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ANALYSIS OF THE $K\pi$ -INTERACTION
IN THE ENERGY REGION UP TO 1 GEV

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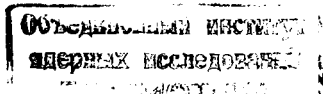
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Submitted to ЯФ



1. Introduction.

Dispersion relation approach is one of the most fundamental methods for exploring the strong interactions of elementary particles. In the present paper the dispersion relations for S and P waves obtained in ref.^{1/} are exploited in order to investigate the $K\pi$ interactions in energy region $\lesssim 1$ GeV.

Experimental data analyzed here are taken from the review talk^{1/2/}. The work lists $\delta_0^{1/2}$, $\delta_0^{3/2}$ and $\delta_1^{1/2}$ phase shifts of the $K\pi$ -scattering obtained from an analysis of experimental data on the K^+N scattering.

The phase shift analysis was carried out under the following assumptions^{1/2/}:

- a) Only S and P waves contribute to the cross sections for energies $\lesssim 1.1$ GeV (this assumption was then checked by a subsidiary analysis involving d waves).
- b) Below 1.1 GeV no inelastic channels contribute.
- c) The phase shift $\delta_1^{3/2}$ is negligible in the energy region from threshold to 1.1 GeV.
- d) In the energy region under consideration the phase $\delta_1^{1/2}$ is determined well via the Breit-Wigner formula (with resonance situated at the point 891 MeV

and with the width 50 MeV i.e. the K^* resonance). Four solutions have been obtained for $\delta_c^{1/2}$ ("down", "up-down", "small" and "up-small") and two solutions have been obtained for $\delta_c^{3/2}$ ("gentle" corresponding to the "down" and "up-down" solutions $\delta_0^{1/2}$ and "large" corresponding to "small" and "up-small" solutions for $\delta_0^{1/2}$). However the "small" and "up-small" solutions for $\delta_0^{1/2}$ and "large" solutions for $\delta_0^{3/2}$ give the very large values for cross sections of the reactions $K^+\pi^+ \rightarrow K^+\pi^+$ and $K^-\pi^- \rightarrow K^-\pi^-$ (i.e. 30 mb instead of the expected 2-3 mb) and thus it can be neglected. The remaining two solutions for the $\delta_0^{1/2}$ phase shift and "gentle" for the $\delta_c^{3/2}$ phase shift are shown in Figs. 1,2. These phases together with the Breit-Wigner $\delta_i^{1/2}$ -phase shift (see Fig.3.) have been employed here, to verify their self-consistency using the dispersion relations for $T_c^{1/2}$, $T_0^{3/2}$ and $T_1^{1/2}$ -partial waves taken from the work¹⁾:

$$T_e^T(\omega) = \frac{p}{\pi} \int_1^\infty \frac{\text{Im} T_e^T(\omega')}{\omega' - \omega} d\omega' + \frac{1}{3\pi i} \int_1^\infty \frac{\Phi_e^T(\omega')}{\omega' + \omega} d\omega' + \Pi_e^T(\omega) + \quad (1)$$

$$+ N_e \int_c^\infty \left\{ \sum_{\substack{i=0,1 \\ \kappa=1/2, 3/2}} \text{Im} T_i^\kappa(\omega') \cdot \mathcal{F}_{(e,T)}^{(i,\kappa)}(\omega', \omega) \right\} d\omega'$$

where $\ell = 0, 1$; $T = \frac{1}{2}, \frac{3}{2}$; $T_e^T(\omega) = \frac{1}{q} e^{i\delta_e^T(\omega)} \sin \delta_e^T(\omega)$
 and $\omega = \sqrt{\nu + \mu^2}$ the total energy of pion in the
 c.m.s. of πK -system; $\mu = 1$; $\nu = q^2 -$
 the 3-momentum of pion squared; $N_0 = \frac{1}{2\pi}$; $N_1 = \frac{1}{6\pi}$;

$$\Phi_e^{1/2}(\omega') = 4 \text{Im} T_e^{3/2}(\omega') - \text{Im} T_e^{1/2}(\omega'),$$

$$\Phi_e^{3/2}(\omega') = 2 \text{Im} T_e^{1/2}(\omega') + \text{Im} T_e^{3/2}(\omega')$$

$\Pi_e^T(\omega)$ is the sum of the pole contributions from ρ - and σ -mesons (the σ -meson is the resonance state in the $\pi\pi$ -system with the isospin $T=0$ and $\ell=0$) and from the pole which describes the high-energy contribution. Explicit functions $\Pi_e^T(\omega)$ and $F_{(e,T)}^{(i,K)}(\omega', \omega)$ are presented in the work¹⁾. We have calculated the real and imaginary parts of the $T_0^{1/2}$, $T_0^{3/2}$ and $T_1^{1/2}$ amplitudes from the experimental phase shifts given in work²⁾. The imaginary parts of partial amplitudes obtained in this way were inserted into the dispersion relations (1), and then the theoretical values for the real parts of the amplitudes $T_0^{1/2}$, $T_0^{3/2}$ and $T_1^{1/2}$

were computed. Next, the ratio

$$R = \frac{\text{Re } T_e^T (\text{theoretical})}{\text{Re } T_e^T (\text{experimental})} = 1$$

was checked in the energy interval from threshold up to 1 GeV. We did not pursue to get the exact value of the ratio $R = 1$ so far as this is not required by the current situation in the $K\pi$ -interaction.

As our calculation indicate, the ratio $= 1$ can be in principle achieved for the whole energy interval under consideration. We confine ourselves to 20% accuracy for large $S_0^{1/2}$ and $P^{1/2}$ waves and to 50% accuracy for small $S_0^{3/2}$ -wave. An examination has shown that the $\delta_1^{3/2}$ phase shift remains negligible (smaller than 1°) throughout. In analyzing the $K\pi$ interaction, an attention has been drawn to:

- 1) the possibility of theoretical selection of one from the two investigated solutions for the $\delta_0^{1/2}$ phase shift;
- 2) validity of the Breit-Wigner approximation in the vicinity of threshold and as a possible consequence, to a rather unnatural growth of the $\delta_0^{1/2}$ and $\delta_0^{3/2}$ phase shifts within the same energy domain (see experimental points in Fig.3 in the energy region $\lesssim 800$ MeV).

We have not succeeded in finding strict arguments against one of the solutions for $\delta_c^{1/2}$ (namely, the "up-down" solution) though objections contained in the work are convincing enough.

We think that in the vicinity of threshold the Breit-Wigner approximation gives the value of the $\delta_1^{1/2}$ phase shift around two times smaller. An increasing of $\delta_1^{1/2}$ contribution near threshold would decrease in an appropriate way the $\delta_0^{1/2}$ and $\delta_0^{3/2}$ phase shifts, and possibly would smooth out the unnatural growth of those phases in the energy region $\lesssim 800$ MeV.

2. Scattering Lengths and Coupling Constants.

The dispersion relations (1) include three parameters: two products of coupling constants $g_{\pi\bar{\pi}p}g_{\rho k\bar{k}}$ and $g_{\pi\lambda\sigma}g_{\sigma k\bar{k}}$ and the constant $\chi^{(+)}$ describing the high-energy contribution, $\chi^{(+)} > 0$. These parameters are contained in the pole terms Π_e^T only. The parameters should be chosen in such a way that it would be possible to get reasonable values for the scattering lengths $a_0^{1/2}$ and $a_c^{3/2}$, in addition to consistency of the energy behaviour of the $\delta_0^{1/2}$, $\delta_0^{3/2}$ and $\delta_1^{1/2}$ phase shifts.

As our calculations show, the values of the integrals from imaginary parts of the $T_0^{1/2}, T_0^{3/2}, T_1^{1/2}$ partial amplitudes depend weakly on the choice of the scattering length. Therefore one can obtain bound on the scattering lengths and on the above-mentioned three parameters, computing values for all dispersion integrals in the point $\omega = 1$.

As a result of such calculations, the following system of equations (or sum rules) has been obtained for "down" $\delta_0^{1/2}$ -solution:

$$a_0^{1/2} - 0,116 = \frac{1}{2} \left(\frac{g_{\pi\pi\rho} g_{\rho\kappa\bar{\kappa}}}{4(v_\rho+1)} + \frac{g_{\pi\pi\sigma} g_{\sigma\kappa\bar{\kappa}}}{4\sqrt{6}(v_\sigma+1)} - \lambda^{(+)} \right)$$

$$a_0^{3/2} - 0,083 = \frac{1}{2} \left(-\frac{g_{\pi\pi\rho} g_{\rho\kappa\bar{\kappa}}}{8(v_\rho+1)} + \frac{g_{\pi\pi\sigma} g_{\sigma\kappa\bar{\kappa}}}{4\sqrt{6}(v_\sigma+1)} - \lambda^{(+)} \right) \quad (2)$$

$$0,048 = \frac{1}{6} \left(\frac{g_{\pi\pi\rho} g_{\rho\kappa\bar{\kappa}}}{4(v_\rho+1)} + \frac{g_{\pi\pi\sigma} g_{\sigma\kappa\bar{\kappa}}}{4\sqrt{6}(v_\sigma+1)} + \lambda^{(+)} \right).$$

Here v_σ and v_ρ are parameters determining the position of σ and ρ resonances via the formula $M_{\sigma,\rho}^2 = 4(v_{\sigma,\rho} + 1)$.

In the system (2) the members in the left-hand side are correct within an accuracy up to 15-20%.

By solving the system (2) we obtain the following relations and restrictions:

$$a) \lambda^{(+)} = 0,26 - a_0^{1/2};$$

$$b) \frac{3}{16} \frac{g_{\pi\pi\rho} g_{\rho\kappa\bar{\kappa}}}{v_p + 1} = a_0^{1/2} - a_0^{3/2} - 0,033; \quad (3)$$

$$c) \frac{g_{\pi\pi\sigma} g_{\sigma\kappa\bar{\kappa}}}{4\sqrt{6} (v_\sigma + 1)} = a_0^{1/2} + 0,028 - \frac{g_{\pi\pi\rho} g_{\rho\kappa\bar{\kappa}}}{4(v_p + 1)}$$

(All the constants in (1) and (2) are normalized to the factor 4π).

Now, choosing an interval of the allowed values $1 \leq g_{\pi\pi\rho} g_{\rho\kappa\bar{\kappa}} \leq 4$ and taking $v_p = 6,5$ (which corresponds to the position of the ρ -resonance at the point $M_\rho = 760$ MeV) and $v_\sigma = 9$ (the position of the σ resonance at the point $M_\sigma = 890$ MeV), we find the corresponding interval for difference of the scattering lengths:

$$0,06 \leq a_0^{1/2} - a_0^{3/2} \leq 0,13 \quad (4)$$

Supposing $a_0^{1/2} > 0$ and $a_0^{3/2} < 0$ we obtain

$$0 \leq a_0^{1/2} \leq 0,13; \quad -0,13 \leq a_0^{3/2} < 0;$$

$$0,26 \geq \lambda^{(+)} \geq 0,13; \quad g_{\lambda\lambda\delta} g_{\sigma\kappa\bar{\kappa}} \leq 2,7. \quad (5)$$

The system (2) for the "up-down" solution of the phase shift has not been analyzed for the reasons we shall speak later, in the following section. The quantity $g_{\lambda\lambda\delta} g_{\sigma\kappa\bar{\kappa}}$ turned out to be the most sensitive to small variations of other parameters. This quantity becomes negative in a number of cases. Since we want to preserve its value equal to about unity and positive we have taken the following values for the scattering lengths:

$$a_c^{1/2} = 0,08, \quad a_c^{3/2} = -0,02. \quad (6)$$

3. Consistency of Phases of the $K\pi$ Scattering and Choice of Parameters of the Problem.

An analysis of the "up-down" solution for $\delta_0^{3/2}$ was carried out for three different variants of the phase shift behaviour in the high-energy region

(see Figs. 4a, b, c). In (1) the contributions from $\text{Im } T_1^{3/2}$ have been neglected. For variant 4a) the values for contributions of the individual terms at the point $\omega = 1$ from the dispersion relations (1) for $\text{Re } T_0^{1/2}$ are as follows:

$$\begin{aligned}
 \text{the first integral term } (J_1) &= 0.180 \\
 \text{the second integral term } (J_2) &= 0.015 \\
 \text{the pole term } (\Pi) &= 0.123 \\
 \text{the third integral term } (J_3) &= 0.008 \\
 \text{the scattering length } a_0^{1/2} &= 0.08
 \end{aligned} \tag{7}$$

(Contributions from analogous terms for variant 4b) differ slightly from (7)). For the values of coupling constants $g_{\pi\pi\rho} g_{\rho K\bar{K}} = 3,6$ and $g_{\pi\pi\sigma} g_{\sigma K\bar{K}} = 1$ it is possible to determine the contribution $\lambda^{(*)}$ from the pole term:

$$0,123 = \frac{1}{2} \left(- \frac{g_{\pi\pi\rho} g_{\rho K\bar{K}}}{4(v_p+1)} - \frac{g_{\pi\pi\sigma} g_{\sigma K\bar{K}}}{4\sqrt{6}(v_s+1)} + \lambda^{(*)} \right) \tag{8}$$

Hence, (at $v_p = 6,5$ and $v_s = 9,0$) $\lambda^{(*)} = 0,38$ follows. Comparing the value $\lambda^{(*)} = 0,38$ with any value from (7) we see that the $\lambda^{(*)}$ term contribution dominates over the contributions not only from the ρ and σ mesons but also from the resonance $\delta_0^{1/2}$ phase shift (J_1 term).

In variant 4c) the $\lambda^{(+)}$ term contributes more than in the previous two variants. The contribution from far singularities to the scattering length might be of course considerable but have not to be definitely large. The magnitude of the $\lambda^{(+)}$ contribution depends mainly on a difference of values of J_1 and the scattering length. If the $\delta_0^{1/2}$ phase is taken to be resonant and the resonance point not far from a threshold then the quantity J_1 is always several times larger than the scattering length, and this difference can be compensated only through large values of $\lambda^{(+)}$. Thus, the choice of the resonance "up-down" solution for $\delta_0^{1/2}$ results in too large contribution from far singularities to the low-energy region. The large values of $\lambda^{(+)}$ complicate considerably a selection of such $\delta_0^{1/2}$, $\delta_0^{3/2}$ and $\delta_1^{1/2}$ that $R = 1$ throughout the energy domain $\lesssim 1$ GeV. We have not succeeded in fitting such phases for the "up-down" solution of the $\delta_0^{1/2}$ phase shift.

For this reason we consider the "up-down" solution for $\delta_0^{1/2}$ to be unacceptable, from theoretical point of view, and therefore we did not analyze the system of equations (2) corresponding to the "up-down" $\delta_0^{1/2}$ solution.

For the "down" $\delta_0^{1/2}$ solution there have been found the variants of $\delta_c^{1/2}$, $\delta_0^{3/2}$ and $\delta_1^{1/2}$ which ensure $R = 1$ to be fulfilled for all three phases simultaneously through a wide energy interval (from 640 MeV up to 1000 MeV).

These variants have been obtained by varying the experimental phase shifts²⁾.

Tables 1, 2 and 3 list values of J_1, J_2, J_3, Π and R for three partial amplitudes $T_0^{1/2}$, $T_0^{3/2}$ and $T_1^{1/2}$ which are determined by phase shifts presented in Figs 1, 2, 3 (solid lines). Parameters take the following values:

$$g_{\pi\pi\rho} g_{\rho K\bar{K}} = 3,11;$$

$$g_{\pi\pi\sigma} g_{\sigma K\bar{K}} = 1,18; \lambda^{(4)} = 0,138; a_c^{1/2} = 0,08; a_0^{3/2} = -0,02.$$

For the "experimental" curves (dotted lines in Figs. 1, 2, 3) the ratio R changes with increasing energy unacceptably (see Tables 1, 2, 3 column R (dotted)).

To preserve the ratio $R \approx 1$ the phase shift $\delta_0^{1/2}$ should be deformed in the way shown in Fig. 1 (i.e. should be decreased in the energy region $\lesssim 900$ MeV).

Relatively small values of $\delta_0^{3/2}$ (as compared with $\delta_c^{1/2}$) are obtained because all but one J_1 contributions to $\text{Re } T_0^{3/2}$ computed by using the dispersion relations (1) are slowly varying functions of

ω and they are relatively large compared with J_1 (as it can be seen from Table 2). Therefore the experimental values of $\delta_0^{3/2}$ cannot change rapidly otherwise the ratio R will turn out to be smaller than unity in that energy region in which the fast growth of the phase shift will occur (see dotted line in Fig. 2 and the corresponding numbers in Table 2, column $R_{(dot)}$). For this reason it is necessary to take the $\delta_0^{3/2}$ phase shift of the slow growth.

The partial wave $T_1^{1/2}$, which is described by the Breit-Wigner formula corresponding to the K^* meson of mass $M_{K^*} = 891$ MeV and width $\Gamma_{K^*} = 50$ MeV, satisfies well the ratio $R = 1$ throughout all energies except for the vicinity of threshold (see Table 3, column $R_{(dotted)}$). The increase of the Breit-Wigner curve at the threshold around two times does not change in practice the value of J_1 ^{but does} increase $Re T_1^{1/2}(exp)$ approximately twice which makes it possible to correct the ratio R at the point $\omega = 1.31$ (see Table 3, columns R and $R_{(dotted)}$).

Thus, the set of $\delta_0^{1/2}$, $\delta_0^{3/2}$ and $\delta_1^{1/2}$ phase shifts given in Figs. 1, 2 and 3 through the solid lines, agrees very well with the dispersion relations (1).

$\delta_c^{1/2}$

4. Resonance Behaviour of the Phase Shift

in the Region $M(K\pi) > 1$ GeV.

The ratio R at $\omega = 3.21$ (which corresponds to the total energy of πK -system $M(K\pi) = 1120$ MeV) for the $T_0^{1/2}$ amplitude is equal to -0.092 . And at higher energies this ratio remains negative. If $\delta_0^{1/2}$ does not cross $\frac{\pi}{2}$ at the energies $M(K\pi) > 1$ GeV then the experimental value of $Re T_0^{1/2}(\omega)$ remains positive with increasing energy. However \Im_1 at an energy $M(K\pi) > 1$ GeV, as from eq.(1) for $T_0^{1/2}$ follows, changes necessarily its sign (becomes negative) and this occurs in the energy range $1.05 \lesssim M(K\pi) \lesssim 1.2$ GeV. Thus, at the energy $M(K\pi) > 1$ GeV and under the condition that the $\delta_0^{1/2}$ phase shift remains smaller than $\frac{\pi}{2}$, the quantity R becomes negative.

To conserve the quantity R positive and equal to unity it is necessary either a) to decrease strongly the $\delta_0^{1/2}$ phase shift in the domain $M(K\pi) < 1$ GeV and set it large and equal to about $\frac{\pi}{2}$, in a wide energy range $M(K\pi) > 1$ GeV; or b) to assume that in the region $1.05 \text{ GeV} \lesssim M(K\pi) \lesssim 1.2 \text{ GeV}$ the $\delta_0^{1/2}$ phase shift crosses $\frac{\pi}{2}$.

The assumption a) results in considerable disagreements with the $\delta_0^{1/2}$ phase shift data available in the low-energy region. In practice the only assumption remains that the $\delta_0^{1/2}$ phase shift possesses the resonant behaviour and the resonance is situated in the region $1.05 \text{ GeV} \lesssim M(K\pi) \lesssim 1.2 \text{ GeV}$. This conclusion agrees with the results of works^{1,3)}.

5. Conclusion.

In the work an analysis of the $K\pi$ scattering phase shifts has been performed in the energy region from 640 MeV up to 1 GeV. The analysis was performed by means of the dispersion relation method.

Our results are as follows:

1. Connections between parameters of the problem and their restrictions have been derived (see the relations (3) - (5)).

The following values for the parameters were employed:

$$\alpha_0^{1/2} = 0,08; \quad \alpha_0^{3/2} = -0,02; \quad g_{\pi\pi\rho} g_{\rho K\bar{K}} = 3,11; \quad g_{\pi\pi\sigma} g_{\sigma K\bar{K}} = 1,18; \\ \chi^{(+)} = 0,138.$$

2. "Theoretical" $\delta_0^{1/2}$, $\delta_0^{3/2}$ and $\delta_1^{1/2}$ phase shifts of the $K\pi$ scattering have been found which are

denoted by solid lines in Figs. 1, 2, 3. These agree well with dispersion relations.

3. An approximation of the threshold behaviour of the $\delta_1^{1/2}$ phase shift by the Breit-Wigner formula with the constant value of the width Γ_{κ^*} is not successful. If it is necessary to increase about twice a contribution from the $\rho^{1/2}$ wave near the threshold compared with that of Breit-Wigner. This results in a smoothing of unnaturally large values of the $\delta_0^{1/2}$ and $\delta_0^{3/2}$ phase shifts at the threshold. An analogous phenomenon must be observed also in analyzing the $\pi\bar{\pi}$ scattering phase shifts.

4. Theoretical arguments are given against the "up-down" solution for the $\delta_0^{1/2}$ phase shift.

5. In the energy region $1.05 \text{ GeV} \lesssim M(\pi\pi) \lesssim 1.2 \text{ GeV}$ the $\delta_0^{1/2}$ phase shift has the resonance behaviour.

Table 1 The value of J_1, J_2, J_3, Π and R for $T_0^{1/2}$ amplitude.

ω	M(kx) Gev	J_1	J_2	J_3	Π	R	$R_{(dott)}$
							(for dotted curve Fig. 1)
1.01	0.64	0,0833	-0,0121	0,0156	-0,0114	1,00	1,00
1.31	0,69	0,103	-0,0113	0,0153	-0,0011	1,13	0,947
1.91	0,82	0,156	-0,0099	0,0147	+0,0078	1,18	0,829
2.33	0,91	0,209	-0,0093	0,0143	+0,0113	1,12	0,715
2.91	1,05	0,100	-0,0093	0,0138	+0,0100	0,97	0,323

Table 2. The value of J_1, J_2, J_3, Π , and R for $T_0^{3/2}$ -amplitude.

ω	M(kx) Gev	J_1	J_2	J_3	Π	R	$R_{(dott)}$
							(for dotted curve Fig. 2)
1,01	0,64	0,00393	0,0278	0,0334	-0,0892	1	0,89
1,31	0,69	0,00516	0,0259	0,0312	-0,0935	0,7	0,43
1,91	0,82	0,00310	0,0228	0,0268	-0,101	0,65	0,14
2,33	0,91	-0,000765	0,0211	0,0238	-0,103	1,03	0,11
2,91	1,05	-0,00307	0,0192	0,0201	-0,103	1,7	0,32

Table 3. The value \mathcal{J}_1 , \mathcal{J}_2 , \mathcal{J}_3 , Π , and R
for $\mathcal{T}_1^{1/2}$ -amplitude.

ω	$M(K\pi)$ Gev	\mathcal{J}_1	\mathcal{J}_2	\mathcal{J}_3	Π	R	$R_{(dott)}$ (for dotted curve Fig. 3)
1,01	0,64	0,0394	-0,0056	0,0086	-0,0422	I	I
1,31	0,69	0,0637	-0,0052	0,0083	-0,0454	I,14	I,91
1,91	0,82	0,153	-0,0046	0,0078	-0,0489	0,94	I,09
2,33	0,91	-0,132	-0,0043	0,0068	-0,0497	I,03	0,955
2,91	1,01	-0,0668	-0,0034	0,0070	-0,0493	0,99	0,90

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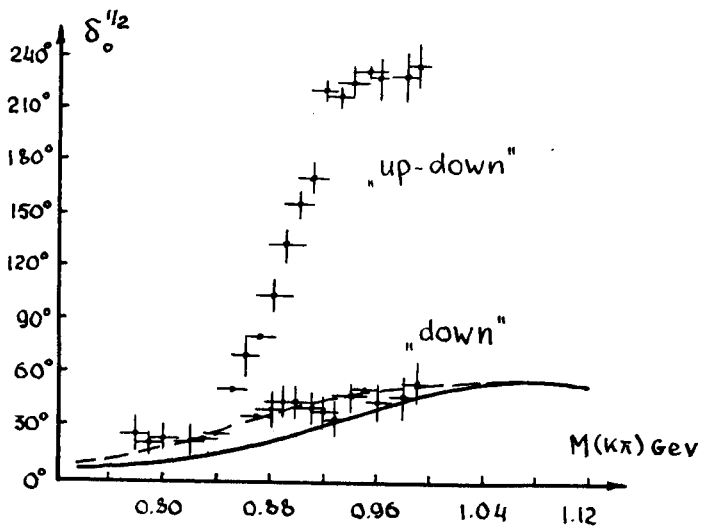


Fig. 1.

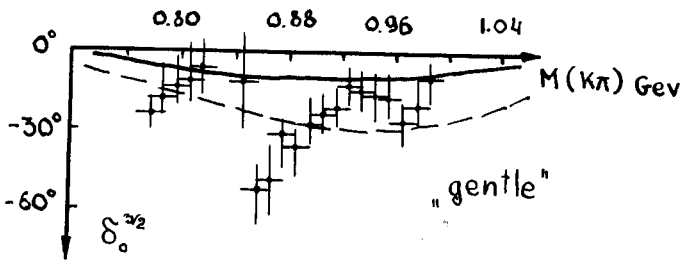


Fig. 2.

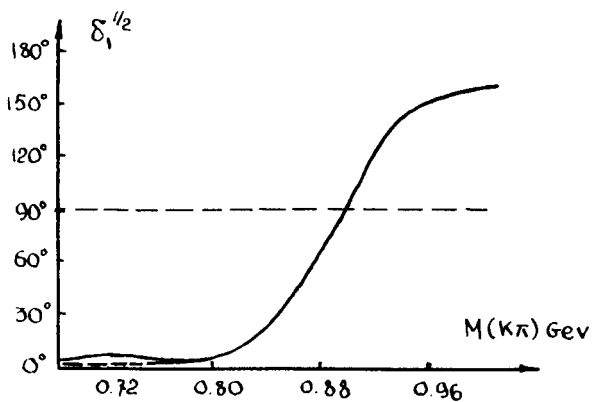


Fig. 3.

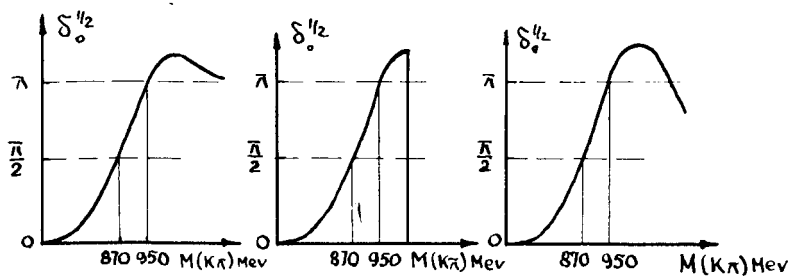


Fig. 4.