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POSITIVITY RESTRICTIONS ON THE IMAGINARY PART OF THE FORWARD PHOTON-PHOTON SCATTERING AMPLITUDES

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It is well known that measuring the processes

$$e\, e \to e\, e\, (\mu\, \mu) \; + \text{ hadrons} \tag{1}$$

we can study the imaginary part of the forward photon-photon scattering amplitude /1,2,3/. This imaginary part of the forward photon-photon scattering amplitude can be given as a symmetric, gauge-invariant Lorentz tensor $W^{\mu\nu\lambda\sigma}(q_1, q_2)$ which (assuming P, T, PT invariance) can be expanded in terms of 8 invariant functions ($q_i^2 \neq 0$, see /1/and /4/).

$$W^{\mu\nu\mu'\nu'}(q_1, q_2) = \sum_a E_a^{\mu\nu\mu'\nu'}(q_1, q_2)\, W_a(q_1^2, q_2^2, q_1 q_2), \tag{2}$$

where q_1 (q_2) is the momentum, μ and μ' (ν, ν') are the Lorentz indices of the first (second) virtual photon.

From the positivity of the cross sections we obtain that

$$c^*_{\mu\nu}\, W^{\mu\nu\lambda\sigma}(q_1, q_2)\, c_{\lambda\sigma} \geq 0 \tag{3}$$

for arbitrary choices of the coefficients $c_{\mu\nu}$. This implies that the $W^{\mu\nu\lambda\sigma}$ matrices must have non-negative eigenvalues. Taking into account that the matrix $W^{\mu\nu\lambda\sigma}$ is gauge-invariant we have to diagonalize only a 9x9 submatrix (the *zero* components can be eliminated).

Using the helicity basis /1,4/

$$E_{TT}^{\mu\nu\mu'\nu'} = R^{\mu\mu'} R^{\nu\nu'}, \tag{4a}$$

$$E_{(TT)_\tau}^{\mu\nu\mu'\nu'} = \frac{1}{2}\left(R^{\mu\nu}R^{\mu'\nu'} + R^{\mu\nu'}R^{\mu'\nu} - R^{\mu\mu'}R^{\nu\nu'}\right), \tag{4b}$$

$$E^{\mu\nu\mu'\nu'}_{(TT)_a} = R^{\mu\nu} R^{\mu'\nu'} - R^{\mu\nu'} R^{\mu'\nu} , \quad (4c)$$

$$E^{\mu\nu\mu'\nu'}_{ST} = \frac{Q_1^{\mu} Q_1^{\mu'}}{Q_1^2} R^{\nu\nu'} ,$$

$$E^{\mu\nu\mu'\nu'}_{TS} = \frac{Q_2^{\nu} Q_2^{\nu'}}{Q_2^2} R^{\mu\mu'} , \quad (4e)$$

$$E^{\mu\nu\mu'\nu'}_{(TS)_\tau} + E^{\mu\nu\mu'\nu'}_{(TS)_{\tau a}} = \frac{1}{\sqrt{Q_1^2 Q_2^2}} (Q_1^{\mu'} Q_2^{\nu'} R^{\mu\nu} + Q_1^{\mu} Q_2^{\nu} R^{\mu'\nu'}) , \quad (4f)$$

$$E^{\mu\nu\mu'\nu'}_{(TS)_\tau} - E^{\mu\nu\mu'\nu'}_{(TS)_{\tau a}} = \frac{1}{\sqrt{Q_1^2 Q_2^2}} (Q_1^{\mu} Q_2^{\nu'} R^{\mu'\nu} + Q_1^{\mu'} Q_2^{\nu} R^{\mu\nu'}) , \quad (4g)$$

$$E^{\mu\nu\mu'\nu'}_{SS} = \frac{1}{Q_1^2 Q_2^2} Q_1^{\mu} Q_1^{\mu'} Q_2^{\nu} Q_2^{\nu'} , \quad (4h)$$

where

$$R^{\mu\nu} = - g^{\mu\nu} - \frac{1}{I^2} [q_1^{\mu} q_1^{\nu} q_2^2 + q_2^{\mu} q_2^{\nu} q_1^2 - \nu (q_1^{\mu} q_2^{\nu} + q_2^{\mu} q_1^{\nu})] , \quad (5a)$$

$$Q_1^{\mu} = q_2^{\mu} - \frac{\nu}{q_1^2} q_1^{\mu} , \quad Q_2^{\nu} = q_1^{\nu} - \frac{\nu}{q_2^2} q_2^{\nu} , \quad Q_i \, q_i = 0 , \quad (5b)$$

$$\nu = q_1 q_2 , \quad I^2 = \nu^2 - q_1^2 q_2^2 \quad (5c)$$

it is easy to diagonalize the matrix $W^{ijk\ell}$. We have obtained that the positivity restriction (3) gives the following and only the following inequalities

$$W_{TT} \geq \frac{1}{2} (W_{TT} + W^a_{TT}) \geq \frac{1}{2} | W^{\tau}_{TT} | , \quad (6a)$$

$$\epsilon W_{SS} \left(W_{TT} + W^{\tau}_{TT} + W^{a}_{TT} \right) \geq \epsilon \left(W^{\tau a}_{TS} + W^{\tau}_{TS} \right)^2 , \tag{6b}$$

$$\epsilon W_{TS} W_{ST} \geq \epsilon \left(W^{\tau a}_{TS} - W^{\tau}_{TS} \right)^2 , \tag{6c}$$

$$\mathrm{sign}\; q_1^2 \; W_{ST} \leq 0 , \qquad \mathrm{sign}\; q_2^2 \; W_{TS} \leq 0 , \tag{6d}$$

where $\epsilon = \mathrm{sign}\; q_1^2 \cdot \mathrm{sign}\; q_2^2$.

We note the following:

i) The sign factors in (6b,c,d) appear because of the kinematic zeros of the structure functions as follows

$$W_{SS} \sim q_1^2 q_2^2 \bar{W}_{SS} , \qquad W_{ST} \sim q_1^2 \bar{W}_{ST} , \qquad W_{TS} \sim q_2^2 \bar{W}_{TS}$$
$$W^{\tau a}_{TS} \sim \sqrt{q_1^2 q_2^2} \; W^{\tau a}_{TS} . \tag{7}$$

For the process (1) $\epsilon = 1$.

ii) From the inequalities (6) we can derive the trivial conditions that $W_{TT} > W^{a}_{TT}$ and $\epsilon W_{SS} > 0$.

iii) Measuring the process (1) with unpolarized electron beams, the structure functions W^{a}_{TT} and $W^{\tau a}_{TT}$ can not be determined, therefore in practice only the inequalities

$$W_{TT} \geq \frac{1}{2} \left| W^{\tau}_{TT} \right| , \tag{8a}$$

$$\left(W_{SS} W_{TT} + \frac{1}{2} W_{TS} W_{ST} \right) \geq \left(W^{\tau}_{TS} \right)^2 \tag{8b}$$

will be useful in estimating the size of the structure functions W^{τ}_{TT} and W^{τ}_{TS} .

iv) If the sea-gull contributions are important (see /4/), the inequality (6a) will saturate.

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