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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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AUTOMODELITY IN STRONG INTERACTIONS

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1. Introduction

All the main experimental facts on deep inelastic lepton-hadron interactions can be understood by means of usual dimensional analysis and automodelity principle for electromagnetic and weak interactions[1].

In the present note we study the asymptotic behaviour of the hadron-hadron collision processes at high energies by means of generalized dimensional analysis and automodelity principle for strong interactions. We take into account the fact, that contrary to deep inelastic lepton-hadron interactions, which are analogous to the phenomenon of point explosion[3], the strong hadron-hadron interactions are analogous to the effect of plane explosion in gas dynamics.

2. Automodelity principle in strong interactions

We shall consider the process of hadron-hadron collision at high energies in the center-of-mass system. There are two experimental rules about the dynamics of particle production in these collisions.

1. Smallness of transverse momenta: Most secondary particles have limited transverse momenta $|\vec{q}_\perp| < 0.4$ GeV. When the energy increases, the longitudinal momenta can increase infinitely. For fixed initial energy the allowed phase space in the $q_z, |\vec{q}_\perp|$ plane is a circle with a radius

$\frac{\sqrt{s}}{2}$, while dynamics show that most events take place within a strip in Fig.1:

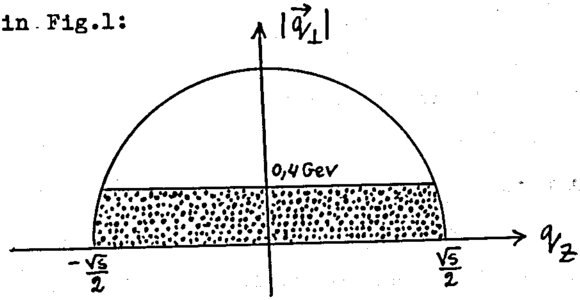

Fig.1. Kinematically and dynamically allowed regions in the phase space.

2. Low multiplicity of particles produced: The average number of particles produced $\bar{n}$ grows slowly as energy increases. This means, together with the rule 1, that most collision energy is transformed to kinetic energy of longitudinal motion and multiplicity of particles produced is growing much less rapidly than the available energy would allow. These two facts allow to make assumption that there is a strong dynamical difference between longitudinal and transversal directions. Therefore it is natural to introduce two different scales of length; x)

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x) The vector units of length $\vec{L} = \{L_x, L_y, L_z\}$ were introduced in the last century and were usefully applied in the dimensional analysis[4]. Taking into account the natural asymmetry of a problem, by means of such a generalized dimensional analysis it is possible to obtain much more information, than using the usual (scalar) unit of length.

L_z - along collision axis,

L_T - in the transverse plane.

Any physical quantity F ,which is measured in hadron collision experiments, can be characterized by definite dimensions in longitudinal and transverse scales:

$$[F] = L_z^n L_T^m \tag{1}$$

Now let us formulate our basic hypothesis:

At high energies there is no fixed dimensional constant with longitudinal dimension. All essential constants, such as masses, effective radii and other unknown parameters have pure transversal dimension.

Therefore, under scale transformations of the form

$$q_z \to \lambda q_z \quad , \quad \vec{q}_\perp \to \vec{q}_\perp \tag{2}$$

all the physical quantities must transform as homogeneous functions of appropriate dimension in the longitudinal scale, that is

$$F \to \lambda^{-n} F \tag{3}$$

This is the formulation of automodelity principle for strong interactions.

The following intuitive picture can be used. In the center-of-mass system, due to Lorentz contraction, the hadron can be represented at high energies as an infinitely thin disk with finite transverse sizes. If there is a fixed constant with longitudinal dimension, it is possible to ascribe to the disk a finite nonzero width. In such a case it will be impossible to make scale transformation in the longitudinal direction.

3. Consequences of automodelity principle for strong interactions

During the past years in the investigation of strong interaction dynamics the importance of inclusive type experiments was recognized . The advantage of this approach was first indicated in[5], where this process was considered in the framework of the general principles of quantum field theory. From the phenomenological point of view these ideas were intensively developed in[6].

Let us consider concrete observable quantities:

A) Total cross sections

The most inclusive experiment is the measurement of the total cross section $\sigma_{tot}(s)$ in hadron-hadron collision $P_1 + P_2 =$ ... where dots represent any number of particles and $s = (P_1+P_2)^2 \approx 4P_2^2$ is the square of the center-of-mass energy. By definition the total cross section is characterized by a definite area transversal to the collision axis. Therefore, the dimensionality of $\sigma_{tot}(s)$ is expressed in the transversal scale as follows:

$$[\sigma_{tot}(s)] = L_T^2 \tag{4}$$

But the invariant variable s at high energies has pure longitudinal dimensionality

$$[s] = L_z^{-2} \tag{5}$$

Under longitudinal scale transformations $P_z \to \lambda P_z$ according to automodelity principle we have

$$\sigma_{tot}(s) = \sigma_{tot}(\lambda^2 s) \tag{6}$$

Hence it appears that σ_{tot} may not depend on S, that is

$$\sigma_{tot} = const. \tag{7}$$

Let us notice that the same result follows from the reggeization of elastic $P_1 + P_2 = P_1 + P_2$ amplitude with exchange of Pomeranchuk trajectory.

B. Single-particle distributions

We consider the inclusive reaction (Fig.2)

$$P_1 + P_2 \to q + \dots \tag{8}$$

where ... represents any number of particles.

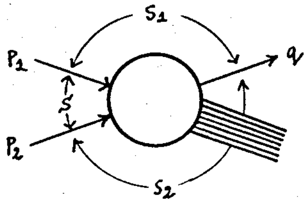

Fig.2. Reaction $p_1 + p_2 \to q + \ldots$ and its kinematical variables.

Let us consider the center-of-mass system in which

$$\begin{aligned} p_1 &\approx \{ p_z, 0, 0, p_z \}, \\ p_2 &\approx \{ p_z, 0, 0, -p_z \}, \\ q &= \{ q_0, \vec{q}_\perp, q_z \}, \quad q_0 = \sqrt{m^2 + \vec{q}_\perp^{\,2} + q_z^2}, \end{aligned} \tag{9}$$

Reaction (8) corresponds to four-point function of Fig.2, with one external leg off mass shell. Hence it depends on three invariant variables, which can be taken as

$$\begin{aligned} s &= p_1 \cdot p_2 \approx 2 p_z^2 \\ s_1 &= p_1 \cdot q \approx p_z (q_0 - q_z) \\ s_2 &= p_2 \cdot q \approx p_z (q_0 + q_z) \end{aligned} \tag{10}$$

The invariant differential cross section for reaction (8) can be represented as follows

$$d\sigma = \frac{d\vec{q}}{q_0} f(s, s_1, s_2) \tag{11}$$

P — "Pionization" region. In this case $|q_z| \sim |\vec{q}_\perp|$ and only the product $s_1 \cdot s_2$ has definite dimension $[s_1 \cdot s_2] = L_z^{-2} L_T^{-2}$ and $[s] = L_z^{-2}$.

From the requirement of automodelity under scale transformations $p_z \to \lambda p_z$, $q_z \to \lambda q_z$ and $\vec{q}_\perp \to \vec{q}_\perp$ the following equality must take place in the F_1 region:

$$f(s, s_1, s_2) = f(\lambda^2 s, s_1, \lambda^2 s_2) \tag{12}$$

It is possible, if

$$f(s, s_1, s_2) = f\left(\frac{s_2}{s}, s_1\right) = f\left(\frac{q_z}{p_z}, |\vec{q}_\perp|\right) \tag{13}$$

The same result follows from Mueller-Regge analysis of the three-particle elastic amplitude $p_1 + p_2 + \bar{q} = p_1 + p_2 + \bar{q}$ with single $\alpha_P(0) = 1$ trajectory exchange.

The F_2 region can be obtained from F_1 simply by substitution $1 \rightleftarrows 2$.

In "pionization" region, from automodelity principle, we have

$$f(s, s_1, s_2) = f\left(\frac{s_1 \cdot s_2}{s}\right) = \varphi(|\vec{q}_\perp|) \tag{14}$$

This result coincides with the double Regge expansion of three-particle amplitude with pomeranchukon exchange [7].

C. Two-particle distributions

The new interesting phenomena occur, when we consider the inclusive processes with two singled out particles in the final state (Fig.3):

$$P_1 + P_2 \to q + q' + \dots \tag{15}$$

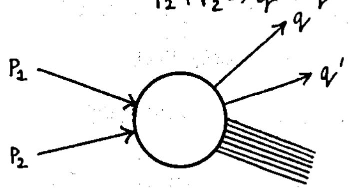

Fig.3. Kinematics of reaction $P_1 + P_2 = q + q' + \dots$

The differential cross section will be function of 6 independent variables x):

$$d\sigma = \frac{d\vec{q}}{q_0}\,\frac{d\vec{q}\,'}{q_0'}\,f(s, s_1, s_2, s_1', s_2', q\cdot q') \tag{16}$$

where for our purpose a natural choice of variables is

$$\begin{aligned}
s &= P_1\cdot P_2 \approx 2P_z^2 \\
s_1 &= P_1\cdot q \approx P_z(q_0 - q_z) \\
s_2 &= P_2\cdot q \approx P_z(q_0 + q_z) \\
s_1' &= P_1\cdot q' \approx P_z(q_0' - q_z') \\
s_2' &= P_2\cdot q' \approx P_z(q_0' + q_z') \\
q\cdot q' &= q_0 q_0' - \vec{q}_\perp\cdot\vec{q}_\perp{}' - q_z q_z'
\end{aligned} \tag{17}$$

x) The general inclusive process $P_1 + P_2 \to q_1 + q_2 + q_3 + \dots + q_n + \dots$ will be a function of 3n independent variables.

The dimension of f is pure transversal $[f] = L_T^6$.

Let us consider different physical situations:

F_1F_1 - the momenta of both detected particles lie in the fragmentation region of particle 1. In this case $q_z \to +\infty$, $q_z' \to +\infty$ and all six variables have definite dimension

$$[s] = [s_2] = [s_2'] = L_z^{-2}, \quad [s_1] = [s_1'] \doteq [q q'] = L_T^{-2}$$

Therefore, according to automodelity principle we find

$$f = f\left(\frac{s_2}{s}, \frac{s_2'}{s}, s_1, s_1', q\cdot q'\right) = \tag{18}$$

$$= f\left(\frac{q_z}{p_z}, \frac{q_z'}{p_z}, |\vec{q}_\perp|, |\vec{q}_\perp'|, \vec{q}_\perp\cdot\vec{q}_\perp'\right)$$

F_1F_2 - particle q is a fragment of 1, q' - fragment of 2. In this case $q_z \to +\infty$, $q_z' \to -\infty$ and asymptotic dimensions of invariant variables are

$$[s] = [s_2] = [s_1'] = [q\cdot q'] = L_z^{-2}, \quad [s_1] = [s_2'] = L_T^{-2}.$$

Hence, from automodelity principle we find

$$f = f\left(\frac{s_2}{s}, \frac{s_1'}{s}, s_1, s_2', \frac{q\cdot q'}{s}\right) = \tag{19}$$

$$= f\left(\frac{q_z}{p_z}, \frac{q_z'}{p_z}, |\vec{q}_\perp|, |\vec{q}_\perp'|\right)$$

In the last expression there is no dependence on the product $\vec{q}_\perp \cdot \vec{q}_\perp'$ so that in this case correlation among produced particles is absent.

PP - Both particles are products of "pionization". In this case $q_z \sim q_z' \sim |\vec{q}_\perp| \sim |\vec{q}_\perp'|$

Only $[s] = L_z^{-2}$ and products $[s_1 \cdot s_2] = [s_1' \cdot s_2'] = L_z^{-2} L_T^{-2}$ have definite dimensions. Therefore, from automodelity requirement

$$f = f\left(\frac{s_1 s_2}{s}, \frac{s_1' s_2'}{s}, q \cdot q'\right) =$$

$$= f\left(|\vec{q}_\perp|, |\vec{q}_\perp'|, \vec{q}_\perp \cdot \vec{q}_\perp'\right) \tag{20}$$

$F_1 P$ - Particle q is a fragment of 1, q' - product of "pionization":

The dimensions are:

$[s] = [s_2] = L_z^{-2}$, $[s_1] = L_T^{-2}$, $[s_1' \cdot s_2'] = L_z^{-2} L_T^{-2}$

Hence, in this case

$$f = f\left(\frac{s_2}{s}, s_1, \frac{s_1 s_2}{s}, q \cdot q'\right) =$$

$$= f\left(\frac{q_z}{p_z}, |\vec{q}_\perp|, |\vec{q}_\perp'|, \vec{q}_\perp \cdot \vec{q}_\perp'\right) \tag{21}$$

Let us remark, that results (18) - (21) are in accordance with the multi-Regge analysis of elastic $p_1+p_2+\bar{q}+\bar{q}'=p_1+p_2+\bar{q}+\bar{q}'$ amplitude[8]. The extension of our consideration to the case of multiparticle distributions is possible. Now let us proceed to the other quantities, measured in high energy experiments.

D. Differential and total cross sections of elastic scattering

The dimension of the differential elastic cross section in the limit of high energies and fixed momentum transfers, when $[t]=[\vec{q}_T^{\,2}]=L_T^{-2}$ is defined by

$$\left[\frac{d\sigma}{dt}\right]=L_T^4 \tag{22}$$

and, consequently, may not depend on s having longitudinal dimension. Thus, it follows from the automodelity principle that

$$\lim_{\substack{s\to\infty \\ t-\text{fixed}}} \frac{d\sigma(s,t)}{dt}=f(t) \tag{23}$$

It is interesting to note, that relation (23) is a consequence of the droplet model and is analysed in detail in ref.[9].
From the relation (23) it also follows that the total elastic cross section $\sigma_{el}(s)$ and the slope of the diffraction peak $b(s)=\frac{d}{dt}\ln\frac{d\sigma}{dt}\Big|_{t=0}$ are constant

$$\sigma_{el}(s) = const. \quad , \quad b(s) = const. \tag{24}$$

It is obvious, that the relation (24) can directly be obtained from the dimensional analysis and automodelity requirement.

E. Other applications

The list of automodelity predictions may, in principle, be extended. Here we oonsider some of them. The ratio of the real and imaginary parts of the elastic scattering amplitude $\alpha(s) = \frac{Re\, T(s,t=0)}{Im\, T(s,t=0)}$ is a dimensionless quantity and therefore in the high energy limit it behaves like oonstant (may be equal to zero):

$$\alpha(s) = const. \tag{25}$$

We note that the relation (25), under the condition of constancy of the total cross section, on the basis of the general principles of quantum field theory, results in the asymptotic equality of the total oross sections of interaotion of partioles and antiparticles [10].

The dimensional considerations lead to the following asymptotic behaviour of the average multiplicity $\bar{n}(s)$, the average transverse momentum $\langle \vec{q}_T(s) \rangle$ of secondaries and the inelastioity coefficient $K(s)$

$$\bar{n}(s)=\text{const.}\,, \quad \langle \vec{q}_T(s) \rangle = \text{const.}\,, \quad K(s) = \left\langle \frac{E'}{E} \right\rangle = \text{const.} \tag{26}$$

where E' is the energy carried away by secondaries in processes $P_1 + P_2 = P_1 + P_2$ + anything.

4. Conclusions

At present there is no definite way for estimating corrections due to violation of automodelity principle. In the cases, when the automodel behaviour hypothesis leads to the constancy of any quantity, we may expect in reality only a weak energy dependence, usually logarithmic dependence. However, the mechanism of the appearance of such a factor is unknown. Nevertheless, the method suggested in this note covers many problems of strong interaction dynamics and enables one to obtain the results of different current models in a simple and uniform way.

In conclusion it is interesting to cite a statement made by Rayleigh in 1915:

"I have often been impressed by the scanty attention paid even by original workers in physics to the great principle of similitude. It happens not infrequently that results in the form of "laws" are put forward as novelties on the basis of elaborate experiments, which might have been predicted a priori after a few minutes consideration".

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References:

1. V.A.Matveev, R.M.Muradyan, A.N.Tavkhelidze. preprint JINR P2-4578, Dubna (1969); Problems of Elementary Particle and Atomic Nucleus Physics, v.2, part 1, p.7, Moscow, Atomizdat, 1971; A.N.Tavkhelidze, Report on Coral Gables Conference, 1970; L.D.Soloviev, XY Int.Conference on High Energy Physics, Kiev, 1970, "Naukova Dumka" publ. 1972, p.513; R.M.Muradyan, ibid. p. 558; T.D.Lee, Physics Today, April, 1972; p. 23; H.Kastrup, Particles, Currents and Symmetries, ed. P.Urban (Springer Verlag, Wien-New-York, 1968), p. 407.

2. V.A.Matveev, R.M.Muradyan, A.N.Tavkhelidze, Preprint JINR E2-5962, Dubna , 1971.
A.M.Baldin, Preprint JINR, P7-5808, Dubna, 1971.

3. L.I.Sedov, Metody podobija i razmernosti v mekhanike, GITTL, Moscow, 1957;
K.P.Stanyukovitch . Neustanovivshiesya dviczenia sploshnoi sredy, GITTL, Moscow, 1957.

4. H.E.Huntley, Dimensional Analysis, Dover Publications, N.Y. 1967.