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PHENOMENOLOGY OF HIGH ENERGY COLLISIONS AND SCALING LAW AT SMALL DISTANCE

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PHENOMENOLOGY OF HIGH ENERGY COLLISIONS AND SCALING LAW AT SMALL DISTANCE

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Феноменология высокоэнергетических процессов и масштабное поведение на малых расстояниях

Показано, что предложение о масштабном поведении на малых расстояниях в γ^5 -теории позволяет понять с единой точки зрения феноменологию различных явлений (упругое рассеяние на большие углы, рассеяние в дифракционной области и глобоко неупругие процессы) и некоторые модификации феноменологических моделей.

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Phenomenology of High Energy Collisions and Scaling Law at Small Distance

It is shown that the assumption of the scaling behaviour at small distance in the γ^5 -field theory allows to understand from an unique point of view the phenomenology of different processes (large angle elastic scattering, scattering in the diffraction region and deep inelastic processes). Some modifications of this phenomenology are suggested.

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The most attraotive hypothesis of the last time is the scaling behaviour at small distanoe, whioh is based on the assumed absenoe of any dimentional parameter in the theory aside from the mass. This means that under the special kinematioal oonditions, when the masses of the external particles are non-essential, i.e. when any of the scalar products of the external momenta $p_i p_j$ (inoluding also $p_i^2 \gg m^2$) amplitudes (or Green funotions) are homogeneous funotions of the momenta

$$T(\Lambda p) = \Lambda^{2\varkappa} T(p) \quad , \qquad (1)$$

the index $\varkappa$ depending only on the number and the kind of the external lines.

What are the consequences of this hypothesis for the physioal processes?

To answer this question we turn to the diagrams of the simplest theory whioh oould pretend to the description of interaction of hadrons. This is the γ^5- theory with the interaotion Lagrangian of the type

$$\mathcal{L}_{int} = g\bar{\psi}\gamma^5\psi\varphi + h\varphi^4 \qquad (2)$$

We shall show that the assumptions, whioh are the bases of our method of summation [1,2] of all logarithmio terms of all diagrams in this theory, are just the direot consequences of the scaling hypothesis. This method makes it possible to answer the above formulated question.

The main result of this report (which is in fact a short review and revision of our activity during the last years) is the statement that the scaling hypothesis allows to understand on a unique basis such different high energy phenomena as the scattering in the diffraction region (Regge-behaviour and the modifications required), large-angle scattering (Wu-Yang-model and possible corrections) and the automodel behaviour of the deep-inelastic ep-scattering. We have to warn the reader from the very beginning that all what we could pretend to, is to clarify the basis of the correct phenomenological description, because we are not able at this stage to calculate the parameters, entering the final results.

1. The general expression for the convergent diagrams in the theory (2) has the well-known form

$$T(p) \sim \int \frac{\prod d\alpha}{\mathcal{D}^2(\alpha)} \, Z(\alpha, p, m) \exp i \{ Q(\alpha, p) - M(\alpha, m) \} , \qquad (3)$$

where $Q(\alpha, p) = d_{ik}(\alpha) p_i p_k$, $M(\alpha, m) = \sum_{\sigma} \alpha_{\sigma} m_{\sigma}^2$

and $Z(\alpha, p, m)$ is some polynomial function of p which emerges due to the numerator of the spinor propagators.

It is clear that homogeneous behaviour of this function in asymptotics, when $\Lambda^2 >> m^2$ could be expected under $p \to \Lambda p$. This asymptotics is determined by the most right singularity of the Mellin transform of

$$T(\Lambda p) = \frac{1}{2i} \int_{\delta - i\infty}^{\delta + i\infty} dj \, \frac{\Lambda^{2j}}{\sin \pi j \, \Gamma(j+1)} \, \Phi(j, p) \tag{4}$$

$$\Phi(j,p) \sim \int \frac{\Pi d\alpha}{\mathfrak{D}^2(\alpha)} \, g(j,\alpha,m) \, \big(Q(\alpha,p)\big)^j \exp\big(iM(\alpha,m)\big)$$

$g(j,\alpha,m)$ being polynomial funotion of j.

The scaling law means that such leading singularity in the j-plane is a simple pole at a point $j = \varkappa$. It is easy to understand that the singularity of $\Phi(j, p)$ is due to the integration region over α , where $Q(\alpha,p) \approx 0$ However, in the Euclidean region, where Q is positively defined function, this region of integration is near its lower bound, where some set of α parameters is close to zero. Topologically this means the contraotion of the corresponding lines into a point. Thus, the set of parameters has to be of suoh sort that the oontraction of corresponding subgraph makes the initial graph independent of eaoh of the variables $p_i p_j$, i.e. all internal lines converge in one point (Fig.1).

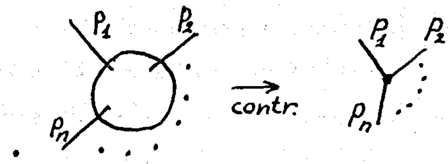

Fig.1

Using the method of investigation of singularities of diagrams [3], one can show that, independently of an order of a diagram, the most right singularity is a simple pole at the point $j_0 = 2 - b/2 - 3f/4$ (b and f are the numbers of boson and fermion external lines), which is generated by the "asymptotical regime" of the whole graph, i.e. comes from the integration region, where all α parameters are small. (In other words, from the region where all the momenta on the lines are simultaneously large). This subdivides the whole contribution of the diagram into the scaling part

$$\Phi_\infty(j, p) = \frac{R(p)}{j - j_0} \tag{5}$$

generated by the asymptotical regime of the whole diagram, and some nonscaling remainder.

The scaling hypothesis states that this result does not change by the summation over the perturbation theory. It is just one of the assumptions of our summation method [1,2].

The divergent parts of the diagram lead to additional powers of log Λ in asymptotics, i.e. to the increase of the order of the pole of Φ_∞. What are the results of the summation of these additional logarithms? This question was answered, in fact, in the earlier works on renormalization group [4] in the following way:

$$T_\infty^{dis}(\Lambda) \to T_\infty^{conv}(\Lambda)\, I^n(\Lambda)\, S^{-f/2}(\Lambda)\, d^{-\ell/2}(\Lambda), \qquad (6)$$

where $I(\Lambda)$ is the invariant charge (the γ^5- theory (2) has two invariant charges, but in the limit $\Lambda\to\infty$ they are reduced to one [5]), S_∞ and d_∞ are the numerators of the fermion and boson propagators. For the S_∞, d_∞ and $I(\Lambda)$ the renormalization group leads to the equations [6]

$$\frac{dI(\Lambda)}{d\ln\Lambda} = F(I(\Lambda));\ \frac{d\ln S_\infty(\Lambda)}{d\ln\Lambda} = G(I(\Lambda));\ \frac{d\ln d_\infty(\Lambda)}{d\ln\Lambda} = H(I(\Lambda))$$

the only scale solution of it being

$$I(\Lambda)\to g_0\ ,\quad S_\infty\to(\Lambda^2)^{\sigma = G(g_0)},\quad d_\infty\to(\Lambda^2)^{\delta = H(g_0)}$$

As a result, anomalous dimensions appear, which shift the pole, according to (6), to the point

$$\varkappa = j_0 - \frac{f}{2}\sigma - \frac{\ell}{2}\delta$$

The second assumption of our method [1,2] was just $I(\Lambda)\to const$ and $\sigma = \delta = 0$. The latter of them is non-essential, but the statement that "Nature reads the book of the free fields" means at least the smallness of σ and δ and hints at the smallness of the bare coupling constant g_0.

Now let us turn to the consequence of the scaling hypothesis for the physical processes.

2. Let us consider first the large angle elastic scattering, when $|S| \sim |t| >> P_i^2, m^2$. The exponent of the expression (3) in this case can be represented in the form

$$S\left(A(\alpha) + \xi B(\alpha)\right) + J(\alpha, P^2, m) \quad ,$$

where $\xi = \frac{t}{S}$ is some fixed value of an order of one. Mellin transformation of the amplitude with respect to the large variable S leads to the following expression

$$\Phi^{\pm}(j, \xi) \sim \int \frac{\Pi d\alpha}{\mathcal{D}^2(\alpha)} g(j, \alpha, m) \, |A(\alpha) + \xi B(\alpha)|^{j} \times \exp\{i J(\alpha, P^2, m)\} \left[\theta(A + \xi B) \pm \theta(-A - \xi B)\right] \frac{e^{i\pi j} \pm 1}{2} \quad , \tag{8}$$

where the signature divisions are necessary because of the two cuts in the complex S-plane.

In distinction with the previous case, the singularities in the j-plane can be generated now by the zero of $A + \xi B$ at the low bound of integration region, as well as somewhere inside the region, because of the cancellation of different terms. The second of the mechanisms is not connected with the scaling behaviour and probably plays no essential role because it contributes to the negative signature amplitude only,[3] and can be responsible partially for the difference of

s-u-crossed reactions, which is known to be of no importance. The singularities, due to the first mechanism, are generated by the asymptotical regime of the subgraphs, each of which being contracted makes the diagram independent of s and t simultaneously (Fig.2).

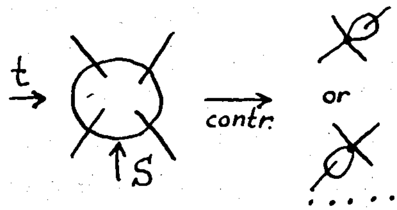

Fig.2

It can be shown again that the most right singularity here is due to the asymptotical regime of the whole graph. According to the scaling hypothesis, it has to be a pole at the point $j = \varkappa + f/4 = -\frac{1}{2}(\ell\delta + f6) = \varkappa_4$ The additional term $f/4$ emerges from the wave functions of the external fermion lines.

For the amplitudes of the non-polarized proton-proton scattering, for instance, at the angle near 90^0 (c.m.s.) and large s this results in

$$T'(s,\xi) \sim g_0^4 R(\xi, g_0)\, s^{-26}$$

We single out here g_0^4 to stress the special role of Born diagrams. Two of the three external lines of vertex functions, entering these diagrams, are on the mass shell. For this reason they work as the pion formfactor of the nucleon and result in the amplitude

$$T_B(t,\xi) \sim g^2 F_\pi(t) t^{\delta}, \quad (s \approx t)$$

Which of the two mechanisms prevails at the accessible energies, depends on the value of the bare coupling constant g_0. However, a rather good agreement of the Wu-Yang [7] model with the experiment $\left(\frac{d\sigma}{dt} \sim F_\gamma^4(t)\right)$ under the condition $F_\pi \approx F_\gamma$ is one of the arguments in favour of small g_0.

From this point of view additional more detailed measurements of the energy dependence of large angle differential cross section, first of all in the Serpukhov energy range, would be of great interest. They could supply, in fact, the first indirect information on the scaling behaviours and anomalous dimensions.

3. Let us turn now to the scattering in the difraction region. The exponent (3) in this case can be rewritten as

$$S A(\alpha) + J(\alpha, t, p^2, m), \qquad S >> |t|, p^2, m^2$$

and the expression of the type (8), with the replacement $A + \xi B \rightarrow A$ is valid. Now the singularities of $\Phi(j)$ in the j-plane are generated by the asymptotical regime of the subgraphs contraction each of them makes the diagram independent of S, i.e. convert it into weakly connected diagram of the type, shown in Fig.3. However, in distinction with the previous case, many subgraphs of such a type are

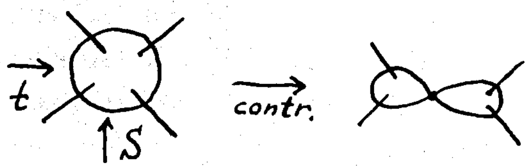

Fig.3

possible. The leading singularity is due to each of such subgraphs with four external lines, the scattering diagram as a whole being among them. (These external lines can be either fermion, if $\sigma < \delta$ or boson, if $\sigma > \delta$ or both, if $\sigma = \delta$). The scale invariance hypothesis means, according to item 1, that the asymptotical regime of each of these subgraphs, as a whole, generates a simple pole at the point $j = \varkappa_4$ However, the large number of such subgraphs increases the order of this pole.

For instance, the maximal order of the pole of the diagrams, drawn in Fig.4, is due to the asymptotical regime of

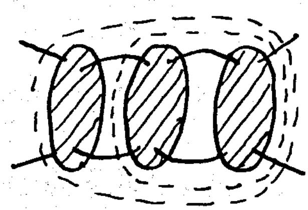

Fig.4

eaoh of the Bethe-Solpeter kernels (shaded blocks), of any of the oonnected unions of two kernels and of the whole diagrams. The nonasymptotical regime of some of the objects (i.e. the nonsoale part of its oontribution) determines junior orders of the pole.

Independence of this pioture on the concrete kind of the kernel permits to sum up all such poles of all diagrams of the perturbation theory [1,2] . This provides the answer

$$\Phi(j,t) = C(t)\left(v(j) - B(t)\right)^{-1} C^{T}(t)$$

where C and B are some matrioes in the spin spaoe, known as a series in the renormalized ooupling oonstant g and $v(j)$ is a function with a square root branoh points, the positions of which are determined by the bare coupling oonstant g_0 and by the anomalous dimensions σ and δ In the simplest oase of small g_0

$$v(j) = \frac{1}{2 r g_0^2}\left[(j+2\sigma) + \sqrt{(j+2\sigma)^2 - 4 r g_0^2}\right],$$

where the number r is determined by the t-channel quantum numbers.

Thus, the direot consequence of the soale hypothesis is the existence of the fixed branch points, which aooompany the moving Regge poles, due to $\det\left[v(j) - B(t)\right] = 0$.

Phenomenologically this is a model close to the one of Van Hove-Durand [8].

The appearance of the fixed branch points is easy understandable, if one remembers that the only scale invariant potential at small distance in quantum mechanics is $V(r) \sim \frac{\lambda}{r^2}$ which gives as is well known a square root branch points in a partial wave amplitude.

It is interesting to note that taking into account the special character of the Pomeron and identifying it with the most right branch point, we could obtain in the approximation of small g_0 : $g_0^2 \approx 0.12$. So, this approximation is self-contained. The same assumption allows to calculate the slopes of the Pomeron residues [9] in good agreement with the experiment.

The same branch points in the reactions with the natural exchange of isospin $I=1$ (and the unnatural with $I=0$) are situated on the imaginary axis ($r< 0$). This allows to obtain at medium energies an additional contribution to the real part of the nonflip amplitudes. Such contribution is necessary [10] for resolving the difficulties of an ordinary phenomenological approach in explanation of polarization in the πN-charge exchange, of difference of the cross section of the s-u crossed reaction $pn \to np$, $p\tilde{p} \to n\tilde{n}$ and $K^- p \to \pi^- \Sigma^+$, $\pi^+ p \to K^+ \Sigma^+$ and of behaviour of the charge asymmetry $A = \frac{d\sigma_{\pi+}}{d\sigma_{\pi-}}$ in the pion photoproduction [11].

4. At last let us turn to the <u>deep inelastic- ep-scattering</u>. It is known, that the summation of the senior logarithmic terms [12] in the region $|q^2| \simeq |S| >> m^2$ in the γ^5 theory (and in the gluon model) cannot explain the automodel behaviour observed in the experiment. However, in this approximation this can hardly be expected, because this approximation leads to the obviously nonscaling zero bare charge situation of Landau et al. For the scaling solution with finite charge renormalization the situation looks like the following [13].

The projection of the asymptotical contribution of diagrams on the gauge invariant transversal (T) and longitudinal (L) parts of $W_{\mu\nu}$ with the same assumption as in previous case, leads to the apparent breaking of the automodel behaviour (up to $(\log S)^{3/2}$ accuracy)

$$W_{T,L} \sim \varphi_{T,L} \; (\omega)^{\beta} \cdot (-q^2)^{d(\beta, g_0) - 2\xi(g_0)}, \quad (\omega >> 1),$$

where the numbers d and ξ are some series (probably asymptotical) in g_0. However, these two terms seem to cancel as a consequence of gauge invariance, which, as known, does not allow an anomalous dimension of the electromagnetic current. Really, the projection of the same contribution $W_{\mu\nu}$ to the "time-like photon states" give

$$\frac{1}{q^2}\left(W_{\mu\nu} q_\mu q_\nu\right) \sim \varphi_0 \cdot (d - 2\xi) \cdot (\omega)^{\beta} \cdot (-q^2)^{d-2\xi}$$

which has to be identical zero. One of the possibility is cancellation of d and $2h$

The same consideration leads also to

$$\frac{\sigma_L}{\sigma_T} \to const \approx g_0^2 \qquad (g_0^2 \ll 1)$$

The experimental value of this ratio 0.15 serves as one more argument in favour of small g_0 .

Note in conclusion that the same mechanism leads for the deep inelastic ee- and $e\tilde{e}$- scattering [13] (fig. 5) in the limit of large S', q_1^2 and q_2^2 to the

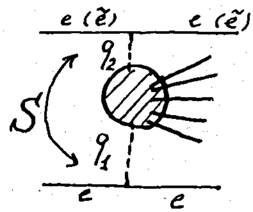

Fig.5

dependence on the dimensional parameter

$$W_{ac} = F\left(\frac{S'}{q_1^2 q_2^2}\right) \qquad ,$$

where a, c are L- or T-states of the photons.

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