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# CONSTRAINTS GIVEN BY THE ATMOSPHERIC NEUTRINO INDUCED MUON FLUX ON THE NEUTRINO NUCLEON INTERACTIONS AT HIGH ENERGIES

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# CONSTRAINTS GIVEN BY THE ATMOSPHERIC NEUTRINO INDUCED MUON FLUX ON THE NEUTRINO NUCLEON INTERACTIONS AT HIGH ENERGIES

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* On leave from the Institute of Atomic Physics, Eotvos Unisersity, Budapest.

## I. Introduction

More than ten years ago M.A. Markov [1] and K. Greisen [2] have pointed out that the muon flux produced by atmospheric neutrinos might be observed. From the quantitative estimates of the muon rate [3,4,5,6] it was clear that this $\nu$ -induced muon flux can be considered as a possible tool to obtain information for the high energy properties of the $\nu N$ interactions, the existence of the intermediate vector boson (IVB) and perhaps on some extraterrestrial sources [2] .

The huge background of the primary cosmic ray muons which is present at the sea level can be reduced by two methods:

a) Measuring the muons produced by neutral particles inside the detector by use of anticoincidence technique

b) Locating the detector at such a great depth where the primary $\mu$ -flux is absorbed giving only a negligible background. So muons deep underground travelling in upward or horizontal direction can be identified by the $\nu_\mu$ -induced muon.

Untill the neutron flux at the sea-level is known with the present accuracy the first method can not be applied since it is impossible to decide whether the muons were produced by neutrons or neutrino. Measurements using this arrangement were done by Cowan et al. [7] and Ashton et al. [8] .

The approach b) was adopted by three groups: in the experiments performed in the Kolar Gold Fields [9] (KGF) and in the East Rand Propriatery Mine (ERPM) [10] the detectors were located at 7000 and 8710 meter water equivalent (m.w.e.) depth+, respectively, where the primary cosmic ray muon intensity is acceptably low; the group at the Utah University [11] , however, used moderate depth (2000 m.w.e.) in conjunction with the experiment that the $\nu_\mu$ -induced muon be travelling upward.

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+If we know the effective range-energy relations of muons for a rock, then the depth can be labeled by the energy which corresponds to effective range equal to the depth. If effective range-energy relations are known for various rocks (among others for standard rock $Z = 11$ , $A = 22$ , $Z^2/A = 5.5$ , $\rho = 2.65\ gcm^{-3}$) then every depth value can be translated into the one for standard rock. In standard rock the depth in KGF is 7600 m.w.e. . The ERPM rock is very similar to the standard rock, in standard rock the depth where the detectors were located in ERPM is 8740 m.w.e. .

The values of the muon flux obtained in these experiments are given in Table I+.

It is natural to ask whether or not these experiments having produced the numbers for the muon rate are able to give information about the $\nu N$ interactions at high energies, as it has been expected? We shall see that we can definitely give positive answer for this question.

Table I

| Experiments | Data of measurements | Events | $\mu$-flux |
|---|---|---|---|
| EPRM | 1964-67 | 36 | 3,7  0,6 |
| KGF | 1965-69 | 16 | 2,6  0,7 |
| UTAH | 1969 | 2 | 5  3 |
| | Total number of events: 54,in average:3,2± 0,1 | | |

Experimental fluxes of muons produced by neutrinos

+To facilitate the comparisons, the flux is given by assuming isotropic neutrino induced muon distribution. The horisontal flux in ERPM and in KGF are $(4.2 \pm 0.7) \times 10^{-13} cm^{-2} sec^{-1} st^{-1}$ and $(3.5 \pm 0.9) \times 10^{-13} cm^{-2} sec^{-1} st^{-1}$, respectively.

The muon flux induced by atmospheric neutrinos can be calculated by use of the integral formula as follows

$$I(\theta,\varepsilon) = \frac{N_A}{A}\int_{\varepsilon}^{\infty} N(\nu+\bar{\nu}, E_\nu, \theta)\, dE_\nu \int_{\varepsilon}^{\infty} E_\mu \frac{d\sigma^A}{dE_\mu}\left(-\frac{dE_\mu}{dx}\right)^{-1} dE_\mu , \qquad (1)$$

where $N(\nu+\bar{\nu}, E_\nu, \theta)$ is the intensity of the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ at horizontal angle $\theta$ (measured to the vertical), $d\sigma^A/dE_\mu$ is the differential cross section per nucleus for the reaction $\nu_\mu + Z \longrightarrow \mu$ + anything$^+$, $\left(-\frac{dE_\mu}{dx}\right)$ is the average energy loss of muons in the rock, $\varepsilon$ is the threshold energy for the detection, $N_A$ is the Avogadro's number, $A$ is the number of nucleons in the nucleus. If we assume that $dE_\mu/dx$ = const, then we get instead of (1) a simpler formula

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$^+$Assuming that the nuclear effects (Fermi motion, Pauli principle, rescattering and absorption of recoiling nucleons) can be neglected $d\sigma_A = Z\, d\sigma_p + (A-Z)\, d\sigma_n$

$$I_{\mu}(\theta, \varepsilon) \approx \frac{N_A}{A} \int_{\varepsilon/k}^{\infty} N(\nu+\bar{\nu}, E_\nu, \theta)\, \sigma_A^T(E_\nu)\, R(kE_\nu - \varepsilon)\, dE_\nu , \quad (2)$$

where $\sigma_A^T(E_\nu)$ is the total cross section per nucleus for the reaction $\nu_\mu + Z \to \mu +$ anything, $R$ is the effective range energy function of muons in the given rock and $k$ is the average energy ratio transferred to the muon from the neutrino defined as

$$k = \langle E_\mu / E_\nu \rangle = \frac{1}{\sigma_T(E_\nu)} \int_0^{E_\nu} \frac{E_\mu}{E_\nu} \frac{d\sigma}{dE_\mu} dE_\mu . \quad (3)$$

In order to estimate the muon flux produced by atmospherio neutrino we have to know

i) the intensity of the atmospherio neutrinos $N(\nu+\bar{\nu}, \theta, E_\nu)$

ii) the average energy loss of the muons $-\frac{dE_\mu}{dx}$

iii) the differential cross section $\frac{d\sigma}{dE_\mu}$ for the reaction $\nu + \mu \to Z +$ anything.

At muon energies less then 5o GeV we can use the formula (2), that is instead of ii) and iii) we have to know only the effective range of the muons, the total oross section and the parameter $k$ defined by (3).

It is clear that the strength of the restrictions which may be derived on the cross section $d\sigma^{\nu}$ at very high energies (not accessible for accelerators) depends on the uncertainties which we have in the $\nu_\mu$ -intensity, in the average energy loss of the muons and in the informations obtained by accelerators at lower energies. On the other hand, we could derive more conclusive constraints if we would have not only the rate but the energy spectrum of the $\nu_\mu$ -induced muons, as well.

The flux, the intensity, the average energy of the $\nu_\mu$ -induced muons have been estimated by various authors [12, 13, 14] . For energies less than 10 GeV the linearly rising total cross section obtained in the CERN heavy liquid bubble chamber experiment [15] was used. At higher energies it was assumed that it increased linearly up to a critical energy value where the total cross section became to be constant. From the measured muon-rate the authors were able to derive restrictions on the value of the energy where the saturation began. The energy transfer ratio to the muon from the neutrino $k$ was assumed to be constant.

The theoretical developments concerning the deep-inelastic lepton-nucleon scattering stimulated by the SLAC-MIT experiments [16,17,18,19] , however, called for the revision of these analyses.

The first estimation which included the "scale-invariance" of the deep-inelastic scattering was done more than two years ago [20] . For the structure functions of the $\nu N$ deep-inelastic scattering the scale invariance was assumed in the form as it was suggested by a revised analysis of the CERN experimental data [21] . It was noticed that the scale invariance restricts the value of the energy transfer ratio $k$ to the limits $0.5 \leq k \leq 0.75$ . Similar bounds were found by Bjorken [20] using the locality of the lepton current [22] . Especially, if the constituents of the nucleon have spin $1/2$ and the sign of the third structure function is negative we can get that $k = 0.53 \pm 0.03$ (in agreement with the data of the CERN experiments (see Ref. [19] ) ). It was shown that if the linear rise of the total cross section could be attributed to the scaling behaviour of the structure functions, then at confidence level more than 99% the slope of the linear rise had to be less than 0,8 ( $\sigma = cE \times 10^{-38}\ cm^2\ GeV^{-1}$ ).

The saturation of the cross section was simulated by assuming that intermediate vector boson (IVB) existed. It was pointed out that it might be possible to derive both <u>upper</u> and <u>lower</u> limits on the mass of the IVB ( $M_W$). At one standard deviation it was obtained that

$$12\ \mathrm{GeV} > M_W > 3.5\ \mathrm{GeV} \tag{4}$$

( $\sigma^{\nu} = \sigma^{\bar{\nu}}$ was assumed). At confidence level more than 99% percent, only a lower limit can be given

$$M_W > 2.0\ \mathrm{GeV}. \tag{5}$$

In the caloulations, the formula (2) was used, which was necessary if the saturation energy is large ( $\geq$ 5oo GeV).

Similar analysis was performed by the ERPM group [23]. They assumed scale invariance, introduced the IVB propagator as well and obtained results in fair agreement with the one's of the above mentioned work. They carefully investigated the possible role of the high-mass resonances and the modifications introduced by the inequality $\sigma^{\nu} > \sigma^{\bar{\nu}}$. It was found that if $\sigma^{\nu} = 3\sigma^{\bar{\nu}}$ e.g. the calculated value of the muon flux would oonsiderably decrease.

We think it is encouraging that the main oonclusions are the same albeit slightly different assumptions were used.

The paper is organized as follows. In section II we give a short discussion of the calculated atmospheric neutrino intensities, the average rate of energy loss and the range energy relations for muons, in Section III we review the experimental and theoretical results on $\nu N$ interactions, Sections IV is devoted to modifioations introduced by assuming the existenoe of the IVB, finally Section V oontains the comparison of the estimated and the measured muon flux and the conclusions.

## II. Neutrino Spectra; Average Rate of the Energy Loss of the Muon

### a) Neutrino Speotra

The neutrino spectra have been calculated by a number of workers. Neutrinos are produoed in the atmosphere from the decay of the secondary $\mu, \pi$ and $K$ mesons. For neutrino spectra up to $10^3$ GeV the general prooedure is to calculate the speotra of the parent pions and kaons as a function of depth in the atmosphere from the measured energy speotrum of muons at sea level. In the derivation an assumption must be used for the ratio of kaons to pions in high energy interactions. For energies above $10^3$ GeV the production speotra of pions and kaons are deduced from an adopted primary spectrum and interaction model.

Calculations taking into acoount all the possible ( $\pi, K, \mu$ ) contributions have been performed by Volkova et al., Coswik et. al. and Osborne et.al. The neutrino intensities were calculated separately for $\nu_e, \tilde{\nu}_e, \nu_\mu$ and $\tilde{\nu}_\mu$ [24, 12] at horizontal and vertical directions and the $\nu_\mu + \tilde{\nu}_\mu$ speotra were derived at intermediate angles as well [26] . For energies between 1-1ooo GeV we use the values of the $\nu_\mu + \tilde{\nu}_\mu$ intensity calculated by Osborne et.al. for a $K/\pi$ ratio of 2o%. Because of uncertainty in this ratios the neutrino intensities are uncertain by $^{+17\%}_{-21\%}$ in the vertical direction and $^{+8\%}_{-11\%}$ in the horizontal direction at 1o GeV. These uncertainties are inoreasing with energy, typical error bars at 1ooo GeV are ± 5o% in the vertical direction and ± 3o% in the horizontal direction. (See Fig.1). In view of this it is suffeoiently aoourate to take a straight line extrapolation on a log-log plot to get the intensities beyond 1o$^4$ GeV [27] . The horizontal speotrum is larger; e.g. at 1ooo GeV its ratio to the vertical one is 3,7 (see Fig.1). These features suggest that it is better to study the induoed muon flux at horizontal direotions. For energies between 0.1-1 GeV we used (with an appropriate normalization) the $\nu_\mu$-spectra calculated by Tam and Yang [ 25] . The errors in this region are large ( ~ 5o%) but its oontribution to the muon flux is comparatively small.

The $\nu_\mu + \bar{\nu}_\mu$ intensity used in the paper [20] is given in Table II.(See Fig.2). For energies $E_\nu > 10^4$ GeV the spectrum was extrapolated assuming that it follows a power low $3.0\ E^{-3.5}\ \mathrm{ster^{-1}\,cm^{-2}\,sec^{-1}\,GeV^{-1}}$.

b) Average Rate of the Energy Loss of the Muons

The main processes by which muon loses energy penetrating rock are as follows: i) ionization and excitation; ii) electron-pair production, iii) bremstrahlung, iv) nuclear interactions.

For the average rate of the energy loss of the muon in standard rock (see footnote on p. 2 ), Hayman et.al. [28] proposed an approximate equation as

$$-\frac{dE}{dx} = 10^{-3}\left(2.053 + 0.154 \ln E - 0.077 \ln(10.93+E) + b \cdot 10^{-3}\, E/\mathrm{GeV}\right) \times \mathrm{GeV\,cm^2\,gr^{-1}}. \qquad (6)$$

The coefficient $b$ can be splitted into a fluctuating and nonfluctuating components. Fluctuations in the rate of the energy loss arise for bremstrahlung and nuclear interactions, since the cross sections for these processes are not small for large energy transfer. The cross sections of ionization and pair production, however, fall off rapidly with increasing energy transfer.

For standard rock in average $b = b_f + b_{nf}$

$$b_f = 2.4 \ cm^2 gr^{-1} , \quad b_{nf} = 1.6 \ cm^2 gr^{-1} \tag{7}$$

for KGF rook $b_f = 2.8 \ cm^2 gr^{-1}$ , $b_{nf} = 2.0 \ cm^2 \ gr^{-1}$ (the pair produotion and bremstrahlung terms are proportional to $z^2/A$ ). We note that the range fluotuations [29] are important only for muons with energy more than a few thousand GeV. Since the majority of contribution to $I(\theta)$ comes from muons with energy less than loo GeV, the enhancement produoed by the range fluotuations is smaller than 5% (even if the saturation energy is large , $E_c > 10^3 GeV$). The damping effeot of the term $bE$ in (6) can be properly treated by usihg the formula (1). But onoe again, sinoe this effect appears above a few hundred GeV, the use of the formula (2) (with the proper value of $R$ ) would result in unoertainties less than 5%.

Summarizing the uncertainties involved in the estimation of the muon flux $I(\theta)$ we can say

1) The differences between the rooks have a completely

negligible effect (less than 5%) on the predictions compared with all the other uncertainties (e.g. the uncertainty in the value of $\sigma_T^A$ for a given $Z$ and $A$ ).

ii) A more significant uncertainty in the predicted $I(\theta)$ is produced by the uncertainty in the neutrino flux. This is $\approx$ 15% at 1oo GeV in the horizontal direction and increases with energy.

iii) Even the uncertainty in the neutrino flux is not very important while the error in the measured cross sections is more than $\pm$ 3o%. (See Section III.). Using the expression (6), the range-energy relation of the muons can be calculated by the formula as follows

$$R(E) = R(E_0) + \int_{E_0}^{E} dE' \left(-\frac{dE'}{dx}\right)^{-1}. \qquad (8)$$

Sternheimer [3o] have calculated the range up to 1o GeV; for energies above 1oo GeV have been published in [28] . In Table III we give the range-energy relation for standard rock in the energy region $0.13 - 2\cdot10^{14}$ GeV. The effect of the range-fluctuations can roughly be seen by modifying the average value of b.

## III. Cross Sections for $\nu N$ Scattering

### a) Experimental results obtained at CERN and ANL

In the neutrino energy region 0.12–12 GeV information about the neutrino nucleon interactions is obtained in the bubble chamber and spark chamber experiment at CERN [15] and in a spark chamber experiment at ANL [31] . The important points of the results relevant here are as follows.

i) Only $\nu_\mu$ events were evaluated. The available data on $d\sigma^{\bar{\nu}}$ are meager and imprecise.

ii) The dominant processes at low energies $(E_\nu < 4\ \mathrm{GeV})$ are the quasi-elastic reaction

$$\nu_\mu + n \longrightarrow \mu^- + p \tag{9}$$

and the $\Delta$ (1236) production

$$\nu_\mu + p(n) \longrightarrow \mu^- + \Delta^{++}(\Delta^+) . \tag{10}$$

In the energy region 1–4 GeV their cross sections are

$$\sigma_\nu^{el} = (0.6 \pm 0.2) \times 10^{-38}\ \mathrm{cm}^2/\mathrm{neutron} \tag{11}$$

and

$$\sigma_\nu^{\Delta^{++}} = (1.12 \pm 0.28) \times 10^{-38}\ cm^2/proton , \quad (12)$$

respectively. Due to isospin invariance we get $\sigma_\nu^{\Delta^+} = \frac{1}{3}\sigma_\nu^{\Delta^{++}}$.

iii) 74o events with 12 GeV $> E_\nu >$ 1 GeV, were used to determine the total cross section for the reaction

$$\nu_\mu + N \longrightarrow \mu^- + \text{anything} . \quad (13)$$

Its value from the propan run is (see Fig.3)

$$\sigma^\nu = (0.80 \pm 0.20) E_\nu \times 10^{-38}\ cm^2/\text{nucleon} . \quad (14)$$

vi) The average energy ratio transferred to the muon from the neutrino $k$ has values for the quasi-elastic (9), $\Delta$ production (1o) and inelastic reaction (13) as follows

$$k_{el} = 0.95 \pm 0.05, \quad E_\nu = 0.2\text{–}5 \text{ GeV}, \quad (15a)$$

$$k_\Delta = 0.70 \pm 0.05, \quad E_\nu = 1\text{–}4 \text{ GeV}, \quad (15b)$$

$$k_{in} = 0.54 \pm 0.06, \quad E_\nu = 1\text{–}10 \text{ GeV} . \quad (15c)$$

v) The measured data are consistent with the scaling hypothesis. (See Seo.III.c and Ref. [21] ).

Using the results ii)-iv) the contributions of the neutrinos with energy less than lo GeV, albeit with large error bars can be calculated. Having supported by the data even at lower energies (v) the scaling hypothesis might be used to estimate the contributions to $\underline{I}(\theta)$ of the high energy neutrinos.

### b) Muon flux produced by neutrinos with energy less than lo GeV

The muon flux is calculated by use of the formula (6). We propose to use the cross sections as given on Fig 4. It is assumed that the reaction (13) is dominated at $E_\nu < 1$ GeV by the processe (9) and (lo). Similarly for the corresponding $\bar{\nu}_\mu$ reactions at energis up to 3 GeV. Although the $\bar{\nu}_\mu$ oross-sections are not measured it is well known that for the prooess (9) at the threshold $\sigma^{\bar{\nu}} < \sigma^{\nu}$, since

$$\frac{d\sigma^\nu}{dq^2} - \frac{d\sigma^{\bar{\nu}}}{dq^2} \propto -q^2 F^A G_M^V \,, \text{ where } F_A(0)=1.23 \,, \; G_M^V(0) = 4.71$$

( $q^2$ is the square of the four-momentum transfer). At high energies, however, $\sigma_{el}^{\nu} = \sigma_{el}^{\bar{\nu}}$. At low energies it is assumed $\sigma^\nu = 3\sigma^{\bar{\nu}}$. In the interval $E_\nu = 1-10$ GeV the value (14) is used for $\sigma_T^\nu$. For the $\bar{\nu}_\mu$

reaction in the energy range 3-1o GeV we assumed a linearly rising cross section with a slope increased by the requirement that at $E_\nu = 10\,GeV$ $\sigma_T^{\nu} = \sigma_T^{\bar\nu}$ . The slope values can be read off from Fig.4. Values used for the average energy ratio transferred by the neutrino to the muon are given in Table II.

Table II

| reaction / neutrino energy(GeV) | quasi-elastic(9) | | $\Delta$ production(1o) | | inclusive (13) | |
|---|---|---|---|---|---|---|
| | $\nu_\mu$ | $\bar\nu_\mu$ | $\nu_\mu$ | $\bar\nu_\mu$ | $\nu_\mu$ | $\bar\nu_\mu$ |
| <1 | 0.8 | 0.8 | 0.2 | 0.2 | - | - |
| 1-3 | - | 0.8 | - | 0.5 | 0.5 | - |
| 3-1o | - | - | - | - | 0.5 | 0.5 |

Average energy transfer ratios $k$ for the various processes in the corresponding energy range.

Taking into account that there is uncertainty $\sim$ 30% in the slope values and neglecting all the other source of errors (especially the uncertainty in the neutrino-flux is $\sim$ 20%) and using the $\nu_\mu$ and $\bar{\nu}_\mu$ intensities calculated by Coswik et. al [12] we obtain that the contribution of the low-energy neutrinos to the muon flux is

$$I_1(E_\nu < 1\,GeV) = (0.40 \pm 0.20) \times 10^{-13}\,cm^{-2}sec^{-1}sr^{-1} \qquad (16a)$$

$$I_2\,(E_\nu = 1-10\,GeV) = (1.70 \pm 0.60) \times 10^{-13}\,cm^{-2}sec^{-1}sr^{-1} \qquad (16b)$$

and their sum is

$$I_L\,(\pi/2) = I\,(E_\nu < 10\,GeV) = (2.10 \pm 0.80) \times 10^{-13}. \qquad (16c)$$

We note that uncertainties due to the unknown $\sigma^{\bar{\nu}}/\sigma^{\nu}$ ratio may give corrections $\sim$ 15%. (decreasing the value of $I_L$ ). We expect that the low energy neutrinos ($E_\nu < 10\,GeV$) produce about one-half or less portion of the measured muon flux. It is clear that more precise measurements in this energy region could considerably increase the predicting power of the deep- mine experiment.

c) <u>Contributions of high energy neutrinos ( $E_\nu > 10$ GeV )</u>

In accelerator experiments the neutrino energy range extends only up to 10 GeV (although it will soon be vastly extended at NAL), therefore if we want to estimate the contributions of the high energy neutrinos to $I(\theta)$ we are forced to abstract those features of the low energy data and some other measured processes (deep-inelastic electroproduction e.g.) which might hopefully be valid for the high energy neutrino nucleon interactions as well.

In interest of clarity we should here dwell on defining the kinematics (see Fig.4). In notation we follow Llewelyn Smith [32] .

Let us consider the process

$$\nu(k)/\bar{\nu}(k') + N(p) \longrightarrow \mu^-(k')/\mu^+(k') \text{ +hadrons,} \qquad (17)$$

where $k, k'$ and $p$ are the four-momenta of the initial neutrino, the final muon and the target nucleon. The cross section for this process is determined by a second rank Lorentz tensor as follows

$$W_{\mu\nu}^{\nu/\bar{\nu}}(q,p) = \frac{1}{4\pi} \sum \int d^4x\, e^{iqx} \langle p | [ J_\mu^{\mp}(x), J_\nu^{\pm}(0) ] | p \rangle , \qquad (18)$$

where $\overline{\sum}$ denotes averaging over the initial nucleon spin, $q$ is the four momentum transfer $q = k - k'$ and $\mathcal{J}_\mu^{\pm}$ is the isospin lowering (rising) weak currents of hadrons. We use Cabibbo current+

$$\mathcal{J}_\mu^{\pm}(x) = \left(V_\mu^{\pm}(x) + A_\mu^{\pm}(x)\right)^{\Delta S=0} \cos\theta_c + \left(V_\mu^{\pm}(x) + A_\mu^{\pm}(x)\right)^{\Delta S=1} \sin\theta_c \,. \quad (19)$$

Assuming $P$, $T$, $PT$ and Lorentz invariance $W_{\mu\nu}^{\nu/\bar{\nu}}$ can be expanded in terms of five invariant functions

$$W_{\mu\nu}^{\nu/\bar{\nu}}(p,q) = -g_{\mu\nu} W_1^{\nu/\bar{\nu}}(q^2,\nu) + \frac{1}{M^2} p_\mu p_\nu W_2^{\nu/\bar{\nu}}(q^2,\nu) -$$

$$- \frac{i\varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta}{2M^2} W_3^{\nu/\bar{\nu}}(q^2,\nu) + \frac{q_\mu q_\nu}{M^2} W_4^{\nu/\bar{\nu}}(q^2,\nu) + \frac{p_\mu q_\nu + p_\nu q_\mu}{2M^2} W_5^{\nu/\bar{\nu}}(q^2,\nu), \quad (20)$$

where $M$ is the mass of the protons and $\nu = pq$. The vector-axial-veotor interference terms contribute only to $W_3$. To order $m^2/EM$ ($m$ is the muon mass) the cross section is determined by $W_1$, $W_2$ and $W_3$,

$$\frac{d^2\sigma^{\nu/\bar{\nu}}}{d\nu\, dQ^2} = \frac{G^2}{2\pi M^2} \cdot \frac{E'}{E}\Big[\cos^2\theta/2\, W_2^{\nu/\bar{\nu}}(q^2,\nu) + 2\sin^2\theta/2\, W_1^{\nu/\bar{\nu}}(q^2,\nu) \mp$$

$$\mp \frac{E+E'}{2M} W_3^{\nu/\bar{\nu}}(q^2,\nu)\sin^2\theta/2\Big] + O(m^2/ME), \quad (21)$$

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+In the following the strangeness ohanging part will be ignored i.e. we take $\theta_c = 0$.

where $Q^2 = -q^2 = 4EE' \sin^2 \theta/2$ , $\nu = M(E-E')_{lab}$ , $\theta$ is the angle between the direction of the initial and final leptons in the laboratory frame. From positivity of the tensor $W_{\mu\nu}^{\nu/\bar\nu}$ it follows that

$$\frac{1}{M^2}\sqrt{\nu^2+M^2Q^2}\,|W_3^{\nu/\bar\nu}| \leq W_1^{\nu/\bar\nu} \leq \left(1+\frac{\nu^2}{M^2Q^2}\right) W_2^{\nu/\bar\nu} . \tag{22}$$

The charge symmetry condition gives

$$W_i^{\nu p} = W_i^{\bar\nu n} , \quad W_i^{\bar\nu p} = W_i^{\nu n} \quad (i=1,2,\dots,5; \ \Delta S=0) . \tag{23}$$

On the basis of the Regge theory we might expect that as $\nu \to \infty$ and $Q^2$ fixed

$$W_1 \longrightarrow \beta_1(q^2)\,\nu^{\alpha_\rho(0)} , \tag{24a}$$

$$W_2 \longrightarrow \beta_2(q^2)\,\nu^{\alpha_\rho(0)-2} , \tag{24b}$$

$$W_3 \longrightarrow \beta_3(q^2)\,\nu^{\alpha_\omega(0)-1} , \tag{24c}$$

where $M\alpha_p(0)$ and $\alpha_\omega(0)$ are the intercepts of the Pomeranchuk and $\omega$ -trajectories at $t=0$, respectively.

Introducing new dimensionless structure functions $(\omega = x^{-1} = 2\nu/Q^2)$

$$W_1(\nu, q^2) = G_1(\omega, Q^2) , \tag{25a}$$

$$\frac{1}{M^2}\nu W_i(\nu, q^2) = G_i(\omega, Q^2) , \quad i=2,3,4,5 \tag{25b}$$

the differential cross section may be written in the form as

$$\frac{d^2\sigma^{\nu|\bar\nu}}{d\omega\, dy} = \frac{G^2ME}{\pi}\left[\left(1-y-\frac{My}{2\omega E}\right)\frac{G_2^{\nu|\bar\nu}}{\omega^2} + y^2\frac{G_1^{\nu|\bar\nu}}{\omega^2} \mp \right. \tag{26}$$

$$\left. \mp\, y(1-\tfrac{1}{2}y)\frac{G_3^{\nu|\bar\nu}}{\omega^3}\right],$$

where $y = \nu/ME$ . Assuming that the "scale invariance" found in the SLAC-MIT experiments [16] for the structure functions of the deep-inelastic electroproduction $(20\,GeV^2 > Q^2 > 1\,GeV^2,\ \nu/M < 12\,GeV)$ can be extended for the invariant functions of the process (17), we obtain that as $Q^2 \longrightarrow \infty$ at $\omega$ fixed (Bjorken limit or automodelity [17], [19] ) the $G_i$ functions approach to a finite nonzero value at all $\omega$

$$\lim_{Q^2\to\infty,\ \omega\ \text{fixed}} G_i(\omega, Q^2) = F_i(\omega) \tag{27}$$

This scaling law gives simple asymptotic forms both for the total cross section (see [17] ) and the average energy transfer ratio of neutrino to the muon $k$ [20]. The point is that if at high $E_\nu$ the low $Q^2$ contributions can be neglected, we can integrate over $y$ in (26) and we obtain [17]

$$\begin{aligned}\lim_{E_\nu\to\infty} \sigma_T^{\nu/\bar\nu}(E_\nu) &= \frac{G^2 ME}{\pi}\left[\tfrac{1}{2}K_2^{\nu/\bar\nu} + \tfrac{1}{6}K_1^{\nu/\bar\nu} \mp \tfrac{1}{3}K_3^{\nu/\bar\nu}\right] = \\ &= \frac{G^2 ME}{\pi}\, C^{\nu/\bar\nu} K_2^{\nu/\bar\nu},\end{aligned} \tag{28}$$

where+

$$K_2^{\nu/\bar\nu} = \int_1^\infty \frac{d\omega}{2\omega^2} F_2^{\nu/\bar\nu}(\omega) \tag{29a}$$

$$K_1^{\nu/\bar\nu} = \int_1^\infty \frac{d\omega}{\omega^3} F_1^{\nu/\bar\nu}(\omega) \tag{29b}$$

$$K_3^{\nu/\bar\nu} = \int_1^\infty \frac{d\omega}{\omega^3} F_3^{\nu/\bar\nu}(\omega) \tag{29c}$$

+Note that due to the factor $1/\omega^2$ the coefficients obtain important contributions from the low (non-Regge) region. (See the Regge behaviour also eq. (24) ).

since due to (22)

$$K_2 \geq K_1 \geq |K_3| \tag{30}$$

we can write

$$1/3 \leq C^{\nu/\bar{\nu}} \leq 1. \tag{31}$$

Similar expression can be obtained for $k$

$$k^{\nu/\bar{\nu}} = \frac{1}{\sigma_T}\int_0^1 z \frac{d\sigma}{dz}^{\nu/\bar{\nu}} = \frac{8 + \frac{K_1^{\nu/\bar{\nu}}}{K_2^{\nu/\bar{\nu}}} \mp 3 K_3^{\nu/\bar{\nu}}/K_2^{\nu/\bar{\nu}}}{12 + 4 K_1^{\nu/\bar{\nu}}/K_2^{\nu/\bar{\nu}} \mp 8 K_3^{\nu/\bar{\nu}}/K_2^{\nu/\bar{\nu}}}, \tag{32}$$

where $z = 1-y$ . Due to the inequalities (30), we can derive

$$0.5 \leq k^{\nu/\bar{\nu}} \leq 0.75 \tag{33}$$

Now we shall be able to discuss the high energy aspects of the results of the CERN heavy liquid bubble chamber experiment.

The most striking result is that the cross section (14) in the energy range 1-12 GeV rises linearly with energy

( Fig.3) in agreement with (28). In this energy range, however, we do not expect scale invariant behaviour+, unless the scaling occurs in an average sense even in the non-asymptotic region. Bloom and Gilman [34] and more extensively Rubinstein et al. [35] have pointed out that in an average sense <u>all</u> the electroproduction and photoproduction data can be fitted with a universal scaling curve in the variable

$$\omega' = \frac{2\nu + M_c^2}{Q^2 + \alpha^2}\ ,\quad M_c^2 \approx 1.5\ ,\quad \alpha^2 \approx 0.4 \tag{34}$$

which approaches to $\omega$ if $Q^2, \nu$ are large. In analogy Myatt and Perkins [21] assumed in their analysis of the combined propane and freon data that scaling in $\omega' = \frac{2\nu + M^2}{Q^2}$ occurs in the non-asymptotic region and so they used all the events with $E_\nu > 1$ GeV. For the present discussion the following is important from their results

i) The scaling hypothesis in the energy region $E = 1\text{-}12$ GeV is supported by the linear rise of the cross section (28) and the constancy of the energy transfer ratio $k$ (see eq. (32), (33) ); $k_{exp} = 0.52 \pm 0.06$.

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+ $Q^2_{max} = 2.5\ \mathrm{GeV}^2$ and for forty percent of the events $\bar{Q}^2 \leq 0.5\ \mathrm{GeV}^2$

ii) The data are compatible with the Callan-Gross relation [36] $2F_1 = \omega' F_2$ , which is also suggested by the electroproduction data.

iii) The data suggest that $K_3$ is negative; it could even be close to the minimum allowed value $K_3 = -K_2$ $(h = 0.5)$ . This implies that $\sigma^{\nu p} + \sigma^{\nu n} > \sigma^{\bar{\nu} p} + \sigma^{\bar{\nu} n}$ .

iv) The scaling functions $F_2(x')$, $x' F_1(x')$ and $x' F_3(x')$ can be fitted by the formula $(1-x'^2)^3$.

If in the description of the high energy $\nu N$ interactions we accept scale invariance and the Callan-Gross relation, only two free parameters remain in the calculation especially $K_3^N = \frac{1}{2}(K_3^{\nu p} + K_3^{\nu n}) = 1/2\,(K_3^{\bar{\nu} p} + K_3^{\bar{\nu} n})$ and $K_1^N = K_2^N = \frac{1}{2}(K_{1,2}^{\nu p} + K_{1,2}^{\nu n}) = 1/2\,(K_{1,2}^{\bar{\nu} p} + K_{1,2}^{\bar{\nu} n})$ .

These parameters, however, in this framework are measured in the CERN experiment: from the measured slope of the total cross section (14) and the energy transfer $k_{exp}$ (33) we obtain

$$K_2^N > -K_3^N \geq 0 \;, \qquad 0.54 \pm 0.13 \leq K_2^N \leq 0.81 \pm 0.20 \tag{35}$$

here the lower limit and the upper limit on $K_2^N$ correspond to $K_3^N = -K_2^N$ and $K_3^N = 0$, respectively.

In the gluon quark parton model from positivity conditions and the experimental data of the electroproduction upper and lower limits can be given on $K_2^N$ [32].

$$(0.18 \pm 0.13) \leq K_2^N \leq 0.50 \pm 0.08. \qquad (36)$$

Furthermore, if we take literally the naive gluon-quark parton model [32] the ratio $d = -F_3(\omega)/F_2(\omega)$ can be interpreted as a measure of the average baryon number of the constituents, therefore we expect that $0 \leq d \leq 1$, in agreement with (35). Especially if we accept that $x \sim \langle \frac{1}{N} \rangle$, where $\langle N \rangle$ is the average number of the partons, and that the main contributions to $K_3$ come from the low $\omega$ region (see footnote on p. 23 ) $K_2^N = 0.34$ can be expected. Using the wave functions discussed by Kuti and Weisskopf [37] one obtains $K_2^N = 0.34$, $k = 0.52$ which implies that $\sigma^{\nu p} + \sigma^{\nu n} \approx 2(\sigma^{\bar{\nu} p} + \sigma^{\bar{\nu} n})$.

We may say that the gluon-quark parton model is in agreement with the CERN data, but the agreement is <u>not</u> trivial, since the quark model favours the lower limit of $K_2^N$ (compare (35) and (36) ) and $d \approx -1$ .

If we insists on e.g. that $d=0$ $(\sigma^{\bar\nu}=\sigma^{\nu})$ then the CERN data can only agree with the prediction of the parton models if lower slope values are assumed for the total cross sections.

We shall see that the same conclusion can be drawn from the independent cosmic ray data: the slope value $0.8$ together with $d=0$ is in contradbtion, on the other hand the prediotions of the gluon quark model are in agreement with the cosmic ray measurements.

## IV. Contributions and Corrections Induced by Assuming that IVB Exists

If we assume that $W$ -boson exists we have (A) to take into account the damping effect of the $W$ propagator for the reaction (17) and (B) to oaloulate the oontributions to the muon flux produoed by the $W$ -produotion reactions.

A) Should the IVB exist, we must use the factor $G^2/(1-q^2/m_W^2)^2$ instead of $G^2$ in the expression of the cross seotion for the reaction (21) ( $M_W$ is the mass of the $W$ -boson). In the Bjorken limit, assuming scale invarianoe we may write the double differential cross seotion as

$$\frac{d^2\sigma^{\nu/\bar{\nu}}}{d\omega\, dy} = \frac{G^2 ME}{\pi} \cdot$$

$$\cdot \frac{(1-y)\, F_2^{\nu/\bar{\nu}}(\omega) + y^2/\omega\, F_1^{\nu/\bar{\nu}}(\omega) \mp y\,(1-1/2\,y)\, \frac{F_3^{\nu/\bar{\nu}}(\omega)}{\omega}}{(\omega + ay)^2}, \quad (37)$$

where $a = \frac{2ME}{M_W^2}$ .

In the estimation of the underground muon flux we need only the differential cross section $d\sigma/dy$ . Since the factor $(\omega+ay)^{-2}$ cannot be factorized in $\omega$ and $y$ the integration over $\omega$ can be performed only if we know the functional form of the scaling functions $F_i(\omega)$ . In Section III we have seen that the $x'$ dependence of the scaling functions $F_2(x')$, $x' F_1(x')$ and $x' F_3(x')$ can be fitted by the same polynomial behaviour $C_i(1-x'^2)^3$ [21] . An important point is that all the functions $F_2(x')$ $x' F_{1,3}(x')$ have the same $x'$ -behaviour.

The $\omega$ -dependence of the $F_i$'s , however, is not very important. Since performing the integration over $\omega$ for an integrand like $F_2(\omega)/(\omega+ay)^2$ we obtain a function which varies smoothly with $ay$ its value in the origin $(ay=0)$ is $K_2$ and it goes to zero when $ay$ goes to infinity as $F_2(\infty)/ay$ . If we assume that $F_2(\omega)$ is constant, then $M_2(0) = K_2 = F_2(\infty)$ ,

if $F_2(\omega)$, however, is equal to $C_2(1-X^2)^3$ +
then $M_2(0)=K_2$, but $F_2(\omega)=C_2=35/16$
(insisting on that the value of $K_2$ is fixed). In the seoond example $\frac{d\sigma^{\nu|\bar\nu}}{dy}$ obtains less contribution from the low $\omega$ region than in the first case.

Therefore, if we approximate the function $F_2(\omega)$ with constant $K_2$, only the very high energy behaviour $(ME_\nu \gg M_W^2)$ will be modified in $d\sigma/dy$. Reoalling that $M_W > 2$ GeV and that the neutrino induoed muon-flux obtains the most important contribution from neutrinos with energy less than 1oo-1ooo GeV (depending on the value of $M_W$), we can see that the unoertainty induced by negleoting the $\omega$ -dependence of the scaling functions is negligible comparing with the other unoertainties of the analysis.

Similarly it is easy to see that the value of the wnergy transfer ratio $k$ (see (3) ) is not sensitive to the $\omega$ -dependence of $F_i$'s, as well (as it is expe cted sinoe at high energies its value is determined by the local (non-local) nature of the lepton current [22]).

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+In the Kuti-Weisskopf's gluon-quark parton model $F_2(x) = C(1-x)^\gamma$ [37].

In the estimate of the muon flux induced by high energy neutrinos for the differential cross section we can use the expression as follows

$$\frac{d\sigma^{\nu|\bar\nu}}{dy} = \frac{G^2 ME}{\pi} \frac{(1-y) + 1/2 y^2 b \mp y(1-1/2 y)d}{1+ay} K_2 , \qquad (38)$$

where $b = K_1 | K_2$, $d = K_3 | K_2$ ( $K_i$'s are defined by eqs. ( 29 ) ) and $1 \geq b \geq |d| \geq 0$ (see eq. (30) ). Performing the integration over $y$ we obtain

$$\sigma_T^{\nu|\bar\nu}(E_\nu, M_W^2) = \frac{G^2 ME}{\pi} K_2^{\nu|\bar\nu} \frac{1}{a} \left\{ \left(1 + \frac{1 \pm d}{a} + \frac{b \pm d}{2a^2}\right) \ln(1+a) - \right.$$

$$\left. - \left[ (1 - 1/4 b \pm 3/4 d) + \frac{b \pm d}{2a} \right] \right\} . \qquad (39)$$

In the limit $M_W^2 \to \infty$ we recover, of course, the formula (28)

$$\lim_{M_W^2 \to \infty} \sigma_T^{\nu|\bar\nu}(E_\nu, M_W^2) = \frac{G^2 ME}{\pi} K_2^{\nu|\bar\nu} (1/2 + \tfrac{1}{6} b \mp 1/3 d) . \qquad (40)$$

In this approximation it is easy to calculate the average energy ratio transferred to the muon from the neutrino $k$, as well. For example if $b=1$, $d=0$ the value of $k$ is

$$k = \frac{\left(1+\frac{1}{2a}+\frac{3}{2a^2}+\frac{1}{2a^3}\right)\ln(1+a)-\frac{17}{12}-\frac{5}{4a}-\frac{1}{2a^2}}{\left(1+\frac{1}{a}+\frac{1}{2a^2}\right)\ln(1+a)-\frac{3}{4}-\frac{1}{2a}}. \tag{41}$$

If $M_W^2 \to \infty$, i.e. $a \to 0$, we recover that (see (32) ) $k \to 0.56$, if $a \to \infty$, however, we obtain that $k \to 1$.

In paper [20], using the formula (2), the average energy loss relation ( 6 ), with $b = 4.0$ ; the differential cross section (38) with $b=1$, $d=0$ +

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+Note that the uncertainties due to the unknown vector-axial-vector interference term are lowered by the interesting interplay between the slope value of the total cross section and the average energy transferred to the muon: a modification of $d$, which increases the slope, will decrease the value of $k$ and vica versa. Furthermore the ratio of the atmospheric antineutrino intensity to the neutrino one at high energy is less than 1/3 (see [12] ). Therefore the errors from this source will be $\sim$ 15% or less.

and $K_2 = 0.80$, the high energy neutrino contribution to the horisontal muon flux $I_H(\Gamma h_1 M_W^2)$ deep underground were calculated as a function of $M_W$ which can be seen in Table V.

B) $W$ -boson can be produced in the Coulomb field of the nucleus by the reactions

$$\nu_\mu + Z \rightarrow \mu + W + Z' \qquad (42)$$

(coherent and incoherent)

$$\nu + N \rightarrow \mu + W + \text{Hadrons}. \qquad (43)$$

The contributions of the process (42) to the muon rate was calculated by a number of workers [12, 13, 14]. In these estimate cross section values of $W_u$ (at lower energies [38] and von Gehlen (at higher energies $E_\nu > 10^4$ GeV [39]; see also [40]), were used with an interpolation for the cross seotions at intermediate energies. Coswik [41] applied interpolations to the estimate of the $W$-mass dependence of the cross sections, as well.

Uncertainties in these calculations were involved, since the average energy, transferred to the muons from the neutrino

$$k_W = \left\langle \frac{E_\mu^{direct} + B E_\mu^{W}}{E_\nu} \right\rangle \qquad (44)$$

was not calculated and the branching ratio for the $W$ -decay channel $W \to \mu\nu$ was not known.

The energy spectrum of the muons at lower neutrino energies (e.g. if $M_W \sim 2\,GeV$, $E_\nu \sim 10\,GeV$ ) is strongly peaked up at the minimal energy transfer value ( $E_\nu m / M_W + M_N$ ). At very high energies ( $E_\nu > 10^4\,GeV$ ), however, the Lee's asymptotic formula gives $k_W \sim 1/2$ . Since the main contribution to the muon flux comes from neutrinos with energy less than 100 GeV, the value of $k_W$ must be close to its threshold value if $B=0$ . If we accept that $B \leq 1/4$ (it is suggested by model calculations [42] ) we obtain in the energy range of interest that $k_W \simeq 0.10 - 0.20$ Coswik [41] assumed that $k_W = 0.65$ in his estimation and obtained lower limit on the $W$ -mass $M_W > 5\,GeV$ . The above qualitative arguments, however, suggest much less values for $k_W$ . Since the muon flux value is approximately proportional to $k_W$ , the flux values of Coswik can simply be corrected by decreasing them by a factor of $\sim 1/5$ . Flux values obtained in this way were accepted in paper [20] (see Table V. ) and the contributions of the reaction (43) were neglected.

Total cross sections and $k_W$ values for reactions (42)-(43) have recently been calculated for a wide range of $W$ -boson masses and neutrino energies by Brown, Hobbs and Smith [43] +. Chen et al. [23] using these cross sections and $k_W$ values have made a very thorough analysis calculating the contributions of the reaction (42)-(43) to the neutrino induced muon-flux. Since the qualitative arguments given above in the estimate of $k_W$ have been confirmed by the explicit calculations of J. Smith et al., the result of paper [23] is roughly in agreement with the muon-flux values of Coswik if it is corrected by a factor of $\sim 1/5$ .

## V. Results and Discussions

If we assume that IVB exists, the neutrino induced muon flux is the sum of the contributions from the low energy

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+It is assumed that $B=0$ .

neutrinos (16), the high energy neutrinos and the IVB production reactions (42)-(43) (see Table V):

$$I(\theta, M_W) = I_L(\theta) + I_H(\theta, M_W) + I_W(\theta, M_W). \qquad (45)$$

The horizontal muon flux was calculated in [20] for different slope or $K_2$ values in the total cross section (Fig. 5., Table VI.). Assuming for the slope the average value 0.8 (14) (i.e. $K_2 \approx 0.80$, if $\alpha = 0$) we obtain the curve A of Fig.5.

$$I^A(\pi/2, M_W) = [2.3 + \bar{I}_H(\pi/2, M_W) + \bar{I}_W(\pi/2, M_W)] \cdot$$

$$\cdot 10^{-13} cm^{-2} sec^{-1} ster^{-1}, \qquad (46A)$$

where $\bar{I}_H$ and $\bar{I}_W$ denote the data of Table V. We remind that according to (38) $\bar{I}_H$ is proportional to $K_2$, therefore if we want to use $K_2$ values predicted by the gluon-quark parton model ($K_2 = 0.56$, $\alpha = 0$ e.g.) the corresponding values of $I_H$ can be obtained from the muon flux values given in Table V by decreasing them with a factor 0.7. The curve B in Fig.6 is calculated by using the combination

$$I^B(\pi/2, M_W) = (2.1 + 0.7\bar{I}_H + \bar{I}_W) \cdot 10^{-13} cm^{-2} sec^{-1} ster^{-1} \qquad (46B)$$

If the values for $K_2$ and $d$ obtained by Kuti and Weisskopf are used and we choose lower slope value in $I_L$, as well, the curve $C$ is obtained

$$I^{c}(\pi/2, M_w) = (1.5 + 0.6\,\bar{I}_H + \bar{I}_w) \times 10^{-13}\ cm^{-2} st^{-1} sec^{-1}. \tag{46C}$$

Finally, if we assume both in the high and the low energy region that the slope of the total cross section is much less than the average value, i.e. if we use slope value 0.3 we obtain the curve

$$I^{D}(\pi/2, M_w) = (1.2 + 0.4\,\bar{I}_H + \bar{I}_w) \times 10^{-13}\ cm^{-2} sec^{-1} ster^{-1}. \tag{46D}$$

According to the results of the deep-mine experiments (Table I), the horizontal muon-flux at confidence level better than 99% is smaller than $5 \times 10^{-13}\ cm^{-2} sec^{-1} steradian^{-1}$

$$I_{exp}(\pi/2) < 5 \times 10^{-13}\ cm^{-2} sec^{-1} steradian^{-1}. \tag{47}$$

We see that the assumptions used for the curve $A$ are in contradiction with the experiment. Even if we take into acoount the uncertainties in the neutrino intensities ($\sim 15\%$) the contradiction remains. Therefore, the firm conclusion is obtained.

If we assume that IVB exists and the scale invariance occurs in an average sense even in the non-asymptotic region, thus the total cross section must rise at higher energies with the same slope which has been found in the heavy liquid bubble chamber experiment in the region 1-10 GeV, then the slope value of this linearly rising cross section must be less than 0.8.

If the highest slope value allowed by the gluon-quark parton model is used, we obtain both upper and lower limit on the mass of the IVB, according to the curve $B$

$$3.5\ \mathrm{GeV} < M_W < 12\ \mathrm{GeV}. \tag{48}$$

If we assume lower slope value in $I_L$ and use the values for $K_2$ and $\alpha$ as predicted by the Kuti-Weisskopf fit we obtain only a lower limit

$$M_W \geq 2.5\ \mathrm{GeV}. \tag{49}$$

We can say with the present uncertainties that the gluon-quark parton model is in agreement with the result of the deep-mine experiment. The agreement may not be completely trivial as is indicated by the fact that the slope value 0.8 is in disagreement with the measured horizontal neutrino-induced muon flux.

Finally if we accept the values used caloulating the curve D of Fig 5, we obtain

$$M_W > 2\ \mathrm{GeV}. \tag{50}$$

If IVB does not exist but the IVB-propagator is preserved in the formula (38) to simulate the saturation of the cross section at higher energies, a characteristic energy where the cross section will be saturated can be defined as

$$E_c = M_W^2 / M_N . \tag{51}$$

The horizontal muon flux will be the sum

$$I(\pi/2) = I_L + I_H . \tag{52}$$

The curves $A', B', D'$ of Fig 6. can be calculated from the horizontal muon flux values given by the curves $A, B, D$ of Fig.5 by subtracting the contribution of the $W$ -production reactions. We see that in cases $A'$ and $B'$ the upper limits

$$E_c^A < 21 \text{GeV}, \quad E_c^B < 105 \text{GeV}, \tag{53}$$

(respectively) can be obtained on the value of the characteristic energy.

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## References:


[1] Markov M.A., Proc. of Annual Int.Conf. High Energy Physics. Rochester, 578 (1960).

[2] Greisen K., Proc. of Int. Conf. for Instrumentation in High Energy Phys. (Inst. Publ. Inc. N.Y. p.209, 1960).

[3] Markov M.A. and I.M. Zeleznykh, Nucl.Phys. 27, 385 (1961).

[4] Zatsepin G.T. and V. A. Kuzmin, Zurn.Eksp.Teor.Fiz. 41, 1818 (1961).

[5] Zatsepin G.T. and I.M. Zeleznykh, Izv.Akad. Nauk SSSR,Ser. Fiz. 20, 738 (1962).

[6] Cool R., Lee T.D., Robinson H., Schwartz M., Phys.Rev., 32, 1297 (1963).

[7] Cowan C.L., F.L. Talbott and P.W. Hesse Ap.J. 148, L73 (1967).

[8] Ashton F., H.J. Edwards, G.N. Kelly, Proc. 11th Int. Conf. on Cosmic Rays, Budapest, Acta Physica Ac. Scient. Hung. Suppl. Vol.4 p. 347 (1969).

[9] Wolfendale A.W. et. al. Proc. 11th. Int.Conf. on Cosmic Rays, Budapest, Vol.4. p.319 (1969).

[10] Reines F. et. al. Proc. 11th Int.Conf. on Cosmic Rays, Budapest, Vol.4. p.327 (1969) and Phys.Rev. D4, 80 (1971).

[11] Keuffel J.W. et.al. Proc. 11th Conf. on Cosmic Rays, Budapest, Vol.4. p. 313 (1969)

[12] Coswik R., Pal Y., Tandon S.N., Proc.Ind.Acad. Sci., 63, 217 (1966)

[13] Menon M.C.K. et.al. Proc.R.Soc. A 3ol, 137 (1967)

[14] Craig R., Osborne J.L., Wolfendale A.W. and E.C.M. Young, Proc. Phys.Soc. ser.2. vol.1. 61 (1968)

[15] Pattinson J.B.M. Proc. 11th. Int.Conf. on Cosmic Rays Budapest, vol.4. p. 337 (1969)

[16] Bloom E.D. et.al., paper presented at the XVth International Conference on High Energy Physics, Kiev, U.S.S.R. (197o ) see also MIT-Slac Report No. SLAC -PUB-796, (197o).

[17] Bjorken J.D.,Phys. Rev. 179, 1547 (1969)

[18] R.P. Feynman, in High Energy Collisions, Third International Conference, State University of New York, Stony Brook, (1969) (Gordon and Breach, New York, 1969).

[19] V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, JINR Report No. E2-4968 (197o)

[2o] Kunszt Z., Thesis (Ph.D.), Eotvos University, Budapest (197o); Kunszt Z. and G. Vesztergombi, JINR Report No. E2-5o92(197o).

[21] Myatt G. and D.H. Perkins, Phys.Lett., 34B, 542 (1971).

[22] Bjorken J.D., Nuovo Cimento, 68A, 569 (197o).

[23] Chen H.H., Kropp W.R., Sobel H.W. and F. Reines, Phys. Rev., D4, 99 (1971).

[24] Volkova L.V. and G.T. Zatsepin, Izvestia Akademii Nauk S.S.S.R., Ser. fyz. 29, 1765 (1965)

[25] Tam A.C. and E.C.M. Young, in Proc. of 11th Conf. on Cosmic Rays, Budapest, (1969), Vol.4. p.3o7.

[26] Osborne J.I., Said, S.S. and A.W. Wolfendale, Proc. Phys. Soc., 86, 93 (1965).

[27] Osborne J.I., private communioation.

[28] Hayman P.J., Palmer N.S. and A.W. Wolfendale, Proc. R. Soc., A, 275, 391 (1963).

[29] Osborne J.L., Wolfendale A.W. and Young E.C.M., Proc. Phys. Soc. (J. Phys.) Ser.2. Vol. p55 (1968).

[3o] Sternheimer J., Phys.Rev., 115, 137 (1959).

[31] Kustor R.L. et.al. Phys.Rev.Letters, 22, 1o14 (1969).

[32] Llewellyn Smith C.H. SLAC-Report, No. SLAC-PUB-958 (1971).

[34] Bloom E.D. and F.J. Gilman, Phys.Rev.Letters, 25, 114o (197o).

[35] Rubinstein H. et.al., Nuclear Phys.B. 39, 421 (1971).

[36] Callan C.G. and D.J. Gross, Phys.Rev.Letters, 22, 156, (1969).

[37] Kuti J. and V. Weisskopf, Phys.Rev., D4, 13418 (1971).

[38] Wu A.C.T. Yang C.P., Fuchel K. and S. Heller, Phys.Rev.Letters, 12, 57 (1964).

[39] Lee T.D. Markstein P. and C.N. Yang, Phys.Rev.Letters 7, 429 (1961) and Von Gehlen G.V., Nuovo Cimento, 3o, 859 (1963).

[4o] Solovie̓v V.V. and I.S. Thsukeman Zh.Eksp.Teor. Fiz., 42, 1252 (1962).

[41] Coswik R. and Y. Pal in Proceedings of the Eleventh International Conference on Cosmic Rays, Budapest, (1969) Vol.4. p. 367.

[42] Uberall H., Phys.Rev., 133, B444 (1964).

[43] Brown R.W., Hobbs R.H. and J. Smith, Phys.Rev., D4, 794 (1971).

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| $E_\nu$ (GeV) | $N(\nu+\bar\nu, E_\nu, \theta=\pi/2)$ $cm^{-2}s^{-1}str^{-1}GeV^{-1}$ |
|---|---|
| 0.2 | $9.09.10^{-1}$ |
| 0.5 | $1.78.10^{-1}$ |
| 1 | $3.27.10^{-2}$ |
| 2 | $5.30.10^{-3}$ |
| 5 | $6.35.10^{-4}$ |
| 10 | $9.49.10^{-5}$ |
| 20 | $1.33.10^{-6}$ |
| 50 | $9.07.10^{-7}$ |
| 100 | $1.18.10^{-7}$ |
| 200 | $1.50.10^{-8}$ |
| 500 | $7.76.10^{-10}$ |
| 1000 | $8.58.10^{-11}$ |
| 2000 | 9.4[illegible].$10^{-12}$ |
| 3000 | $2.6\ .10^{-12}$ |
| 5000 | 5.2[illegible].$10^{-13}$ |
| 10000 | $6.3\ .10^{-14}$ |

Table III.

| $E_\mu$ (GeV) | $R_\mu^{st.r.}$ g/cm² | $E_\mu$ (GeV) | $R_\mu^{st.r.}$ g/cm² |
|---|---|---|---|
| 0.1283 | 3.955 | 0.3035 | 102.1 |
| 0.1311 | 4.831 | 0.4165 | 175.5 |
| 0.1340 | 5.768 | 0.5000 | 230.0 |
| 0.1360 | 6.761 | 0.6142 | 303.1 |
| 0.1396 | 7.806 | 0.7837 | 410.0 |
| 0.1424 | 8.901 | 0.8967 | 480.1 |
| 0.1453 | 10.04 | 1.000 | 545.0 |
| 0.1509 | 12.44 | 1.519 | 852.3 |
| 0.1622 | 17.67 | 2.000 | 1230 |
| 0.1735 | 23.34 | 2.931 | 1649 |
| 0.1961 | 35.68 | 3.496 | 1957 |
| 0.200 | 37.9 | 5.000 | 2760 |

Table IV.

Effective range-energy relations of muons in standard rock

| $E_\mu$(GeV) | $R^{st.r.}_{b=2.3}$ g/cm$^2$ | $R^{st.r.}_{b=4.0}$ g/cm$^2$ | $R^{st.r.}_{b=6.0}$ g/cm$^2$ |
|---|---|---|---|
| 10 | 5.082.10$^{3}$ | 5.069.10$^{3}$ | 5.053.10$^{3}$ |
| 15 | 7.328.10$^{3}$ | 7.244.10$^{3}$ | 7.253.10$^{3}$ |
| 20 | 9.525.10$^{3}$ | 9.463.10$^{3}$ | 9.389.10$^{3}$ |
| 25 | 1.169.10$^{4}$ | 1.154.10$^{4}$ | 1.148.10$^{4}$ |
| 30 | 1.382.10$^{4}$ | 1.368.10$^{4}$ | 1.352.10$^{4}$ |
| 35 | 1.592.10$^{4}$ | 1.574.10$^{4}$ | 1.552.10$^{4}$ |
| 40 | 1.801.10$^{4}$ | 1.777.10$^{4}$ | 1.749.10$^{4}$ |
| 45 | 2.007.10$^{4}$ | 1.977.10$^{4}$ | 1.943.10$^{4}$ |
| 50 | 2.212.10$^{4}$ | 2.175.10$^{4}$ | 2.134.10$^{4}$ |
| 60 | 2.616.10$^{4}$ | 2.565.10$^{4}$ | 2.507.10$^{4}$ |
| 70 | 3.014.10$^{4}$ | 2.946.10$^{4}$ | 2.871.10$^{4}$ |
| 80 | 3.407.10$^{4}$ | 3.320.10$^{4}$ | 3.225.10$^{4}$ |
| 90 | 3.784.10$^{4}$ | 3.687.10$^{4}$ | 3.570.10$^{4}$ |
| 100 | 4.168.10$^{4}$ | 4.038.10$^{4}$ | 3.898.10$^{4}$ |
| 200 | 7.768.10$^{4}$ | 7.337.10$^{4}$ | 6.902.10$^{4}$ |
| 300 | 1.105.10$^{5}$ | 1.022.10$^{5}$ | 9.426.10$^{4}$ |
| 500 | 1.690.10$^{5}$ | 1.511.10$^{5}$ | 1.352.10$^{5}$ |
| 700 | 2.201.10$^{5}$ | 1.916.10$^{5}$ | 1.679.10$^{5}$ |
| 1.000 | 2.865.10$^{5}$ | 2.421.10$^{5}$ | 2.072.10$^{5}$ |
| 2.000 | 4.537.10$^{5}$ | 3.605.10$^{5}$ | 2.954.10$^{5}$ |
| 3.000 | 5.735.10$^{5}$ | 4.404.10$^{5}$ | 3.526.10$^{5}$ |
| 5.000 | 7.439.10$^{5}$ | 5.489.10$^{5}$ | 4.288.10$^{5}$ |
| 7.000 | 8.658.10$^{5}$ | 6.243.10$^{5}$ | 4.809.10$^{5}$ |
| 10.000 | 1.001.10$^{6}$ | 7.067.10$^{5}$ | 5.373.10$^{5}$ |
| 20.000 | 1.279.10$^{6}$ | 8.717.10$^{5}$ | 6.491.10$^{5}$ |

Table IV. (cont.)

| $M_W$ (GeV) | $\overline{I}_H$ | $\overline{I}_W$ |
| --- | --- | --- |
| 2.0 | 1.33 | 3.28 |
| 2.5 | 1.68 | 2.36 |
| 3.0 | 2.00 | 1.80 |
| 5.0 | 2.93 | 0.78 |
| 8.0 | 3.75 | 0.28 |
| 10 | 4.12 | 0.16 |
| 15 | 4.68 | 0.10 |
| 20 | 4.92 | 0.06 |
| 30 | 5.25 | 0.04 |
| 50 | 5.41 | 0.02 |
| 100 | 5.58 | 0.00 |
| 1000 | 5.60 | 0.00 |

Table V.

$\overline{I}_H$ is the contribution to the horizontal muon flux $\times 10^{13}$ ($cm^{-2}s^{-1}sr^{-1}$) from high energy neutrinos calculated by the formula (1) and (38) assuming that $b=1$, $d=0$ $K_2^{\nu/\bar{\nu}}{}_{nucleon} = 0.8$ .

$\overline{I}_W$ is one fifth of the result of Coswik obtained for the contribution of the reaction (42) to the muon flux $\times 10^{13}$ in $cm^{-2}s^{-1}sr^{-1}$.

| $M_W$ (GeV) | $I^A \times 10^{13}$ ($cm^{-2}s^{-1}st^{-1}$) | $I^B \times 10^{13}$ ($cm^{-2}s^{-1}st^{-1}$) | $I^C \times 10^{13}$ ($cm^{-2}s^{-1}st^{-1}$) | $I^D \times 10^{-13}$ ($cm^{-2}s^{-1}st^{-1}$) |
|---|---|---|---|---|
| 2 | 6.91 | 6.31 | 5.58 | 5.01 |
| 2.5 | 6.34 | 5.64 | 4.94 | 4.23 |
| 3 | 6.10 | 5.3 | 4.50 | 3.80 |
| 5 | 6.01 | 4.93 | 4.04 | 3.15 |
| 8 | 6.33 | 5.01 | 4.03 | 3.00 |
| 10 | 6.58 | 5.14 | 4.13 | 3.01 |
| 15 | 7.08 | 5.48 | 4.40 | 3.13 |
| 20 | 7.28 | 5.62 | 4.01 | 3.23 |
| 30 | 7.60 | 5.82 | 4.69 | 3.34 |
| 50 | 7.73 | 5.91 | 4.77 | 3.38 |
| 100 | 7.88 | 6.00 | 4.85 | 3.40 |
| 1000 | 7.90 | 6.02 | 4.87 | 3.44 |

Table VI.

Horizontal $\nu_\mu$ -induced muon-flux values calculated by the formula (46A,B,C,D).

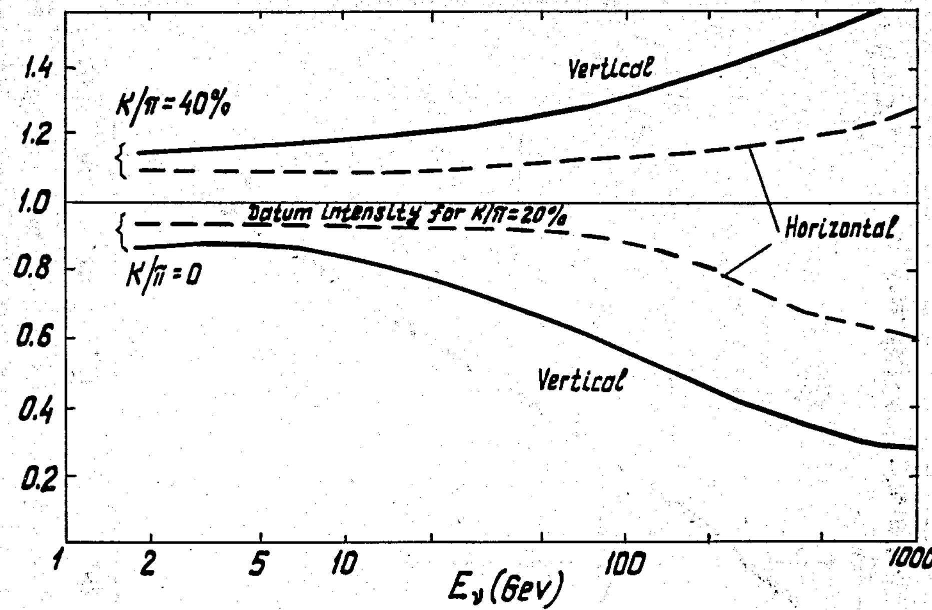


Fig.1. Energy spectra of $\nu_\mu + \bar{\nu}_\mu$ for $K/\pi$ ratios of 0 and plotted with respect to the intensity for a ratio of 20% (taken from [26]).

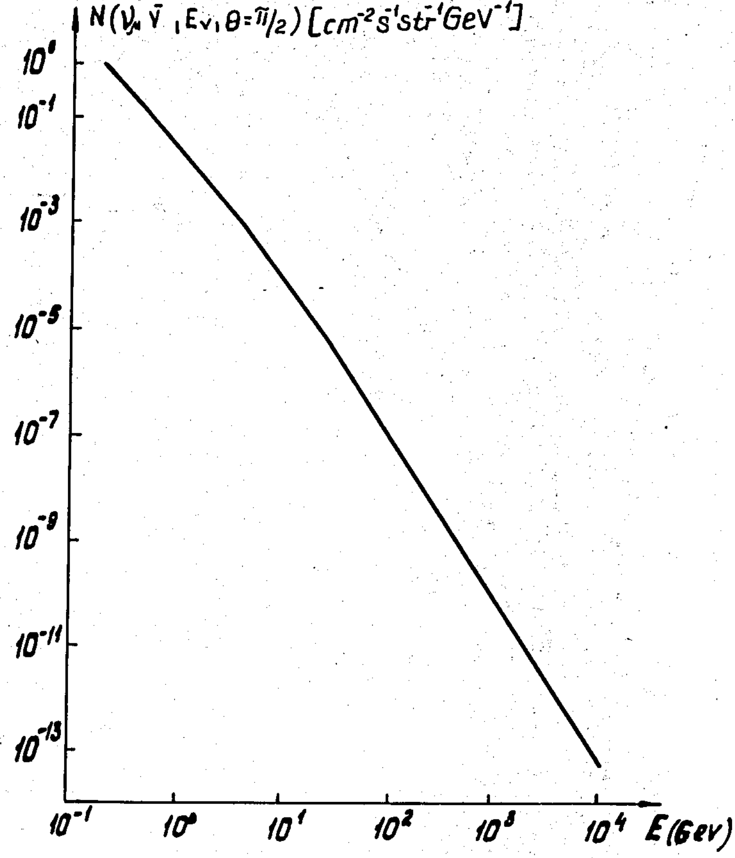


Fig. 2. Intensity of atmospheric neutrinos plus antineutrinos in the horizontal direction as calculated by Osborne et al. [26].

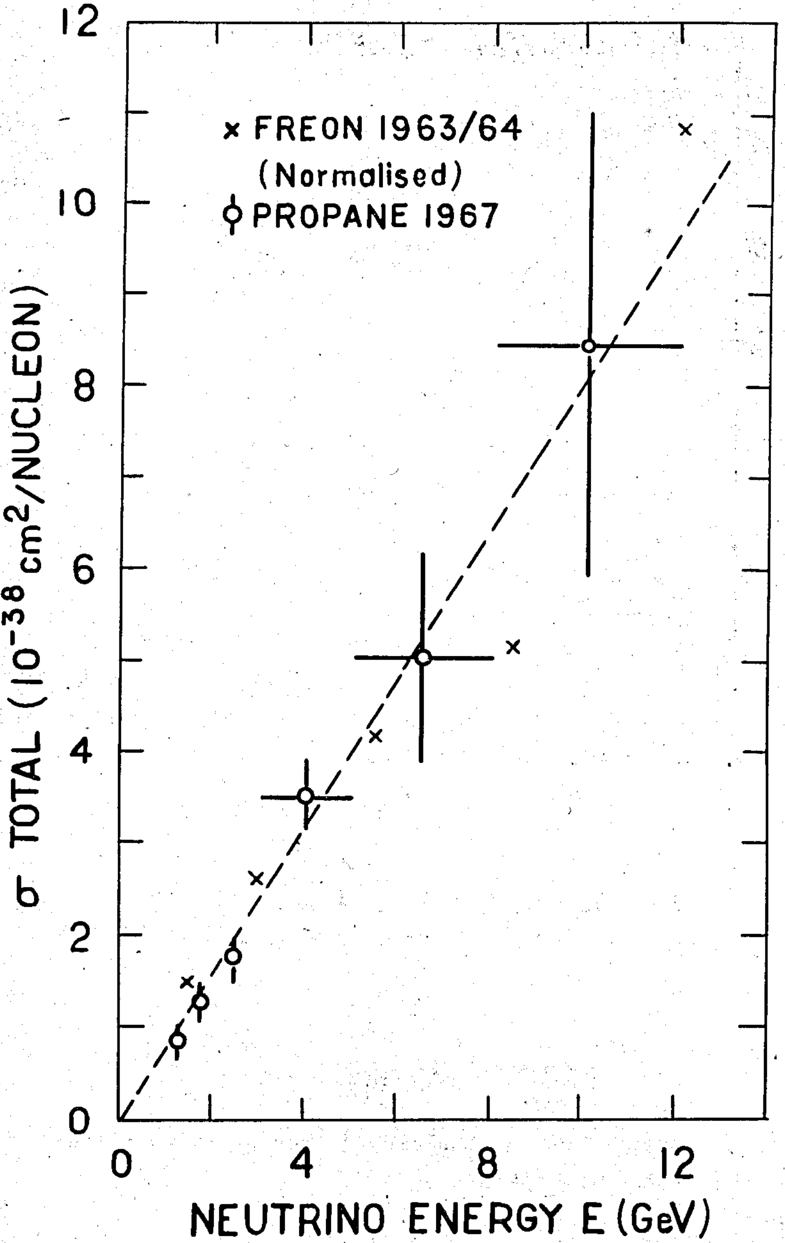


Fig. 3. Total neutrino-nucleon cross section as a function of neutrino energy E ( taken from [21] ).

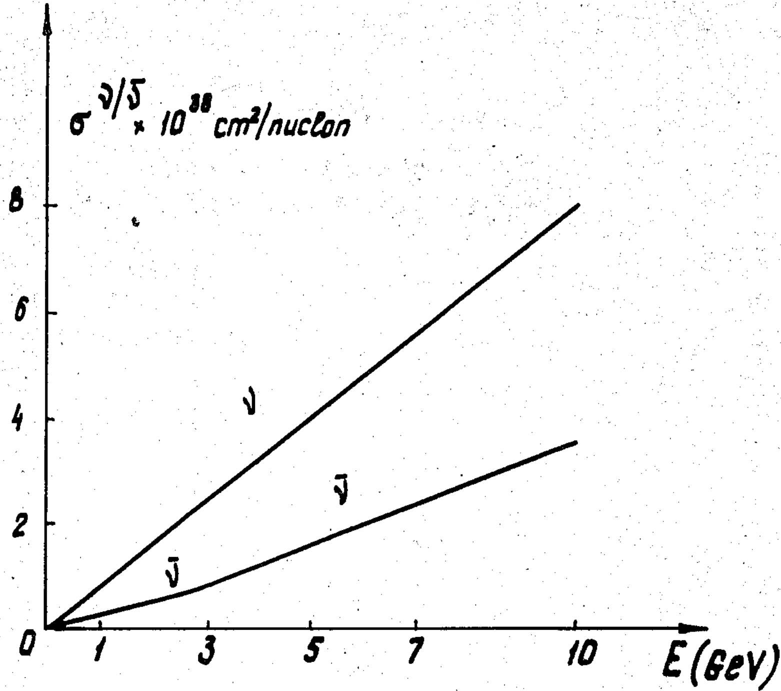


Fig.4. Total cross section values for $\nu_\mu(\bar{\nu}_\mu)N$ scattering used for the estimation of the contribution of the low energy neutrinos to the muon flux.

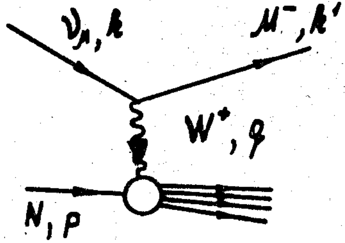


Fig. 5. Inelastic neutrino-nucleon scattering in lowest order of weak interactions.

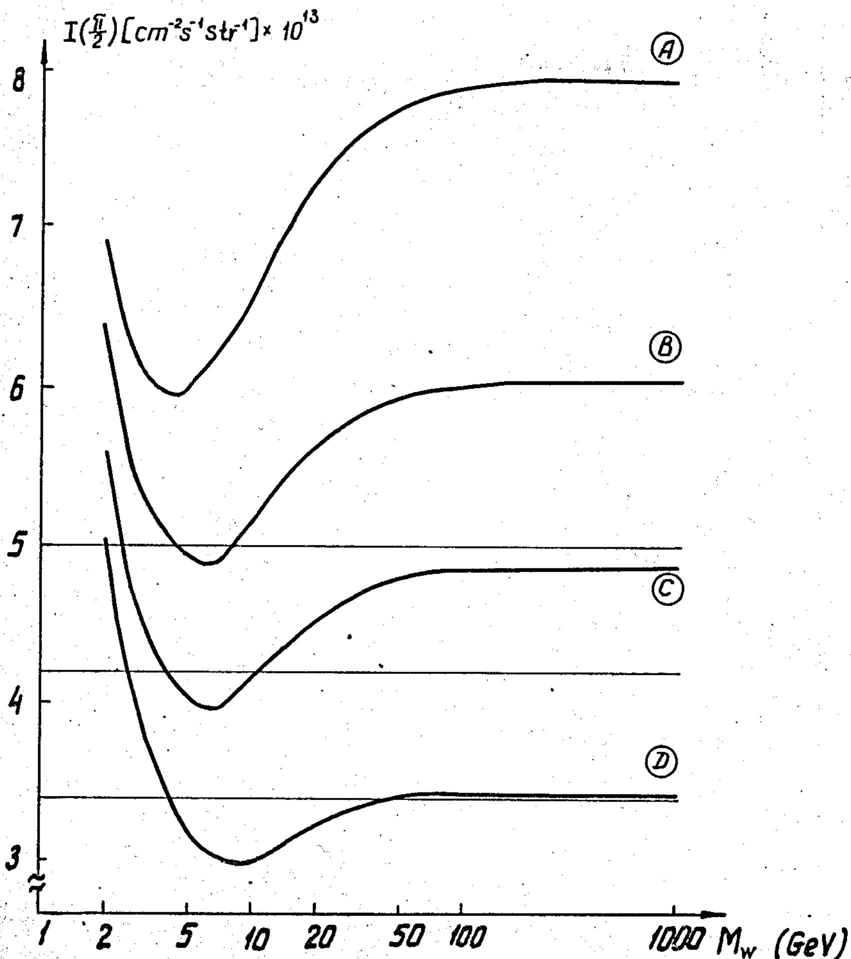


Fig.6. Calculated muon fluxes as a function of the mass of the IVB $M_W$ ( see Table VI).

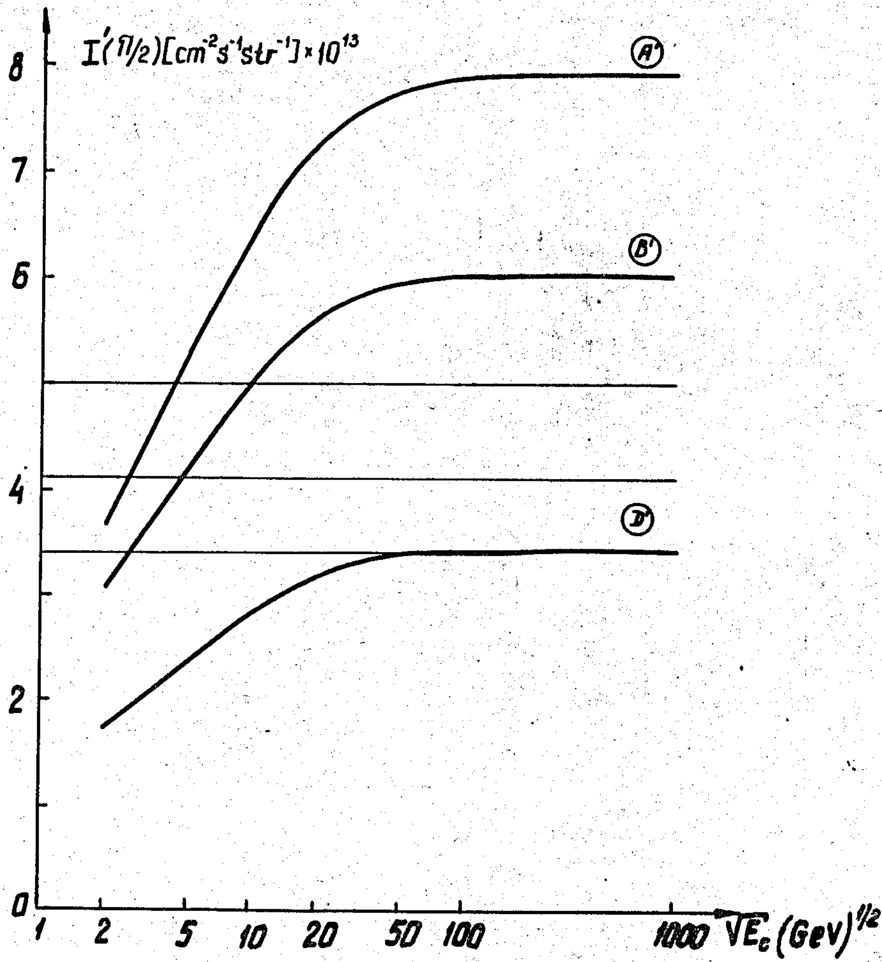


Fig. 7. Calculated muon fluxes as a function of the saturation energy $E_c$

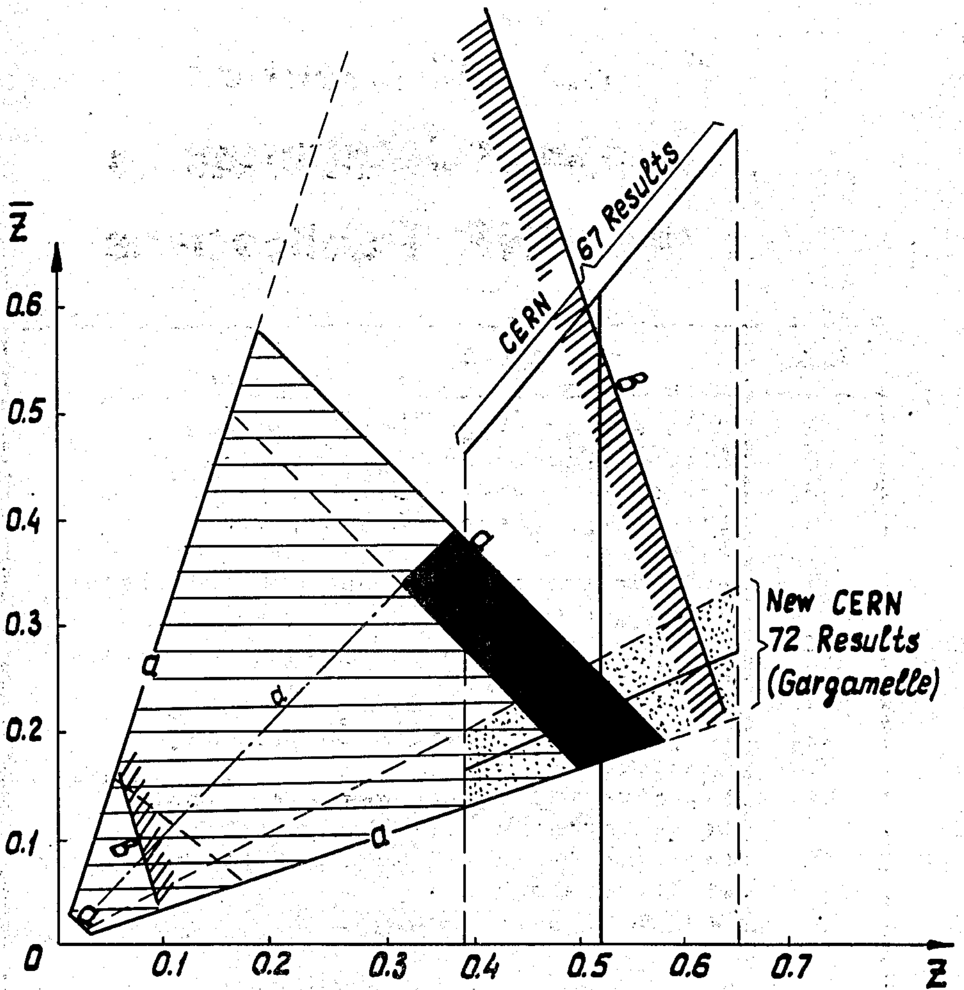


Fig.8.Allowed regions for the slopes of the total cross sections $\left(\sigma^{\nu}(\sigma^{\bar{\nu}}) = \frac{G^2ME}{\pi} Z(\bar{Z})\right)$. The region inside the countour "a" is allowed by the SLAC-MIT experiment using positivity conditions in the parton model; the region inside the contour "b" is allowed by the atmospheric neutrinos induced muon flux. The dark plot is the most probable region in the quark patron model ( the portion of the strange-quark pairs is not too large). See Neutrino 72 Conference, Balatonfured (1972) .