda_F^2}{\ell^2} \gg 1, \tag{42}$$

where $\Lambda_F = \sqrt{\frac{g_F}{\hbar c}} = 0{,}66 \cdot 10^{-16}\,cm$, $\ell \simeq \lambdabar = \frac{\hbar}{p}$.

Hence, it follows that a weak interaction becomes strong at an energy $E \sim \frac{\hbar c}{\Lambda_F}$ 300 GeV (see also refs.[19,20]).

We now consider a decay of a heavy hadron of mass which is due to a weak interaction

$$M \rightarrow m + \ell + \tilde{\nu}.$$

Here m is the nucleon mass, ℓ is a lepton and $\tilde{\nu}$ stands for an antineutrino. The decay constant Γ for this process, when $M \gg m$ is[21]

$$\frac{\Gamma}{M} = \frac{1}{4\pi^3} G_F^2 M^4 N, \tag{43}$$

where $G_F = \frac{g_F}{\hbar c} = 1 \cdot 10^{-5} \frac{1}{m^2}$, $\mathcal{N}$ is the number of channels for different decay modes, which may be large. It is seen from this formula that for the hadron mass

$$M > m_F = \frac{\hbar}{\Lambda_F c} \qquad (44)$$

the decay constant Γ becomes comparable with the hadron mass M and the hadron stops to exist as an elementary partiole since it may not be assigned a definite mass. It is advisable to refer to a particle with mass M_F as weak maximon . As is seen from eqs. (44) and (26?), this restriction begins to work earlier than that imposed by gravitation, since $M_F \ll M_g$. At the same time the restriotion on local theory must then oocur muoh earlier than it is expected from the assumption about the existence of a gravitational maximon M_g.

6. "Blaokness" of Particles and Locality

The elemantary partiole is a oertain medium whioh is desoribed by the creation and annihilation of virtual partioles.

It is natural to raise the question about the condition of propagation of a metrio signal in suoh a peouliar medium. Starting from perturbatinn theory, it is possible to answer this question by means of the Green funotion which, being based on looal theory, guarantees the propagation of an interao- tion with the velooity of light.

However, the situation changes if the interaction becomes strong. In this case there arise nonlinear phenomena and a strong absorption as a result of inelastic processes.

The former group of phenomena occurs in the region of strong fields and small gradients. In refs. [22,23], by the example of a scalar and an electromagnetic fields it was shown that the law of propagation of these fields noticeably changes, up to the disappearance of any possibility of propagating: the characteristics of nonlinear equations become imaginary, and the equation of the hyperbolic type turns to an equation of the elliptic type. This situation was called a "lump" of events. In the spirit or contemporary understanding it is more right to give in the name of "light collapse" [6,22].

In the region of large gradients there occur inelastic processes. A possible limitation of the space-time description of the elementary particle structure was indicated in ref. [25]. This limitation is due to the fact that the cross section for an inelastic process does not decrease with increasing energy but tends to a constant limit or even increases slowly.

At the same time elastic scattering assumes the character of a diffractional scattering on a "black" sphere of dimension a. In particular, on the basis of first papers on pion-nucleon scattering, it was noted [24] that the "effective" potential for this scattering is purely imaginary and is well represented by the formula [x]

[x] This potential is being successfully used as a first approximation in the theory of "quasipotential" [25].

$$\tilde{V}(q) = i A(E)\, e^{-a^2 q^2}. \qquad (45)$$

Here q is the momentum transfer and $A(E)$ a certain function of the energy E defined by the main differential maximum.

When the role of inelastic processes becomes dominating the information is relative not so much to the space-time structure as to the production of new particles. The "blackness" of particles makes it impossible to use elastic scattering for studying space-time distribution of matter. The given example of mesons is a very particular case, therefore it is not of value of principle.

For the problem we are considering here of interest would be only a situation when "blackness" would appear for the most universal signal. The most universal are weak interactions.

Therefore, such a situation can arise if the weak interaction is assumed to increase up to scales dictated by unitary limit.

Thus the possible limitation of local theory by the conditions of the signal propagation inside an elementary particle coincides with the condition following from the existence of a "weak maximon".

7. The Momentum Space

Classic theory deals with the space $\mathcal{R}_4(x)$ and the contravariant tangent spaoe $\mathcal{R}_4(p)$ in a combined manner. Quite a different situation takes place in the domain of quantum phenomena. In quantum motion the trajectory of a material point is nondifferentiable [x] and the spaces $\mathcal{R}_4(x)$ and $\mathcal{R}_4(p)$ are mutually complementary. They belong to two different incompatible classes of measurements.

The both spaces are equal in rights from the theoretical point of view, since the transition from the description in one of them to the description in the other is performed by means of a unitary transformation of the state veotors Ψ and the appropriate transformation of the operators $\hat{\mathcal{L}}$ which represent physical quantities.

However, both the descriptions are unequal in rights in a physioal experiment. The space $\mathcal{R}_4(x)$ figures in an experiment in its macroscopic aspect. The microscopic ordering is not revealed experimentally, in a direot manner, sinoe the observed causality is a macrosoopio causality. In fact, for an event A located in the spaoe-time region $\mathcal{G}_A(x)$ to be oonsidered as a cause of an event B in the domain $\mathcal{G}_A(y)$ it is necessary to be sure that when A had oocured a quantum of an energy $\mathcal{E} = \hbar\omega \geqslant 0$ and momentum $\vec{p} = \hbar\bar{k}$ had been emitted which was then absorbed in the domain $\mathcal{G}_B(y)$ generating thereby the event B.

[x] See ref.[27].

In this description of the causal relationship we have used both the spaces $\mathcal{R}_4(x)$ and $\mathcal{R}_4(p)$: the former is to show the relative position of the events A and B and the latter is to indicate the direction of energy and momentum transfer [x].
The oombined use of the mutually oomplementary spaces $\mathcal{R}_4(x)$ and $\mathcal{R}_4(p)$ leads us to the classic, i.e. macroscopio, field of physics.

Consequently, the space-time description is oarried out with an aoouracy which is far from being sufficient for the restrictions of the type (14) to be appreciable.

Contrary to the space-time description, the momentum-energy description in the spaoe $\mathcal{R}_4(p)$ is realized in an experiment with an aoouracy which seems to be unlimited.
In this description miorosoopio causality expressed by the condition of local commutativity (10) is displayed indirectly, in the behaviour of the amplitudes $T_{if}(p)$ prediated by local theory for different physical processes. Here i labels the initial state and f the final one. In particular, micro-causality is manifested in the analytic properties of the amplitude $T_{if}(p)$ in the complex plane of the variable p_0 [xx].
In the space $\mathcal{R}_4(p)$ the state of free stable partioles is desoribed by the points on a hyperboloid:

---

x See ref.[6].

xxThese properties underlie "dispersion relations" which are important for analysing experimental data[28].

$$p^2 \equiv p_0^2 - \vec{p}^{\,2} = m_0^2 , \tag{46}$$

where m_0 is the particle mass. Each such hyperboloid is the Lobachevsky space $R_3(\vec{p})$ with curvature $R = -\frac{1}{m_0^2}$. In the space $R_4(p)$, due to the fact that its metric (4) is nondefinite, there exists no invariant concept of large or small momenta. Consequently, there also exist no invariant limitations on the frequency ω or the wave vector $|\vec{k}|$. Such limitations would necessarily select any frame of reference. This supplements the assertion about the absence of an invariant measure of spacing of events in the space $R_4(x)$.

The amplitudes T_{if} describing physical processes are the matrix elements of the scattering matrix

$$S_{if} = \delta_{if} + i\, T_{if} . \tag{47}$$

This matrix is known to define the state of particles at a time "moment" $t_f = +\infty$ if it is given at a time "moment" $t_i = -\infty$. From the geometrical viewpoint the S-matrix transforms the state of particles given in a certain direct product of the Lobachevsky spaces $R_3(\vec{p}_1) \otimes R_3(\vec{p}_2) \otimes R_3(\vec{p}_3) .. \otimes R_3(\vec{p}_i)$ to a new state given, generally speaking, by some other product of these spaces $R_3(p_1) \otimes R_3(p_2) \otimes --- R_3(p_f)$. Since the particle moments are specified, the particle coordinates are undetermined. The time "moments" $t = \pm\infty$ are undetermined too. Therefore the ordering of events in $R_4(x)$ obtained by means of the S matrix is minimal one.

Contrary to the common opinion, the description of the microworld phenomena by means of the S matrix is incomplete. The S-matrix cannot be used to describe the behaviour of unstable particle since the specification of the initial conditions, in this case, cannot be formulated for the time moment $t = -\infty$ x .

This statement may be illustrated by a situation with K_o mesons when it is necessary to follow the evolution of the state

$$\tilde{K}^o = \frac{1}{\sqrt{2}} \left[K_s^o(t) - K_L^o(t) \right], \tag{48}$$

where $\tilde{K}_o$ are the state of an antimeson and K_s^o and K_L^o are the states of a short- and long-living mesons, t - is the time. Oldfashioned methods of description seem here to be inevitable since an ordering of events in time with an accuracy $\Delta t \ll \tau_s$ of life-time of a short-lived K^o meson is needed.

These remarks related to the S-matrix do not restrict possibilities of the description in the space $R_4(p)$ which can be extended to the domain of complex p . Moreover, such an extension seems to be necessary for describing the behaviour of unstable particles. Therefore, in spite of formal equality in rights of the descriptions of phenomena in $R_4(x)$ and $R_4(p)$, the description in the latter is less open to the criticism addressed to the local theory which deals with the space-time.

x The exception is some special cases when, e.g., an unstable particle may be thought of as a resonance.

It seems that in this conneotion, as early as in the fourties, H.Snyder[30] suggested an interesting idea according to which the metric of the momentum space $\mathcal{R}_4(p)$ may be more complicated than the Minkovsky metric (4). Instead of (4), he has just proposed a Riemannian metric

$$dp^2 = g_{\mu\nu}\, dp_\mu\, dp_\nu, \tag{49}$$

where the metric tensor $g_{\mu\nu}$ is a function of the momentum

$$g_{\mu\nu} = g_{\mu\nu}(p, p_a) \tag{50}$$

and the parameter

$$p_a = \frac{\hbar}{a}. \tag{51}$$

Here a is a certain "elementary lenghh" and p_a is a momentum playing the role of the curvature scale for the momentum space. The relationship between the space $\mathcal{R}_4(x)$ and $\mathcal{R}_4(p)$ is based on the assumption that the ourved spaoe $\mathcal{R}_4(p)$ is a space of constant curvature x . This limitation makes it possible to consider the coordinates $x_0\ x_1\ x_2\ x_3$ in the space $\mathcal{R}_4(x)$ as displacement operators in the space $\mathcal{R}_4(p)$:

$$x_\mu \longrightarrow \hat{x}_\mu \equiv \left\{ i \frac{\partial}{\partial p_\mu} + A_\mu^\nu(p) \frac{\partial}{\partial p_\nu} \right\}. \tag{52}$$

For $a=0$, $A_\mu^\nu(p)=0$ so that (52) transforms to the usual operator of coordinates which is charaoteristio of local theory.

---

x The Snyder idea has then been developed in papers of I.Gelfand/31/, V.Kadyshevsky/32/ and I.Tamm/33/.

References

I. H.Poincare, La mesure du temps.Rev.de Metaphysique et de Morale, VI (1898).
2. А.Эйнштейн, Собр.научн.трудов, I,7 (1965), Сущность теории относительности, ИЛ (1955).
3. Л.И.Мандельштам, Сбор.соч. У, АН СССР (1965).
4. А.А.Фридман, Мир как пространство и время, Наука (1965).
5. А.А.Тяпкин, УФН 106, 6I7 (1972).
Б.Б.Кадомцев, и др. там же.
6. Д.И.Блохинцев. Пространство и время в микромире, "Наука", (1970).
7. H.Rund, The Differential Geometry of Finsler Space, J. Springer (1959).
8. D.I.Blokhintsev, Nuovo Cim. 16, 382 (1960).
9. K.Menger, Proc.Nat.Ac.Sci.USA 28, 535 (1942).
I0. B.Schweizer, A.Sklar. Pacific Journ.of Math. 10,N.1 (1960).
II. Д.И.Блохинцев, Сообщения ОИЯИ Р2-6094, Дубна (1971).
I2. D.I.Blokhintsev, Acta Phys.Sc.Hung. 22, 307 (1967).
I3. В.С.Попов, Ядерная физика XII, 429 (1970).
I4. Л.Д.Соловьев, Раппорт.доклад на Киевской конференции по физике высоких энергий (1970).
I5. М.А.Марков, препринт Е-2644, Дубна (1966).
I6. М.А.Марков, препринт Е2-527I , Дубна (1970).
I7. I.A.Wheeler, Сб. Гравитация и геометрия, МИР (1965).
I8. Д.И.Блохинцев, УФН XII, 38I (1957).
I9. D.I.Blokhintsev, Roc.Rochester Conf. 1960.
20. D.I.Blokhintsev, Nuovo Cim. X, 9 , 925 (1958), ЖЭТФ 35, I00I (1958).
2I. С.И.Биленький, Введение в диаграммную технику, Атомиздат (1971).
22. Д.И.Блохинцев, ДАН 82, 553 (1952), См.также 6 .
23. Д.И.Блохинцев, В.Орлов, ЖЭТФ 25, 5I3 (1953).
24. D.I.Blokhintsev, Nucl.Phys. 31, 628 (1961).

24. D.I.Blokhintsev, Nucl .Phys. 31, 628 (1961).
25. D.I.Blokhintsev, V.S.Barashenkov, and V.Grishin, Nuovo Cim. X, 9, 249 (1958).
26. A.A.Logunov, A.N.Tavkhelidze, Nuovo Cim. 29, 380 (1963).
27. Р.Фейнман и А.Хиббс. Квантовая механика и интегралы по траекториям, МИР (1968).
28. Н.Н.Боголюбов, Б.В.Медведев и М.К.Поливанов. Вопросы теории дисперсионных соотношений, ГИФМЛ (1958).
29. Н.А.Черников, Научные доклады высшей школы. физ.мат.науки, №2, 158 (1958).

30. H.Snyder, Phys.Rev. 71, 38 (1947).
31. Ю.А.Гельфанд, ЖЭТФ 57,4504 (1959), 43, 256 (1962), 44, 1248 (1963).
32. В.Г.Кадышевский, ЖЭТФ 41, 1885 (1961), ДАН СССР 147, 588 (1962). ДАН СССР 147, 1336 (1963).
33. И.Е.Тамм. XII Межд. конференция по физике высоких энергий, Дубна, т.II, 229 (1964).
34. В.Г.Кадышевский, Препринт Р2-5717 Дубна (1971).

Received by Publishing Department
on August 11, 1972