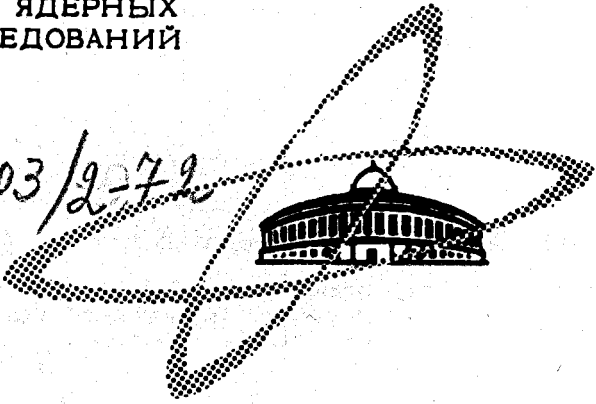


4/ix-72

К-92  
ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ

Дубна.

3003/2-72



E2 - 6499

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

Z.Kunszt , V.M.Ter-Antonyan

POSITIVITY RESTRICTIONS  
ON THE IMAGINARY PART  
OF THE FORWARD PHOTON-PHOTON  
SCATTERING AMPLITUDES

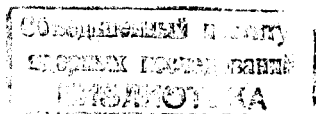
1972

E2 - 6499

**Z.Kunszt\*, V.M.Ter-Antonyan**

**POSITIVITY RESTRICTIONS  
ON THE IMAGINARY PART  
OF THE FORWARD PHOTON-PHOTON  
SCATTERING AMPLITUDES**

*Submitted to Lettere al Nuovo Cimento*



---

\* On leave from the Institute of Atomic Physics,  
Eotvos University, Budapest

It is well known that measuring the processes

$$e e \rightarrow e e (\mu \mu) + \text{hadrons} \quad (1)$$

we can study the imaginary part of the forward photon-photon scattering amplitude <sup>/1,2,3/</sup> This imaginary part of the forward photon-photon scattering amplitude can be given as a symmetric, gauge-invariant Lorentz tensor  $W^{\mu\nu\lambda\sigma}(q_1, q_2)$  which (assuming  $P, T, PT$  invariance) can be expanded in terms of 8 invariant functions ( $q_i^2 \neq 0$ , see /1/ and /4/).

$$W^{\mu\nu\mu'\nu'}(q_1, q_2) = \sum_a E_a^{\mu\nu\mu'\nu'}(q_1, q_2) W_a(q_1^2, q_2^2, q_1 q_2) \quad (2)$$

where  $q_1(q_2)$  is the momentum,  $\mu$  and  $\mu'(\nu, \nu')$  are the Lorentz indices of the first (second) virtual photon.

From the positivity of the cross sections we obtain that

$$c_{\mu\nu}^* W^{\mu\nu\lambda\sigma}(q_1, q_2) c_{\lambda\sigma} \geq 0 \quad (3)$$

for arbitrary choices of the coefficients  $c_{\mu\nu}$ . This implies that the  $W^{\mu\nu\lambda\sigma}$  matrices must have non-negative eigenvalues. Taking into account that the matrix  $W^{\mu\nu\lambda\sigma}$  is gauge-invariant we have to diagonalize only a 9x9 submatrix (the zero components can be eliminated).

Using the helicity basis <sup>/1,4/</sup>

$$E_{TT}^{\mu\nu\mu'\nu'} = R^{\mu\mu'} R^{\nu\nu'} \quad (4a)$$

$$E_{(TT)}^{\mu\nu\mu'\nu'} = \frac{1}{2} (R^{\mu\nu} R^{\mu'\nu'} + R^{\mu\nu'} R^{\mu\nu} - R^{\mu\mu'} R^{\nu\nu'}) \quad (4b)$$

$$E_{(TT)_a}^{\mu\nu\mu'\nu'} = R^{\mu\nu} R^{\mu'\nu'} - R^{\mu\nu'} R^{\mu'\nu}, \quad (4c)$$

$$E_{ST}^{\mu\nu\mu'\nu'} = \frac{Q_1^\mu Q_1^{\mu'}}{Q_1^2} R^{\nu\nu'},$$

$$E_{TS}^{\mu\nu\mu'\nu'} = \frac{Q_2^\nu Q_2^{\nu'}}{Q_2^2} R^{\mu\mu'}, \quad (4e)$$

$$E_{(TS)_r}^{\mu\nu\mu'\nu'} + E_{(TS)_a}^{\mu\nu\mu'\nu'} = \frac{1}{\sqrt{Q_1^2 Q_2^2}} (Q_1^\mu Q_2^{\nu'} R^{\mu\nu} + Q_1^\mu Q_2^\nu R^{\mu'\nu'}) , \quad (4f)$$

$$E_{(TS)_r}^{\mu\nu\mu'\nu'} - E_{(TS)_{ra}}^{\mu\nu\mu'\nu'} = \frac{1}{\sqrt{Q_1^2 Q_2^2}} (Q_1^\mu Q_2^{\nu'} R^{\mu'\nu} + Q_1^\mu Q_2^\nu R^{\mu\nu'}) , \quad (4g)$$

$$E_{SS}^{\mu\nu\mu'\nu'} = \frac{1}{Q_1^2 Q_2^2} Q_1^\mu Q_1^{\mu'} Q_2^\nu Q_2^{\nu'} , \quad (4h)$$

where

$$R^{\mu\nu} = -g^{\mu\nu} \frac{1}{l^2} [ q_1^\mu q_1^\nu q_2^2 + q_2^\mu q_2^\nu q_1^2 - \nu (q_1^\mu q_2^\nu + q_2^\mu q_1^\nu) ] , \quad (5a)$$

$$Q_1^\mu = q_2^\mu - \frac{\nu}{q_2^2} q_1^\mu , \quad Q_2^\nu = q_1^\nu - \frac{\nu}{q_1^2} q_2^\nu , \quad Q_i q_i = 0 , \quad (5b)$$

$$\nu = q_1 q_2 , \quad l^2 = \nu^2 - q_1^2 q_2^2 \quad (5c)$$

it is easy to diagonalize the matrix  $W^{ijkl}$ . We have obtained that the positivity restriction (3) gives the following and only the following inequalities

$$W_{TT} \geq \frac{1}{2} (W_{TT} + W_{TT}^a) \geq \frac{1}{2} |W_{TT}^r| , \quad (6a)$$

$$\epsilon W_{SS} (W_{TT} + W_{TT}^r + W_{TT}^a) > \epsilon (W_{TS}^r + W_{TS}^r)^2, \quad (6b)$$

$$\epsilon W_{TS} W_{ST} \geq \epsilon (W_{TS}^r - W_{TS}^r)^2, \quad (6c)$$

$$\text{sign } q_1^2 W_{ST} \leq 0, \quad \text{sign } q_2^2 W_{TS} \leq 0, \quad (6d)$$

where  $\epsilon = \text{sign } q_1^2 \cdot \text{sign } q_2^2$ .

We note the following:

i) The sign factors in (6b,c,d) appear because of the kinematic zeros of the structure functions as follows

$$W_{SS} \sim q_1^2 q_2^2 \bar{W}_{SS}, \quad W_{ST} \sim q_1^2 \bar{W}_{ST}, \quad W_{TS} \sim q_2^2 \bar{W}_{TS} \quad (7)$$

$$W_{TS}^r \sim \sqrt{q_1^2 q_2^2} W_{TS}^r.$$

For the process (1)  $\epsilon = 1$ .

ii) From the inequalities (6) we can derive the trivial conditions that  $W_{TT} > W_{TT}^a$  and  $\epsilon W_{SS} > 0$ .

iii) Measuring the process (1) with unpolarized electron beams, the structure functions  $W_{TT}^a$  and  $W_{TT}^r$  can not be determined, therefore in practice only the inequalities

$$W_{TT} > \frac{1}{2} |W_{TT}^r|, \quad (8a)$$

$$(W_{SS} W_{TT} + \frac{1}{2} W_{TS} W_{ST}) \geq (W_{TS}^r)^2 \quad (8b)$$

will be useful in estimating the size of the structure functions  $W_{TT}^r$  and  $W_{TS}^r$ .

iv) If the sea-gull contributions are important (see /4/), the inequality (6a) will saturate.

It is a pleasure to thank Prof. R.M.Muradyan for comments and one of the authors (K.Z.) is indebted to Dr. F.Niedermayer for a helpful discussion.

## References

1. V.E. Balakin, B.M. Budnev, I.F. Ginzburg. Pisma v Zurn. Eksp. Teor. Fiz., 11, 559 (1970).
2. S.J. Brodksly, T. Kinoshita, H. Terazawa. Phys.Rev.Lett., 25, 972 (1970);  
N.Arteago-Romero, A. Jaccarini, P. Kessler. College de France preprint (April, 1970).
3. Z. Kunszt, R.M. Muradyan, V.M. Ter-Antonyan. JINR E2-5347, Dubna, 1970.
4. Z. Kunszt, V.M. Ter-Antonyan. JINR E2-6257, Dubna, 1972..

*Received by Publishing Department  
on June 6, 1972.*