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$\eta \to 3\pi$ DECAY AND $\pi\eta$ - SCATTERING IN CHIRAL $SU_2 \times SU_2$ SYMMETRY

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Иванов Е.А., Зупник Б.М. **E2 - 6472**

Распад $\eta \to 3\pi$ и $\pi\eta$ - рассеяние в киральной $SU_2 \times SU_2$ симметрии

Проводится анализ проблемы распада $\eta \to 3\pi$ в киральной $SU_2 \times SU_2$ симметрии. Показано, что известные трудности обычной полюсной модели связаны с недооценкой роли графика с η-мезонным полюсом. Такая модель оказывается пригодной для описания распада $\eta \to 3\pi$: при разумном выборе константы $\pi^0 - \eta$ перехода $|g_{\pi\eta}| \sim 0{,}05$ удается удовлетворить экспериментальным данным как по ширинам, так и по энергетическому спектру распада с помощью параметра вершины $\pi\eta\pi\eta$-взаимодействия в графике с η-полюсом. Наш анализ позволяет получить предсказания для s- и p-волновых длин $\pi\eta$-рассеяния:

$$|a^0_{\pi\eta}| \sim 0{,}2\, m_\pi^{-1}\ ; \qquad |a^1_{\pi\eta}| \sim 0{,}1\, m_\pi^{-3}\ .$$

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Ivanov E.A., Zupnik B.M. **E2 - 6472**

$\eta \to 3\pi$ Decay and $\pi\eta$-Scattering in Chiral $SU_2 \times SU_2$ Symmetry

The problem of the $\eta \to 3\pi$ decay in chiral $SU_2 \times SU_2$ symmetry is investigated. We argue that the well-known difficulties of conventional pole model originate from underestimating the role of the η-meson pole graph. This model is shown to be suitable to describe the $\eta \to 3\pi$ decay: at the reasonable choice of the $\pi^0 - \eta$ transition constant $|g_{\pi\eta}| \sim 0.05$ the experimental data on both decay widths and energy spectrum can be satisfied by fitting the parameter of $\pi\eta\pi\eta$-vertex in the η-pole graph. Our analysis leads to predictions for the s-and p-wave $\pi\eta$-scattering lengths

$$|a^0_{\pi\eta}| \sim 0.2\, m_\pi^{-1}\ ; \qquad |a^1_{\pi\eta}| \sim 0.1\, m_\pi^{-3}\ .$$

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$\eta \to 3\pi$ DECAY AND $\pi\eta$ - SCATTERING IN CHIRAL $SU_2 \times SU_2$ SYMMETRY

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1. Introduction

The conventional approach to the current algebra description of the $\eta \to 3\pi$ decay is based on the following natural assumptions /1-5/:

a) the process is caused by the conventional electromagnetic interaction;

b) the on-mass-shell decay amplitude depends linearly on energy variables.

Sutherland /1/ has shown that under these assumptions the $\eta \to 3\pi$ decay is forbidden in the $SU_2 \times SU_2$ symmetry limit $m_\pi = 0$. Dolgov et al. /2/ and Bardeen et al. /3/ have considered the pole model for the $\eta \to 3\pi$ decay and have argued that this process can only proceed via the $SU_2 \times SU_2$ symmetry violation induced by nonzero mass of pion. They have obtained the prediction for the Dalitz plot distribution in the $\eta \to \pi^+\pi^-\pi^0$ decay in agreement with experiment. However, within the framework of standard scheme all attempts to derive the satisfactory results for the $\eta \to 3\pi$ decay rates were unsuccessful[2-5]. In this connection different suggestions have been made to take the alternative form for the $\eta \to 3\pi$ interaction [6,7] or to modify the concept of PCAC smoothness[8].

The purpose of the present paper is to show that difficulties of the $\eta \to 3\pi$ decay description in the $SU_2 \times SU_2$ current algebra are not of principle; these are because of underestimating the role of the η-meson pole diagram. In fact, the careful analysis indicates that the conventional pole model based on the broken chiral $SU_2 \times SU_2$ symmetry can describe the $\eta \to 3\pi$ decay sufficiently well.

In our analysis the effective Lagrangian method is used. The pole amplitude for $\eta \to 3\pi$ involves the contributions of the contact $\eta \to 3\pi$ graph and of the graphs with π^0 and η-meson poles. The amplitude is proportional to the $\pi^0 - \eta$ transition constant $g_{\pi\eta}$ which can be evaluated through U-spin invariance [2-5]. Note that in the framework of electromagnetic interaction with conventional transformation properties under $SU_2 \times SU_2$ the $\pi^0 - \eta$ transition is described by the effective coupling with derivatives[3] $g_{\pi\eta} \partial_\mu \pi^0 \partial_\mu \eta$. Therefore it is natural to apply the "current-mixing" model for the electromagnetic SU_3 -breaking to evaluate $g_{\pi\eta}$ with respect to U-spin (instead of the "mass-mixing" model usually used). The "current-mixing" model gives the reasonable value $g_{\pi\eta} \approx -0.05$.

We don't assume the contribution of the η-pole graph to be negligible a priori. In our treatment this contribution is determined from the requirement that the pole model should be consistent with the experimental data on the $\eta \to 3\pi$ decay. The η-pole graph contains the strong vertex of $\pi\eta\pi\eta$-interaction which is characterised under $SU_2 \times SU_2$ by one coupling constant q. This constant serves as a parameter. It turns out that both decay widths and the slope of the Dalitz plot for the $\eta \to \pi^+ \pi^- \pi^0$ decay can be fitted in good agreement with experiment by single parameter q. Taking the observable width of $\eta \to 3\pi^0$ as input, we obtain, because of uncertainty in sign of decay amplitude, two alternative values for q :

1) $q = 6 \pm 0.6$; 2) $q = -4.7 \pm 0.6$.

Correspondingly, we have two values for the slope a:

1) $a \approx -0.59$; 2) $a \approx -0.57$.

which agree well with the latest experimental data /9/ $a^{exp} = -0.58 \pm 0.01$.

The present approach leads to reasonable predictions for the parameters of the low-energy $\pi\eta$-scattering amplitude. Using the above values of the $\pi\eta\pi\eta$-coupling constant q we find that the S-and P-wave $\pi\eta$-scattering lengths $a^0_{\pi\eta}$ and $a^1_{\pi\eta}$ must be of order of the $\pi\pi$-scattering lengths given by current algebra calculations:

1) $a^0_{\pi\eta} \approx 0.20\, m_\pi^{-1}$ 2) $a^0_{\pi\eta} \approx -0.16\, m_\pi^{-1}$

$a^1_{\pi\eta} \approx -0.07\, m_\pi^{-3}$ $a^1_{\pi\eta} \approx 0.05\, m_\pi^{-3}$

2. The Pole Model for the $\eta \to 3\pi$ Decay

In constructing the $\eta \to 3\pi$ amplitude we confine our consideration to the following diagrams /2,5/.

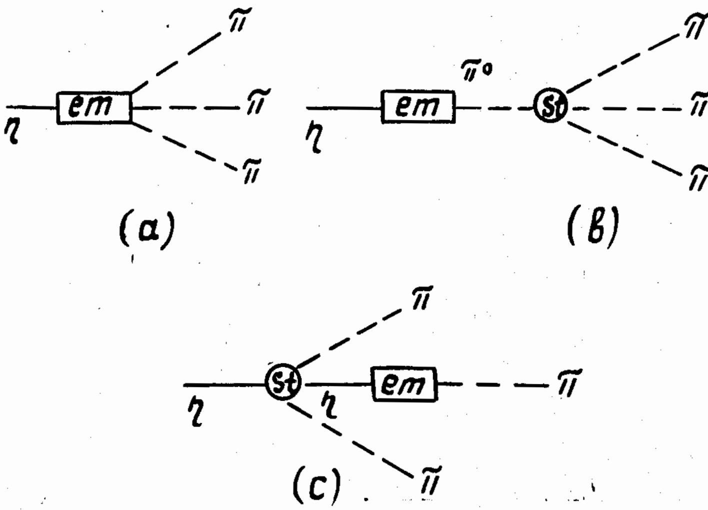

Fig. 1.(a). Contact graph in $\eta \to 3\pi$; (b) Pion pole graph in $\eta \to 3\pi$. (c) Eta pole graph in $\eta \to 3\pi$.

The strong interaction vertices in these graphs can be determined from the effective Lagrangian

$$\mathcal{L}_{st} = \mathcal{L}_0 + \mathcal{L}_{S.B.} = -\frac{1}{2}[(\partial_\mu \vec{\pi})^2 + (\partial_\mu \sigma)^2]\,(1-2qf_\pi^{-2}\eta^2) - \qquad (2.1)$$

$$-\frac{1}{2}(\partial_\mu \eta)^2 - \frac{1}{2} m_\eta^2 \eta^2 + m_\pi^2 f_\pi \sigma ,$$

where $\vec{\pi}$, η are fields of π -and η -mesons[x/], $\sigma = \sqrt{(f_\pi^2 - \vec{\pi}^2)}$, $f_\pi \approx 95$ MeV is the pion decay constant and q is the coupling constant, which characterizes the strong $\pi\eta$ -interaction. This Lagrangian contains the chiral invariant term $\mathfrak{L}_0$ and symmetry breaking term $\mathfrak{L}_{S.B.} = m_\pi^2 f_\pi \sigma$ which belongs to the representation /10/ $(1/2, 1/2)$ of $SU_2 \times SU_2$.

The vertices $\pi\pi\pi\pi$ and $\pi\eta\pi\eta$ are given by terms of the fourth order in fields in the Lagrangian $\mathfrak{L}_{st}$

$$\mathfrak{L}_{int}(\pi, \pi) = -\frac{1}{8} f_\pi^{-2} [\partial_\mu (\vec{\pi}^2) \partial_\mu (\vec{\pi}^2) + m_\pi^2 (\vec{\pi}^2)^2] , \tag{2.2}$$

$$\mathfrak{L}_{int}(\pi\eta) = q f_\pi^{-2} (\partial_\mu \vec{\pi})^2 \eta^2 . \tag{2.3}$$

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x/ The field $\vec{\pi}$ transforms in the following way /10/: $\delta\vec{\pi} = -\vec{\alpha} \times \vec{\pi} + \vec{\beta}\sqrt{(f_\pi^2 - \vec{\pi}^2)}$ where $\vec{\alpha}$ and $\vec{\beta}$ are parameters of $SU_2 \times SU_2$ transformations; η is the chiral scalar: $\delta\eta = 0$

The $\pi^0 - \eta$ and contact $\eta \to 3\pi$ transitions are described by the effective Lagrangian $\mathfrak{L}_{em}(\pi\eta)$ with transformation properties of the conventional electromagnetic interaction. The simplest $\mathfrak{L}_{em}(\pi\eta)$ takes the following form /3/

$$\mathfrak{L}_{em}(\pi,\eta) = g_{\pi\eta} f_\pi^{-1} \partial_\mu \eta J^3_{5\mu} = g_{\pi\eta} \partial_\mu \eta \partial_\mu \pi^0 + \tag{2.4}$$

$$+ \frac{1}{2} g_{\pi\eta} f_\pi^{-2} \partial_\mu \eta [\pi^0 \partial_\mu (\vec{\pi}^{\,2}) - \partial_\mu \pi^0 (\vec{\pi})^2] + \ldots$$

where $J^3_{5\mu}$ is the neutral component of the axial-vector current calculated from the Lagrangian (2.1).

In general, the parameter $g_{\pi\eta}$ of $\pi^0 - \eta$ transition must be of the order of e^2. By $SU(3)$ the $\pi^0 - \eta$ transition may be linked with the electromagnetic mass shift of π - and k-mesons. In the earlier calculations /2,5/ the $\pi^0 - \eta$ transition parameter $g_{\pi\eta}$ was estimated from the "mass-mixing" model for electromagnetic violation of $SU(3)$ symmetry. However, it should be remarked that the electromagnetic $\pi^0\eta$ -interaction (2.4) contains the field derivatives. Therefore, in our opinion, there should be used the model of "current-mixing" rather than "mass-mixing". The simple calculation in the "current-mixing" model gives us the following estimate for the $\pi^0 - \eta$ transition constant

$$g_{\pi\eta} = \frac{1}{\sqrt{3}} m_{\pi}^{2} (m_{\pi^+}^{-2} - m_{\pi^0}^{-2}) + \frac{1}{\sqrt{3}} m_{k}^{2} (m_{k^0}^{-2} - m_{k^+}^{-2}) \approx - 0.05 . \qquad (2.5)$$

The amplitudes of $\eta \to \pi^+ \pi^- \pi^0$ and $\eta \to 3\pi^0$ decays ,involving the contributions of graphs (a), (b) and (c), are of the following form

$$\mathfrak{M}_{+-0} = g_{\pi\eta} f_{\pi}^{-2} \frac{m_{\pi}^{2} m_{\eta}^{2}}{m_{\eta}^{2} - m_{\pi}^{2}} [(1-2q)(1-\frac{2E_{\pi^0}}{m_{\eta}}) + 2q \frac{m_{\pi}^{2}}{m_{\eta}^{2}}] , \qquad (2.6)$$

$$\mathfrak{M}_{000} = g_{\pi\eta} f_{\pi}^{-2} \frac{m_{\pi}^{2} m_{\eta}^{2}}{m_{\eta}^{2} - m_{\pi}^{2}} [1 + 2q (3 \frac{m_{\pi}^{2}}{m_{\eta}^{2}} - 1)] , \qquad (2.7)$$

where E_{π^0} denotes the neutral pion energy. In these amplitudes, the terms proportional to the $\pi\eta$-scattering constant q are just the contribution of the graph with η -meson pole (Fig. 1c)[x/].

[x/] First the η-pole graph was considered in the work /2/.

From the matrix element $\mathfrak{M}_{+-0}$ we find a, the slope of distribution on the Dalitz plot for the $\eta \to \pi^+\pi^-\pi^0$ de - cay x/:

$$a = -2 \frac{m_\eta - m_{\pi^0} - m_{\pi^+} - m_{\pi^-}}{m_\eta} \left(1 + 4 \frac{m_{\pi^+} - m_{\pi^0}}{m_\eta} + 6 \frac{q}{2q-1} \frac{m_\pi^2}{m_\eta} \right)^{-1}. \quad (2.8)$$

It should be noted here that the amplitudes $\mathfrak{M}_{+-0}$ and $\mathfrak{M}_{000}$ are proportional to m_π and therefore tend to zero in the $SU_2 \times SU_2$ symmetry limit $m_\pi \to 0$ in accordance with the result by Sutherland /1/.

3. The $\eta \to 3\pi$ Decay and $\pi\eta$ -Scattering

In this section we show that the $\eta \to 3\pi$ decay can be described well enough in the standard pole model defined above. The crucial point is that the graph with η -pole gives the essential contribution to the $\eta \to 3\pi$ amplitude.

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x/ Our choice of coordinates on the Dalitz plot is the same as in works /9,11/: $\mathfrak{M}_{+-0} \propto 1 + a(T_0 / \bar{T} - 1)$ where T_0 is the kinetic energy of neutral pion; $\bar{T} = \frac{1}{3}(m_\eta - m_{\pi^0} - m_{\pi^+} - m_{\pi^-})$ is the average kinetic energy.

Expressions for both decay rates and slope (2.8) of the Dalitz plot distribution in mode $\eta \to \pi^+\pi^-\pi^0$ involve $\pi\eta\pi\eta$ -coupling constant q as a parameter. In order to fix this parameter more accurately one should use the experimental data on decay rates rather than those on the energy spectrum since the slope (2.8) depends on q weakly. We take the observable $\eta \to 3\pi^0$ decay width /12/ $\Gamma^{exp}(\eta \to 3\pi^0) = (0.80 \pm 0.21)$ keV as input and express the constant q in terms of the matrix element $\mathfrak{M}_{000}$ and the $\pi^0 - \eta$ transition parameter $g_{\pi\eta}$ making use of the formula (2.6):

$$q = 0.616 - 0.276\, g_{\pi\eta}^{-1}\, \mathfrak{M}_{000}. \tag{3.1}$$

It is $|\mathfrak{M}_{000}|^{exp} = 0.98 \pm 0.11$ that corresponds to the partial width $\Gamma^{exp}(\eta \to 3\pi^0)$. Using the estimate (2.5) for $g_{\pi\eta}$, we obtain the following values of the parameter q:

$$1.\ \mathfrak{M}_{000} > 0 \qquad q = 5.96 \pm 0.61 \tag{3.2}$$

$$2.\ \mathfrak{M}_{000} < 0 \qquad q = -4.72 \pm 0.61. \tag{3.3}$$

The theoretical ratio $r = \Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 1.6$ agrees with experiment /12/ so $\Gamma(\eta \to \pi^+\pi^-\pi^0)$ is also fitted well with above choice of the parameter q.

In verifying the applicability of the conventional pole model for description of the $\eta \to 3\pi$ decay predictions for the slope a should be considered crucial. Eliminating the constant q from eqs. (2.6) and (2.8) we obtain a relation between a , $\mathfrak{M}_{000}$ and $g_{\pi\eta}$:

$$a^{-1} = (0.604 - 1.444\, g^{-1}_{\pi\eta}\, \mathfrak{M}_{000})^{-1} - 1.723 \,. \qquad (3.4)$$

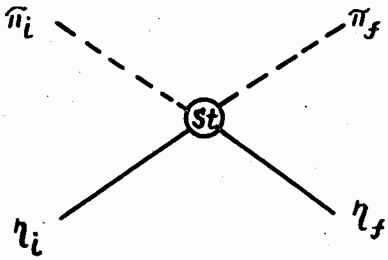
With $g_{\pi\eta} \approx -0.05$ and $|\mathfrak{M}_{000}|^{exp} = 0.98 \pm 0.11$ this formula gives

1. $\mathfrak{M}_{000} > 0$ $\quad a = -0.593 \pm 0.002$
2. $\mathfrak{M}_{000} < 0$ $\quad a = -0.567 \pm 0.002$

These predictions agree well with experimental value[9] $a^{exp} \doteq -0.58 \pm 0.11$.

The matrix element for $\eta \to 3\pi$ is connected with amplitude for $\pi\eta \to \pi\eta$ in the present approach, therefore we can determine low-energy $\pi\eta$ -scattering parameters using the experimental data on the $\eta \to 3\pi$ decay. It is important to stress here that the model under study would be considered fair provided the restrictions on the amplitude $\pi\eta \to \pi\eta$ are reasonable.

The amplitude for the process $\pi\eta \to \pi\eta$ is calculated from Lagrangian (2.3):



$$\mathfrak{M}(\pi\eta \to \pi\eta) = 2f^{-2}q\,(t - 2m_\pi^2)\,, \qquad (3.5)$$

Fig 2. $\pi\eta$ -scattering.

where $t = -(P_{\pi_i} - P_{\pi_f})^2$. The s -and p -wave $\pi\eta$ -scattering lengths are given by:

$$a^0_{\pi\eta} = -\frac{m_\pi^2}{4\pi f_\pi^2 (m_\eta + m_\pi)}\, q \approx -0.033\, q\, m_\pi^{-1}\,, \qquad (3.6)$$

$$a^1_{\pi\eta} = \frac{1}{12\pi f_\pi^2 \, (m_\eta + m_\pi)}\, q \approx 0.011\, q\, m_\pi^{-3}\,. \qquad (3.7)$$

Now it is easy to find $a^0_{\pi\eta}$ and $a^1_{\pi\eta}$ which correspond to values (3.2) of the $\pi\eta\pi\eta$ -coupling constant q :

1. $\mathfrak{M}_{000} > 0$ $\quad a^0_{\pi\eta} = -0.20 \pm 0.02\, m_\pi^{-1}$; $a^1_{\pi\eta} = 0.07 \pm 0.01\, m_\pi^{-3}$;
2. $\mathfrak{M}_{000} < 0$ $\quad a^0_{\pi\eta} = 0.16 \pm 0.02\, m_\pi^{-1}$; $a^1_{\pi\eta} = -0.05 \pm 0.01\, m_\pi^{-3}$.

Thus the $\pi\eta$ -scattering lengths turn out to be of the same order of magnitude as the $\pi\pi$ -scattering lengths calculated in current algebra /13/. This seems to be sensible and serves as an argument in favour of that the conventional pole model based on the $SU_2 \times SU_2$ symmetry is suitable to describe the $\eta \to 3\pi$ decay.

It is necessary to note that the numerical expressions (3.1), (3.6) and (3.7) are sensible to an error in measuring the quantity $|\mathfrak{M}_{000}|$ and to the possible deviations of the value of the constant $g_{\pi\eta}$ from that of calculated using formula (2.5). So the above results for $a^0_{\pi\eta}$ and $a^1_{\pi\eta}$ should be regarded as fairly rough estimates. At the same time the slope a is insensitive to variations of the parameter q because the dependence of the expression (2.8) on q is supressed by factor $m^2_\pi \,/\, m^2_\eta$. This suppression accounts also for that the theoretical value of the slope a differs a little from the experimental one even without involving the η - pole graph, although the decay rates in this case are much smaller than those observed /3,4/.

Finally, we should like to stress that in the model under consideration the use of the $SU_2 \times SU_2$ invariant $\pi\eta\pi\eta$ -interaction (2.3) does not lead to contradictions. However, one can not presently rule out the possibility of the chiral symmetry breaking $\pi\eta\pi\eta$ -interaction:

$$\mathcal{L}_{S.B.}(\pi, \eta) = h\, \eta^2\, \vec{\pi}^2 ,$$

where h is an appropriate coupling constant. For instance, if $\mathfrak{M}_{000} < 0$ and $g_{\pi\eta} \approx -0.05$ the allowed values of the constant h lie within fairly broad boundaries: $-1,2 \le h \le 0$. It is of interest, if one takes the earlier data on the slope[11]: $a = -0.48 \pm 0.04$, then $|h| \sim \frac{m_\pi^2}{2 f_\pi^2} |q|$ is needed for consistency of the pole model with experiment.

4. Conclusion

The performed analysis indicates that the appreciable $\pi\eta$-scattering is needed to exist at low energies for success of the conventional description of the $\eta \to 3\pi$ decay in $SU_2 \times SU_2$ symmetry. As the experimental information on the process $\pi\eta \to \pi\eta$ is not available at the present time, it is an open question to what extent our results for the $\pi\eta$-scattering lengths $a^0_{\pi\eta}$ and $a^1_{\pi\eta}$ correspond to experiment. In this connection, it is natural to try to estimate parameters of the $\pi\eta$-scattering amplitude from theoretical speculations, for example, using $SU_3 \times SU_3$ symmetry of strong interactions[14]. The effective Lagrangian $\mathcal{L}_{em}(\pi,\eta)$ which describes $\pi^0-\eta$ and contact $\eta \to 3\pi$ transitions has the same form under $SU_3 \times SU_3$ and $SU_2 \times SU_2$. However, the form of the strong Lagrangians for the pseudoscalar mesons is different in these two cases. In particular it can be shown that the invariant interaction Lagrangian of Goldstone mesons, under different nonlinear realizations of $SU_3 \times SU_3$, does not contain terms responsible for $\pi\eta$ -scattering. An insertion of symmetry breaking with transformation properties of representation $(3,\bar{3}) \oplus (\bar{3},3)$ of $SU_3 \times SU_3$ /15/ results in the nonderivative $\pi\eta$ -interaction with the constant $h = \frac{1}{12}\frac{m_\pi^2}{f_\pi^2}$;$(q=0)$. We note that this result corresponds to the restrictions on the $\pi\eta$ -scattering amplitude found in work /2/. With such values of the constants q and h the graph with η -pole gives extremely small contribution to the $\eta \to 3\pi$ decay amplitudes. Thus, our approach encounters difficulties in the case of $SU_3 \times SU_3$ symmetry.

A possible way of avoiding these difficulties will be discussed elsewhere. Here we would like only to stress once more that at the level of $SU_2 \times SU_2$ symmetry one succeeds in describing fairly well the process $\eta \to 3\pi$ within the framework of conventional pole model.

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