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PIONIZATION AT ISR-ENERGIES

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PIONIZATION AT ISR-ENERGIES

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1. Introduction

It is an old practice in high-energy phenomenology to ascribe the whole of the multiple particle production to two processes which may be named pionization and fragmentation. Pionization gives rise to many particles with low energies, fragmentation gives rise to a few particles with high energies emitted at small angles to the collision axis (all in the centre-of-momentum system, CMS). It is understood that the separation in energy should be more and more pronounced if we go to higher and higher collision energies. The pionization process ("direct" production) means not only the production of pions but also of kaons ("kaonization") and, in fact, any hadrons. Also, the pionization particles must not necessarily be emitted isotropically. As will be explained below, pionization can be accounted for by some simple modification of the statistical model and hence can be given a precise meaning. Fragmentation, at accelerator energies, is represented by the decay of the colliding nucleons which escape from the collision in an excited state ($\Delta(1236)$, $N(1688)$, $\Sigma^{-}(1197)$? ...) and subsequently decay into a proton or a neutron and a few mesons. Since the colliding particles (supposedly even incident pions) retain an appreciable fraction (≈ 0.5) of their initial energy, the decay products will be emitted with high energies and at small angles. These are just the above characteristics of the "fragmentation" process. In fact, Hagendorn and Ranft/1/ have shown that at accelerator energies the whole of the single particle production spectra can be adequately described by "throughgoing" and "newly created" particles. The newly created particles completely correspond to our pionization particles, and the throughgoing particles represent the collision particles escaping in a superposition of excited states. It seems natural to suppose that excitation and decay of colliding particles will continue to higher energies so that we could always identify fragmentation with this process. However, we do not want here to specify fragmentation more precisely than by the above characteristics concerning energy and emission angle. Thus fragmentation in this wide sense may comprise as well such processes as the non-resonant "limiting fragmentation" of Yang et al./2/, the "novas" of Jacob and Slansky/3/, the "diffraction dissociation" of Good, Walker et al./4/ or of Hwa and Lam/5/. In fact, fragmentation can even be simulated by our two-centre formula with an anisotropy parameter γ_f of the order of the Lorentz factor of the incoming particles (see below).

The existence of pionization is doubted at high energies, e.g.[2]. The purpose of this letter is to show that at ISR energies, which correspond to collision energies E_0 (LS kinetic energy of the bombarding particle) up to 1000 GeV, there is pionization. We shall show that the same formula that describes pionization at accelerator energies also describes most of the ISR production spectra measured by Neuhofer et al.[6], and that the rest can be made plausible to arise from the fragmentation process.

2. Pionization

At accelerator energies pionization can be described by a modification of the statistical model. The modification consists in taking into account the empirical fact that the pionization particles in the CMS are not emitted isotropically but are collimated in forward and backward direction along the collision axis. The combination of this peripherality with the statistical idea and intensive comparison with experimental production spectra has first been done in the "two-temperature statistical model" by Bowen, Wayland et al.[7], in the "thermodynamical model" by Hagedorn and Ranft[1], and by the author[8]. These three models fit the data almost equally well[x]. Our proposal[8] may be called a "semi-statistical" or "two-centre-statistical" model.

[x] See also the formula proposed by Hoang and its fit to some data[9].

Some comments on its relation to the thermodynamical model are given in [8] x/. We have introduced two centres or "fireballs" . These emit particles isotropically and with a Bose- or Fermi-type momentum distribution in their own rest system. There is only one temperature kT , the same in both centres. In the CMS the centres move with the same absolute value of velocity β_f , but in opposite direction along the collision axis xx/. Thus, in a Lorentz system S in which the fireball moves with velocity β_{fS} and Lorentz factor $\gamma_{fS} = (1 - \beta_{fS}^2)^{-1/2}$ the number of particles of rest mass m with momenta in the interval dp emitted into the solid angle $d\Omega$ at an angle θ to the direction of the fireball velocity is given by the formula

$$\frac{d^2N}{d\Omega\, dp} = \frac{MF}{4\pi C N_{\pm}(m,kT)}\, \frac{p^2}{E}\, \frac{\gamma_{fS}(E - p\beta_{fS}\cos\theta)}{\exp(\gamma_{fS}(E - p\beta_{fS}\cos\theta)/kT) \pm 1} \qquad (1)$$

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x/ In [8] on page 7 line 12 it must read $p_t \to (\pi m kT/2)^{1/2}$ instead of $\bar{p}_t \to \pi/4\cdot(3/2\cdot kT + m)$, and on page 11 line 9 from the bottom it must read $3/(16\,\Omega(E))^{1/3}$ instead of $3/\Omega(E)^{1/3}$.

xx/ Originally, a velocity distribution function, expressed in terms of a distribution in $\gamma_f = (1 - \beta_f^2)^{-1/2}$ had been introduced. However, we have convinced ourselves that a δ-distribution, i.e. a single absolute value of the velocity (or equivalently of γ_f) gives data fits of the same quality. This is already indicated by Fig.20 of [8] where the influence of the shape of the velocity distribution function on the paricle spectra is shown to be small.

$E = (p^2 + m^2)^{1/2}$ is the total energy of the particle. $\beta = (1 - 1/\gamma_{fS}^2)^{1/2}$. by means of

$$CN_{\pm}(m, kT) = \int_0^{\infty} \frac{p^2\, dp}{\exp(E/kT) \pm 1} \qquad (2)$$

$$\approx 1.25\,(kT)^3 \exp\left(-\frac{m}{kT}\right)\left[\left(\frac{m}{kT}\right)^{3/2} + 0.86\left(\frac{m}{kT}\right)^{0.75} + \left\{\begin{matrix} 1.44 \\ 1.92 \end{matrix}\right\}\right] \begin{matrix} \text{bosons} \\ \text{fermions} \end{matrix}$$

(maximum error 3%)

The function (1) is normalized to MF: $\iint (d^2N/d\Omega\, dp)\, d\Omega\, dp = MF$ The upper sign refers to fermions (nucleons), the lower sign to bosons (pions, kaons). γ_{fS} (β_{fS}) in system S is obtained by Lorentz transformation from the CMS where γ_f is originally given. In any system the total production spectrum is the sum of two terms (1), one with γ_{fS} and β_{fS} referring to the CMS-forward and the other to the CMS-backward fireball. For example, in the Laboratory System (LS) we have $\gamma_{fS\,1,2} = \gamma_f \gamma_C \pm ((\gamma_f^2 - 1)(\gamma_C^2 - 1))^{1/2}$ with γ_C = Lorentz factor of the CMS in the LS. In the CMS itself we have $\gamma_{fS\,1} = \gamma_{fS\,2} = \gamma_f$, $\beta_{fS\,1} = -\beta_{fS\,2} = \beta_f$.

After specification of collision energy and particle mass formula (1) contains three parameters MF, kT and γ_f. No prediction is made about multiplicity and inelasticity. Therefore these parameters are regarded as fitting parameters. MF represents the total number of particles of mass m emitted from one fireball in a collision, i.e. half the total multiplicity. It fixes the absolute height of the spectrum. In this letter we are only concerned with the shape of the spectrum, this is determined by kT and γ_f. If we confine ourselves to the pionization particles, which at accelerator energies represent the bulk of all produced particles, formula (1) excellently fits the single particle production spectra, as is demonstrated in [8]. The fitting values for MF come close to the observed multiplicities. Typical values for kT and γ_f are plotted in Fig. 2[x]. The data in [8] justify the assumption that both kT and γ_f in the first line depend only on collision energy. Thus formula (1) seems to be simple but powerful fitting formula.

The two-fireball interpretation, however, is regarded only as some helpful pictorial view. The main theoretical background is the statistical model in the sense of a "minimal information" interpretation. The Bose (or Fermi) momentum distributions in the strict sense must be replaced by momentum (phase) space distributions, and kT is to be replaced by the distribution of total energy (mass) and multiplicity of the statistical ensemble. The quantity γ_f is regarded as a formal "anisotropy parameter" incorporating the effect of angular-momentum conservation as well as dynamical effects of strong interaction.

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[x] These fitting values are obtained under the assumption of a distribution for γ_f. They change, however, only slightly, if we take a fixed value for γ_f.

3. ISR-Data Fit

We apply the pionization formula (1) to the ISR-data of Neuhofer et al.[6]. They present energy spectra of γ rays from the reaction $p+p \to \gamma +$(anything) at CMS angles $\theta = 10.2^\circ$, 16.4°, 23.5° and 90° and at CMS energies $\sqrt{s}$ (a) 30.2 GeV (E_0 =484 GeV), (b) 44.7 GeV (E_0 =1063 GeV). At $\sqrt{s}$ =52.7 GeV spectra were measured only at 10.2° and 90°. These data are too scarce for a significant fit, therefore they are left out. We proceed as follows. The observed γ rays arise from decay of produced π°, K_S° and others. We obtain that 67% of the observed γ rays come from $\pi^\circ \to \gamma\gamma$, 12% from $K_S^\circ \to \pi^\circ\pi^\circ \to 4\gamma$ and 18% from the various decay channels of the η if we take into account all hadrons with life times between $\tau = 2.5 \times 10^{-19}$ s (η meson) and $\tau = 3 \times 10^{-10}$ s (Ξ° hyperon)[10] and take their production rate as predicted by thermodynamics at a temperature of kT = 0.14 GeV (cf.[8] section 4.3). Compared with ISR data[11], $K^-/\pi^- \approx 0.08$, our production rate of $K^+/\pi^- = K_S^\circ/\pi^\circ = 0.22$ might seem rather high. However, the results of[11] are obtained only for production angles of $2.3^\circ - 11.5^\circ$, i.e. in the fragmentation region (see below). Cosmic-ray data also suggest higher rates[12]. In order to simplify matters we take into account only the two decays $\pi^\circ \to 2\gamma$ and $K_S^\circ \to 2\pi^\circ \to 4\gamma$. This may be justified because the other chains result in γ spectra which deviate from the shape of the $\pi^\circ \to \gamma\gamma$ spectrum in the same sense as the

$K_S^\circ \rightarrow 2\pi^\circ \rightarrow 4\gamma$ spectrum does. Thus, the K_S° decay is taken as representative of all the other decays and accordingly is taken to contribute at the enhanced rate $q = (K_S^\circ \rightarrow 2\pi^\circ \rightarrow 4\gamma)/(\pi^\circ \rightarrow 2\gamma) = 0.4 \approx (\eta + K_S^\circ)/\pi^\circ$, i.e. $MF(K)/MF(\pi) = 0.4$ in formula (1)[x/].

According to Sternheimer[/13/], in the decay $K_S^\circ \rightarrow \pi^\circ\pi^\circ$ the spectrum of the K°, $d^2N(E,\theta)/dEd\Omega$ (arbitrary Lorentz system), can be expressed through the π° spectrum, $d^2\sigma(E_\pi,\theta)/dE_\pi d\Omega$ by means of

$$\frac{d^2N(E,\theta)}{dE\,d\Omega} = -\kappa\left[\frac{E_\pi/a}{2}\,\frac{\partial}{\partial(E_\pi/a)}\left(\frac{d^2\sigma(E_\pi/a,\theta)}{d(E_\pi/a)\,d\Omega}\right)\right]_{E_\pi=E} \qquad (3)$$

with a =1.088, κ =0.84[/10,13/]. Following Sternheimer we assume that the formula (3) is valid within some 10% if $E \gtrsim$ 1.5 GeV. In the decay $\pi^\circ \rightarrow \gamma\gamma$ the π° spectrum can be expressed via the γ spectrum by means of the same formula, only with $a = \kappa = 1$. The accuracy should then be better than some 10% if $E_\pi \gtrsim$ 0.5 GeV. Then we obtain the K° spectrum in terms of the γ spectrum by inserting formula (3) with $a = \kappa = 1$ into the righ-hand side of formula (3) with a =1.088, κ =0.84. Sternheimer's formula (3) suggests a simple fitting method, namely to compare the spectra of the primaries (K_S°, π°), $d^2N/dEd\Omega$, described by formula (1), with the differentiated spectra

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[x/] For the determination of the value for kT and γ_f the value of q is not critical. E.g., we would obtain $0.13 \lesssim kT \lesssim$ 0.19 GeV, $1.25 \lesssim \gamma_f \lesssim$ 1.50 if we described the 90° and 23.5° spectra by $\pi^\circ \rightarrow \gamma\gamma$ decay only (q =0).

of the secondaries (γ), $d^2\sigma/dE\,d\Omega$, as given by experiment. In order to do so, we fit the experimental γ-ray spectra by exponentials $d^2\sigma/dk\,d\Omega = c(\theta)\exp(-k/k_0(\theta))$. In the second paper of /6/ it has been shown that this gives excellent fits. The fitting values of $k_0(\theta)$ are the following. At $\sqrt{s}$ =30.2 GeV : $k_0(90)$=0.167, $k_0(24)$=0.325, $k_0(16)$=0.402, $k_0(10)$=0.472, and at $\sqrt{s}$ =44.7 GeV: $k_0(90)$=0.161, $k_0(24)$=0.376, $k_0(16)$=0.460, $k_0(10)$= =0.663. Carrying out the differentiations and rearranging terms we finally have

$$\frac{2k_0(\theta)}{E}\,\frac{d^2N(E,\theta)}{dE\,d\Omega} = c(\theta)\exp(-E/k_0(\theta)) \qquad (3')$$

for the contribution of $\pi^\circ \to \gamma\gamma$, and

$$\frac{2a\exp(E(1/a-1)/k_0(\theta))}{\kappa(E/(a\,k_0(\theta))-1)}\,\frac{2\,k_0(\theta)}{E}\,\frac{d^2N(E,\theta)}{dE\,d\Omega} = c(\theta)\exp(-E/k_0(\theta)) \qquad (3'')$$

for the contribution of $K^\circ_S \to 2\pi^\circ \to 4\pi$ (a =1.088, κ =0.84). The right-hand sides represent the original γ spectrum. Therefore we plot the left-hand-side expressions with $d^2N/dE\,d\Omega$ from formula (1) (forward and backward fireball, $E \approx p = k$) on the original γ-ray data plots.The error bars are then indicative of the significance of the fit.Figure 1 a,b presents the results.The curves are not drawn in the low momentum region since the error of Sternheimer's approximation (3) becomes large here.

The curves at 10.2^o fall significantly below the experimental points. The situation resembles very much that at accelerator energies[8]. Therefore also at ISR energies we ascribe the discrepancy to the fact that we have left out the particles from the fragmentation process. Let us estimate this contribution. As mentioned in the introduction we can roughly simulate the fragmentation particles by the pionization formula (1) with large γ_f. AT $\sqrt{s}$ =44.7GeV we obtain $\gamma_f = \gamma_F = 9$ if we identify, for the moment, fragmentation with the decay of an excited particle of mean mass M^* =1.8 GeV/c^2 and attribute an energy of E_p =10.3 GeV to pionization (see section 4.1). For the pionization fireballs we have γ_f = =1.40 (Fig. 2). With $m = 0$ integration over momenta of formula (1) gives the CMS angular dependence (from both fireballs)

$$\frac{dN}{d\Omega} = \frac{MF}{4\pi\gamma_f^2}\left[\frac{1}{(1-\beta_f\cos\theta)^2}+\frac{1}{(1+\beta_f\cos\theta)^2}\right] \qquad (4)$$

Inserting the above values for γ_f shows that pionization particles predominate over fragmentation particles when $\theta > \theta_d$ $=17^o$, provided pionization and fragmentation yield equal numbers of particles. The angle θ_d is not very sensitive to the value of γ_f and γ_F and hence may be applied to both the E_0 =484 GeV and E_0 =1063 GeV data. Actually it is more likely that even at ISR energies pionization yields the bulk of all produced particles[14] so that the discriminating angle θ_d may be lower-ed. Hence we conclude that only the 10.2^{o} spectra are appreciably influenced by fragmentation particles. A more detailed comparsion with $m > 0$ and taking account of the momentum spectra at the respective angles confirms this conclusion. Additional support comes from the large bumps in the 10.2^{o} spectra. A look on Fig. 23 of /8/, where LS decay spectra of excited states are shown, makes it very likely that the bumps arise from decay of different excited states of the escaped collision particles, i.e. from special modes of fragmentation.

Thus, except at 10.2^{o}, where we must add a contribution from fragmentation, we succeed in fitting the ISR data by the pionization formula (1). The fitting values are kT =0.17 GeV, γ_f =1.35 at E_0 =484 GeV, and kT = 0.18 GeV, γ_f = 1.40 at E_0 = 1063 GeV. They are plotted in Fig. 2 together with some typical accelerator data. The tolerance limits indicate the range of the parameter (γ_f or kT , respectively), which would still give a tolerable fit if the other parameter (kT or γ_f , respectively) is kept fixed. Q has always been fixed to Q=0.4, and MF has always been chosen suitably, since we are only concerned with the shape of the spectra, not with their absolute height. The total uncertainty range is thought to be still larger since both parameters may take on extreme values at the same time, the correct error bars are larger than those shown, and the contributions to the γs from the decays of η , Λ , ... are only roughly taken into account.

From Fig. 2 it is seen that there is, if any, only a slow increase in kT and γ_f as collision energy rises[x]. So we conclude that at ISR energies there is pionization.

4. Conclusion and Remarks

4.1. How much of the available CMS energy $E_C = \sqrt{s} - 2m_p$ is spent in pionization ? We write $E_{P(ionization)} = \langle n \rangle \times \times \bar{\epsilon}_{tot}(kT) \times \gamma_f = K_f \times E_C$. At 30 GeV collision energy we have E_C =5.87 GeV, $\gamma_f \approx 1.15$, $\langle n \rangle \approx 6$ and $\bar{\epsilon}_{tot}(0.14)$ = =0.453 GeV[8]. Hence $K_f = E_P/E_C$ =0.53. At 1063 GeV collision energy we have E_C =42.82 GeV, $\gamma_f \approx 1.40$, $\langle n \rangle \approx 13.5$[17] and $\bar{\epsilon}_{tot}(0.18)$=0.551 GeV. Hence K_f =0.24. Some of the 13.5 produced particles may arise from fragmentation, some particles may be heavier than pions (20% nucleons would give K_f =0.27), both effects may cancel each other to some degree. Thus, although the number of pionization particles and also their individual CMS energy increases as the collision energy rises, we conclude that this increase is so slow that the fraction K_f of the available energy which is spent in pionization drops from about 0.53 at 30 GeV to about 0.24 at 1000 GeV: Pionization remains important regarding multiplicity but becomes unimportant regarding energy[xx] [xxx].

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[x] Cf. also some cosmic-ray results on transverse momentum[15] and on γ_f [16] indicating the same tendency.

[xx] The same tendency has been noticed in cosmic-ray physics[18].

[xxx] In[8] we had concluded $\gamma_f \propto E_0^{1/4}$. This was without account of the decrease in pionization energy.

4.2. Ultimately all the collision energy is then kept by the fragmentation process. Imagine that fragmentation corresponds to the decay of the colliding nucleons escaping in an excited state of mean mass M^* and that M^* is limited to some finite value. Then after de-excitation the bombarding nucleon will retain a constant mean fraction of its initial energy, and this fraction may well range around 0.5, depending on the value of M^* and on the M^*- decay characteristics (e.g. $M^* = 1.8$ GeV/c^2, statistical decay). This is just what is found in cosmic-ray extensive-air-shower work/19/.

4.3. The near constancy of mean transverse momentum is accounted for, in the thermodynamical model/1/, by the existence of the highest temperature. An alternative possibility would be the following. The temperature rises with the total energy E of the statistical ensemble as usually, $kT \propto (E/\Omega)^{1/4}$, Ω =interaction volume. The energy E , however, is identified with pionization energy E_p only. Then the increase in temperature is slowed down to a dependence on collision energy that cannot be excluded by the experimental determinations, i.e. to a dependence weaker than $kT \propto \bar{p} = \frac{4}{\pi}\bar{p}_t \propto E_0^{0.12}$ /15/ . E.g., $kT \propto \bar{p}_t \propto E_0^{0.07}$ with $E_p \propto E_C^{0.55} \propto E_0^{0.275}$, Ω =const. Identify fragmentation with nucleon excitation and decay; by inspection of the tables of particle properties/10/ it is seen that the mean momentum of the decay particles is also of the order of 0.5 GeV/c only.

4.4. Finally we compare our view with the conjectures of Feynman[20] and of Benecke et al.[2]. Feynman writes $E\,d\sigma/dp_t^2\,dp_1 = \pi E/p^2 \times d^2N/dp\,d\Omega = f(p_t, x, W)$ in the CMS. $x = p_1/W$, p_1 = CMS longitudinal momentum of the outgoing particle, W =CMS momentum of each of the two incoming particles, $W \sim \sqrt{s}/2 \sim E_C/2 \sim \sqrt{E_0 m_p/2}$. He then suggests in the CMS

$$f(p_t, x, W) \to f(p_t) \quad \text{for } x \to 0, \quad W \text{ fixed} \qquad \text{(A)}$$

$$f(p_t, x, W) \to f(p_t, x) \quad \text{for } W \to \infty, \quad x, p_t \text{ fixed (scaling)} \qquad \text{(B)}$$

The hypothesis of "limiting fragmentation" of Benecke et al.[2] assumes that there are only two groups of particles, which are regarded as fragments of the projectile and the target particle, respectively. For symmetry it suffices to consider the target fragments in the LS . Their LS energies remain finite when the collision energy rises (E_0 , s , W , $E_C \to \infty$). For them a limiting partial cross section is predicted in the LS x/

$$\frac{d\sigma}{d^3p} = \frac{1}{p^2}\,\frac{d^2N}{d\Omega\,dp} = g(\vec{p}), \quad \text{independent of } W \qquad \text{(C)}$$

with $\vec{p}$ = LS three-momentum of the emitted particle of mass m , $p = |\vec{p}|$.

Whether our spectrum, based on formula (1), has the properties (A), (B) and (C) or not depends on the energy dependence of the functions $MF(W)$, $kT(W)$ and $\gamma_f(W)$.

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x/ Strictly speaking, at finite collision energy this cross section is comparable to our distribution only when complemented by a high-momentum cutoff function.

By a reasonable choice (A), (B), (C) can be fulfilled approximately. We shall, however, proceed in another way. We make some definite assumptions about $\gamma_f(W)$ and $kT(W)$ and then study which function $MT(W)$ is needed for (A), (B), (C) to be strictly fulfilled. We consider the two cases of "extreme pionization" and "extreme fragmentation". By "extreme pionization" we mean $\gamma_f(W) = \gamma_f$ =const, $kT(W)$ =const ($CN(W)$ =const). This is the slowest possible dependence of γ_f on W compatible with the fitting data of Fig. 2. In terms of Feynman's variable it corresponds to $|x| \lesssim$ (1 GeV)/W, i.e. to "wee" x. By "extreme fragmentation" we mean $\gamma_f(W) \propto W$, $kT(W)$ =const ($CN(W)$ =const). This energy dependence $\gamma_f(W)$ is like that of the Lorentz factor of the incoming particles; it is the strongest possible energy dependence compatible with energy-momentum conservation and with a finite mass of the outgoing particle. For the study of Feynman's postulates (A) and (B) we have to write formula (1) in terms of p_t and x in the CMS. For the study of limiting fragmentation (C) we have to write formula(1) in the LS , i.e. only to replace by the Lorentz factor γ_{fS2} of the background fireball in the LS (see section 2) and to realize that in the extreme fragmentation case, $\gamma_f \approx c\gamma_C$, we have $\gamma_{fS2} \to (1+c^2)/2c$ = const for $\gamma_C \to \infty$. The table below then sums up the results.

	(A)	(B)	(C)	
$MF(W) \propto$	$\exp(aW)/W$	const	const	Extreme Fragmentation ($\gamma_f \propto W$)
$MF(W) \propto$	const	$\exp(bW)/W$	-	Extreme Pionization (γ_f=const)

Since the dependence of multiplicity on collision energy $MF(W)$ =const is too weak and the dependence $MF(W) \propto \exp(aW)/W$ is far too strong as compared with data[21], the cases considered can fulfil neither the postulates of Feynman nor that of Benecke et al.

Reconciliation between extreme fragmentation and both Feynman scaling (B) and limiting fragmentation (C), could be achieved if we ascribe the increase in multiplicity to pionization and assume constant multiplicity, $MF(W)$= =const, in the extreme fragmentation process. That would mean that the outgoing excited nucleon as representative of fragmentation must be independent of collision energy with respect to (1) mean mass, (2) decay modes, and (3) mean fraction of total collision energy[x]. This would fit with the view that pionization ultimately takes over a vanishing fraction of total collision energy and yet continues to yield the bulk of all produced particles.

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[x] Cf. e.g. Narayan[22] who notes that an excited nucleon of mass M^* =1.45...1.68 GeV/c^2 and decaying according to $M^* \to N + \pi$ would trivially exhibit Feynman scaling.

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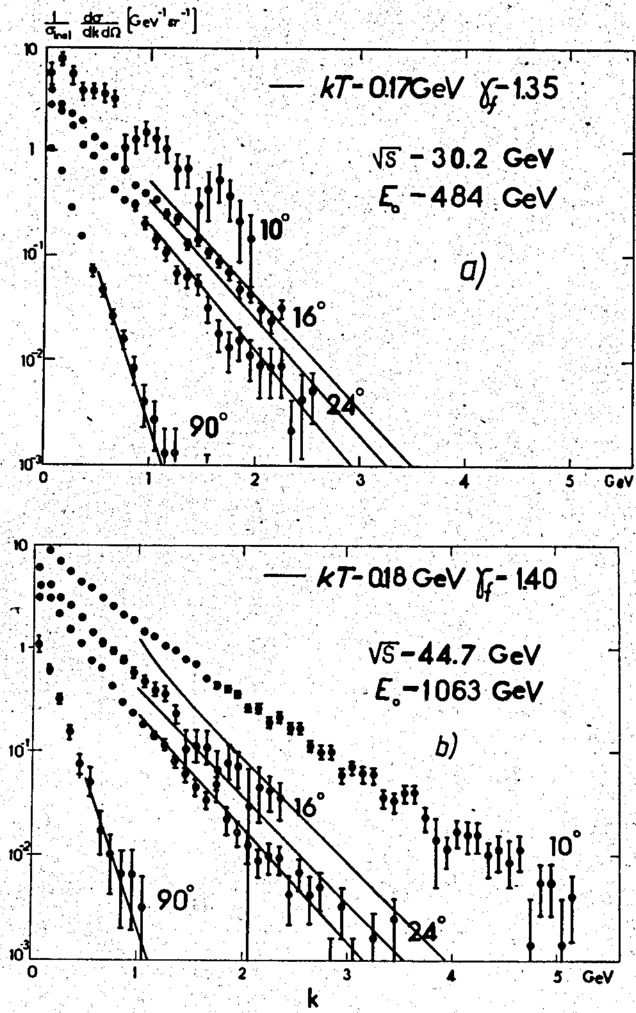

Fig. 1. ISR data of [5] fitted by the pionization formula (1). The disagreement at 10° is attributed to the fact that particles from the fragmentation process have been ignored (see text).

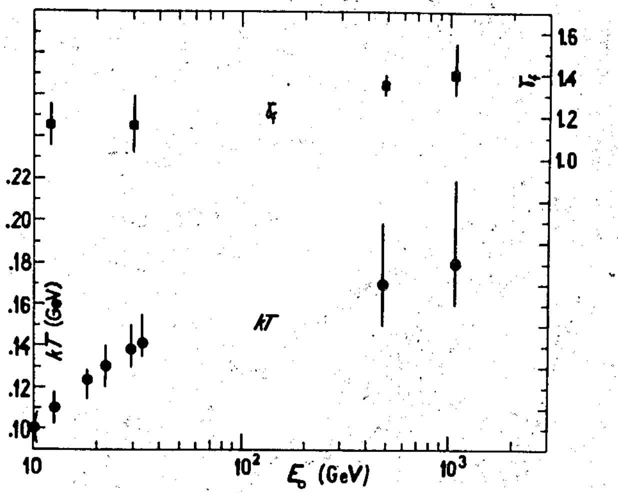

Fig. 2. The values for kT and γ_f of formula (1) as obtained from fitting ISR as well as accelerator single-particle production spectra of pionization particles. The tolerance limits indicate the range which would still give tolerable fits (see text).