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The purpose of this note is to present constraints on the cross sections for the processes

$$N\,(\pi) + N \rightarrow \mu^+\mu^-\;(W^{\pm}) + hadrons. \qquad (1)$$
$$\qquad\qquad\qquad\qquad \llcorner\!\!\rightarrow \nu\mu^{\pm}$$

The cross sections are defined by the tensors /1,2/

$$W^{ab}_{\alpha\beta,\alpha'\beta'}(p_1,p_2,q) = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \bar{\sum_{spin}} \int d^4x\; e^{-iqx} \times \qquad (2a)$$

$$\times < p_1 a \;\; p_2 \beta \;in \mid V^b_\nu(x)\; V^a_\mu(0) \mid p_1 a' \; p_2 \beta' in >_c \;,$$

$$\bar{W}^{ab}_{\alpha\beta,\alpha'\beta'}(p_1,p_2,q) = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) \bar{\sum_{spin}} \int d^4x\; e^{-iqx} \times \qquad (2b)$$

$$\times < p_1 a \;\; p_2 \beta \;\; in \mid A^b_\nu(x)\;\; A^b_\mu(0) \mid p_1 a' \; p_2 \beta' \; in > \;.$$

Neglecting the lepton masses the cross section for the μ -pair production has the expression

$$\frac{d\sigma^{Q^2}}{dq^2} = \frac{2\pi a^2}{3q^2} \frac{1}{(2\pi)^4\sqrt{(p_1p_2)^2 - m_1^2 m_2^2}} \int \frac{d^3q}{2q_0} W^{Q^2}_{\alpha\beta,\alpha\beta} \tag{3a}$$

and for $W^{\pm}_{\nu\mu}$ production it is

$$\sigma^{\pm} = B G\sqrt{2} \frac{1}{(2\pi)^4\sqrt{(p_1p_2)^2 - m_1^2 m_2^2}} \int \frac{d^3q}{2q_0} \Big[\frac{1}{2}\big(W^{1-i2,1+i2}_{\alpha\beta,\alpha\beta} + \overline{W}^{-1-i2,1+i2}_{\alpha\beta,\alpha\beta}\big)\cos^2\theta_c + \frac{1}{2}\big(W^{4-i5,4+i5}_{\alpha\beta,\alpha\beta} + \overline{W}^{4-i5,4+i5}_{\alpha\beta,\alpha\beta}\big)\sin^2\theta_c \Big] , \tag{3b}$$

where α , β , α' , β' denote the isospin of the initial state hadrons, V^a_μ , A^a_μ are the vector and axial-vector components of the nonet of the hadronic currents, θ_c is the Cabbibo angle x/ , Q denotes combination $V^3 + \frac{1}{\sqrt{3}}V^8$, B is the branching ratio $\Gamma(W\to\nu\mu)/\Gamma(W\to all)$, G is the weak coupling constant $(\sim 10^{-5}\, m_p^{-2})$, m_1, m_2 are the masses of the incoming hadrons.

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x/ In the following we use $\theta_c = 0$.

Recently Lipkin and Peshkin /3/ called attention to model independent isospin relations for inclusive reaction cross sections. The crossed channel reactions of the processes (1) were considered by Llewellyn-Smith and Pais /4/. Since in the process (1) the particle selected in the final state has definite charge with isospin $I \leq 1$ and the initial state is in general a coherent sum of states of different isospin, without additional assumption their methods do not give bounds on the cross sections.

At high q^2 and s it might be that these processes (1) are controlled by the light cone singularities of the operator products /2,5/, similarly to the deep-inelastic lepton-nucleon scattering. The arguments for the dominance of the light-cone singularities and for the use of the light-cone algebra of Fritzsch and Gell-Mann/6/ for these matrix elements are however less convincing, since i) the phase of the matrix elements may be large so that the usual stationary phase argument may fail, ii) Wilson's expansion can in general be used only for matrix elements of fixed states /7/, iii) $q^2 > 0$, therefore semiconnected pieces do contribute and we have operator product and not commutator.

Let us forget for a moment these objections and apply Fritzsch-Gell-Mann's operator product expansion to the operator product of eqs. (2). Then we obtain that i) the contributions of the nonconserving pieces of the axial-vector current can be neglected and the vector-vector contributions are equal to the axial-vector contributions, ii) the bilocal operators are members of an $SU(3)$ nonet.

It might happen that these properties remain valid even if we must accept the objections listed above. We assume (without further arguments) that at high $s=(p_1+p_2)^2$ and q^2 ($s > q^2 \gg m_N^2$) the symmetric part of the tensors (2) are dominated by an $SU(3)$ nonet, that is

$$W^{\{a,b\}}_{\alpha\beta,\alpha'\beta'} = d^{abc}\, V^{c}_{\alpha\beta,\alpha'\beta'} , \qquad (4)$$

where d^{abc} is the coefficient of the anticommutators of the Gell-Mann's $SU(3)$ matrices $\frac{1}{2}\lambda^{a}$. By use of this assumption inequalities follow from the positivity conditions

$$C^{\alpha\beta}_{a}\, W^{ab}_{\alpha\beta,\alpha'\beta'} C^{*\alpha'\beta'}_{b} > 0 . \qquad (5)$$

It is clear that our approach gives a generalization of the method of Nachtman /8/ and Callan et al. /9/ proposed for the inelastic lepton-nucleon scattering to the μ -pair and W -boson production.

Assuming isospin invariance for the hadronic states we should use the $SU(3)$ components $C = 1,2,3,8,0$; therefore the $W^{ab}_{\alpha\beta,\alpha'\beta'}$ tensor can be represented as a 20x20 matrix for the NN collisions and as a 30x30 matrix for the πN collisions. By a systematic exploitation of the equations (4) and (5) we can obtain representations for the cross section as follows

$$W^{Q^2}_{pp} = W^{Q^2}_{nn} = \frac{1}{9}(2m + 2p + 8q), \tag{6a}$$

$$W^{Q^2}_{pn}(z) = W^{Q^2}_{np}(-z) = \frac{1}{9}(m + 2n + u(z) + 4u(-z)), \tag{6b}$$

$$W^{+}_{pp} + W^{-}_{pp} = 2(p+q), \tag{6c}$$

$$W^{+}_{pn} + W^{-}_{pn} = u(z) + u(z), \tag{6d}$$

where m , n , p , q and u are positive functions and since the nucleons are identical particles they are symmetric functions of z except u ($z = p_1 q/|p_1|\,|q|$).

On the even part of u we have the additional constraint

$$u(z) + u(-z) \geq p + q. \tag{6e}$$

For πN collisions we obtained

$$W^{Q^2}_{\pi^+ p} = \frac{2}{3}(x + 8v - 3t), \tag{7a}$$

$$W^{Q^2}_{\pi^+ n} = \frac{2}{9}(x + 2y + 8v + 16w - 3s), \tag{7b}$$

$$W^{Q^2}_{\pi^- n} = \frac{2}{3}(x + 2v + 3t) , \quad (7c)$$

$$W^{Q^2}_{\pi^- p} = \frac{2}{9}(x + 2y + 2v + 4w + 3s) , \quad (7d)$$

$$W^{Q^2}_{\pi^0 p} = \frac{2}{9}(2x + y + 16v + 8w - 3r) , \quad (7e)$$

$$W^{Q^2}_{\pi^0 n} = \frac{2}{9}(2x + y + 4v + 2w + 3r) , \quad (7f)$$

$$W^+_{\pi^+ p} + W^-_{\pi^+ p} = W^+_{\pi^- n} + W^-_{\pi^- n} = 12v , \quad (7g)$$

$$W^+_{\pi^+ n} + W^-_{\pi^+ n} = W^+_{\pi^- p} + W^-_{\pi^- p} = 4v + 8w , \quad (7h)$$

$$W^+_{\pi^0 p} + W^-_{\pi^0 p} = W^+_{\pi^0 n} + W^-_{\pi^0 n} = 8v + 4w , \quad (7i)$$

where $x , y , v , w , r , s , t$ are positive functions satisfying the additional inequalities

$$2v \geq t , \quad 2v + w \geq r , \quad v + 2w \geq s . \quad (7j)$$

By use of the representations (6) and (7) it is straightforward to deduce inequalities.

We note the following

i) The equations are valid for the differential cross sections $d^4\sigma / d^4 q$. Due to the symmetry in z we can obtain stronger relations for the total cross section than for the differential cross sections in the case of the NN collisions; e.g. for the differential cross section we obtain

$$4 \geq W_{pn}^{Q^2}(z) / W_{pn}^{Q^2}(-z) \geq 1/4 \quad , \tag{8a}$$

$$W_{pn}^{Q^2} \geq \frac{1}{8} W_{pp}^{Q^2} \quad . \tag{8b}$$

For the total cross sections instead of 4 and 1/4 there will be 1 and instead of 1/8 we obtain 5/16.

ii) Only upper bound can be given on the cross section of the W-boson production (because of the isoscalar contributions):

$$9 W_{pp}^{Q^2} \geq (W_{pp}^{+} + W_{pp}^{-}) \quad , \tag{9a}$$

$$\frac{18}{5} W_{pn}^{Q^2} \geq (W_{pn}^{+} + W_{pn}^{-}) \ . \tag{9b}$$

iii) If we neglect the isoscalar contributions as it is proposed in paper /10/ we can obtain equalities

$$4W^{Q^2}_{pp} = W^{+}_{pp} + W^{-}_{pp} \quad , \tag{10a}$$

$$2W^{Q^2}_{pn} - W^{+}_{pn} + W^{-}_{pn} \quad . \tag{10b}$$

That gives some insight into the nature of this assumption: it is very near to the one that the <u>upper</u> bound is equal to the <u>lower</u> bound x/.

iv) We record some inequalities for the πN collisions:

$$1/4 \leq \frac{W^{Q^2}_{\pi^+p}}{W^{Q^2}_{\pi^-n}} \leq 4 \; , \qquad \frac{5}{24} W^{Q^2}_{\pi^+p} \leq W^{Q^2}_{\pi^+p} \quad , \tag{11a}$$

$$\frac{1}{12} W^{Q^2}_{\pi^+p} \leq W^{Q^2}_{\pi^-p} \; ; \qquad 1/4 \leq \frac{W^{Q^2}_{\pi^+n} + W^{Q^2}_{\pi^+p}}{W^{Q^2}_{\pi^-n} + W^{Q^2}_{\pi^-p}} \leq 4 \; , \tag{11b}$$

$$W^{+}_{\pi^+p} + W^{-}_{\pi^+p} < 9W^{Q^2}_{\pi^+p} \quad , \tag{11c}$$

$$W^{+}_{\pi^-p} + W^{-}_{\pi^-p} < \frac{18}{5} W^{Q^2}_{\pi^-p} \quad . \tag{11d}$$

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x/ This remark applies in a certain sense to our assumption (4) as well, but it can be tested separately in the μ-pair production.

v) The inequalities (9b) and (11d) give the following inequalities for the cross sections (assuming that $W = \bar{W}$)

$$\frac{1}{2}\left(\sigma^{+}_{\pi^-(n)p} + \sigma^{-}_{\pi^-(n)p}\right) \leq 0.21\, q^2 B \, \frac{d\sigma^{Q^2}_{\pi^-(n)p}}{dq^2}\,, \tag{12a}$$

$$\frac{1}{2}\left(\sigma^{+}_{\pi^+(p)p} + \sigma^{-}_{\pi^+(p)p}\right) \leq 0.52\, q^2 B \, \frac{d\sigma^{Q^2}_{\pi^+(p)p}}{dq^2}\,. \tag{12b}$$

vi) It is straightforward to derive similar inequalities for nucleus targets of definite isospin.

We stress the importance of the assumption (4) for this feature must be tested first by use of the given inequalities in order to decide wether there is or not some relevance of the light-cone model of Fritsch and Gell-Mann.

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