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The conventional theory of leptonic weak interactions[1] has the well known deficiency that, due to non-renormalizable nature of its divergences, it is in fact non-universal[2]. The reason for this lies in the non-equality of most divergent terms in matrix elements of different processes. Consider, e.g., the diagrams describing the processes $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ and $\nu_e e^- \to \nu_e e^-$. The highest divergences in the diagrams are proportional to $c_n G^n L^{2n-2}$, where G is the Fermi constant and L is a cut-off parameter which for pure leptonic interactions[3] is presumably of order $L \approx G^{-1/2}$ ("unitary cut-off"). It is not hard to verify that the coefficients c_n are not equal for these two processes[4]. Therefore, the effective coupling constants for $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ and $\nu_e e^- \to \nu_e e^-$ may be quite different, unlike the prediction of the original "universal" Lagrangian. Furthermore, some processes which are forbidden in the first order (say, $\nu_\mu e^- \to \nu_\mu e^-$) become allowed in the second order and the effective coupling constants for such processes are of order $\approx G^2 L^2 \approx G$. These difficulties persist in theories with intermediate vector bosons and seem to be unaviodable even by use of a summation of infinite number of diagrams[5]. So the conventional theory of leptonic weak interactions completely loses its predictive power.

Here we propose a new theory of leptonic weak interactions which is free from these difficulties. The principal idea is that the Lagrangian is invariant with respect to rotations and reflections in a four-dimensional leptonic isospace (or leptospace)[6]. The leptons and antileptons are described by spinors of this space $\psi = (\nu_e, e^-, \nu_\mu, \mu^-)$, $\bar{\psi} = (\bar{\nu}_e, \bar{e}^-, \bar{\nu}_\mu, \bar{\mu}^-)$ where $\bar{l} \equiv l^+ \gamma_0$ and l is the usual Dirac spinor. To construct the theory we introduce four 4x4 hermitian matrices a_i $(i = 1,2,3,4)$ satisfying the relations $a_i a_j + a_j a_i = 2\delta_{ij}$ and six matrices $a_{ij} = (2i)^{-1}(a_i a_j - a_j a_i)$ representing the generators of the rotations in leptospace. Then the weak currents have the form[7] $J^\mu_{ij} = \bar{\psi} a_{ij} O^\mu \psi$ (where $O^\mu = \gamma^\mu (1 + \gamma_5)$) and the Lagrangian describing leptonic weak interaction is[8].

$$\mathfrak{L}_L = \frac{1}{8} G J^\mu_{ij} J^{ij}_\mu = \frac{1}{8} G (\bar{\psi} a_{ij} O^\mu \psi)(\bar{\psi} a^{ij} O_\mu \psi) , \qquad (1)$$

where $A_i \equiv A^i$ and the usual summation rule is used. This Lagrangian is invariant under the transformation of spinors $\psi \to \exp(\frac{i}{2} a_{ij} \phi^{ij})$ and $\psi \to a_l \psi$ representing rotations and reflections of the leptospace. Furthermore, it is invariant under a_5-transformation $\psi \to a_5 \psi$, $\bar{\psi} \to \bar{\psi}$, where $a_5 = a_1 a_2 a_3 a_4$.

Consider now the subgroup of the three-dimensional rotations through the axes 1,2,3 of the leptospace and require the current J^{μ}_{12} to be coupled with the electromagnetic field and J^{μ}_{13}, J^{μ}_{23} - with weak hadronic (charged) currents. To this end it is convenient to choose the following representation for a-matrices:

$$a_n = \tau_3 \otimes \tau_n \ (n = 1,2,3), \qquad a_{mn} = \epsilon_{mnl}\, 1 \otimes \tau_l ,$$

where τ_n are the Pauli matrices. Then the charge matrix $Q_L = \frac{1}{2}(a_{12} - 1)$, connected with rotations in the plane (1,2), and the matrix $F_L = -i a_1 a_2 a_3 = i a_4 a_5$ representing the reflection of the fourth axis, are diagonal, with the diagonal elements $Q_L = (0,-1,0,-1)$ and $F_L = (+1,+1,-1,-1)$. Thus the multiplicatively conserved quantum number F_L may be identified with the multiplicative muonic charge (muonic parity)/9/.

Fixing the representation for the matrices a_4 and a_5, $a_4 = \tau_1 \otimes 1$, $a_5 = -\tau_2 \otimes 1$, and using the identity $\frac{1}{2} J_{ij} J^{ij} = (\bar{\psi}\psi)^2 + (\bar{\psi} a_5 \psi)^2$ and the Fierz identity (for anticommuting spinors l) we get

$$\begin{aligned}\mathfrak{L}_L = \frac{1}{4} G[& 2(\bar{e}\nu_e)(\bar{\nu}_\mu \mu) + 2(\bar{\nu}_e e)(\bar{\mu}\nu_\mu) - 2(\bar{e}\nu_\mu)(\bar{\nu}_e \mu) - 2(\bar{\nu}_\mu e)(\bar{\mu}\nu_e) + \\ & + 2(\bar{e}\nu_e)(\bar{\nu}_e e) + 2(\bar{\mu}\nu_\mu)(\bar{\nu}_\mu \mu) + 2(\bar{\mu}\nu_e)(\bar{\nu}_e \mu) + 2(\bar{e}\nu_\mu)(\bar{\nu}_\mu e) - \\ & - (\bar{e}\mu)^2 - (\bar{\mu} e)^2 + 4(\bar{e} e)(\bar{\mu}\mu) + 4(\bar{\nu}_e \nu_e)(\bar{\nu}_\mu \nu_\mu) + \\ & + (\bar{\nu}_e \nu_e)^2 + (\bar{\nu}_\mu \nu_\mu)^2 - (\bar{\nu}_e \nu_\mu)^2 - (\bar{\nu}_\mu \nu_e)^2 + (\bar{e} e)^2 + (\bar{\mu}\mu)^2] .\end{aligned} \qquad (2)$$

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The following essentially different processes predicted by this Lagrangian can be detected in near future[10]: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, $\mu^- \to e^- \nu_e \bar{\nu}_\mu$, $\nu_e e^- \to \nu_e e^-$, $\nu_\mu e^- \to \nu_\mu e^-$, $e^- e^- \to \mu^- \mu^-$. Our theory predicts the definite relations between these processes which are easily deduced from eq. (2).

Consider now the problem of higher order corrections. The most divergent terms do not depend on masses of leptons and on other interactions. So the highest divergences preserve the summetries of the original Lagrangian (1), including a_5-symmetry. As the most general four-fermion interaction, which is invariant under rotations and reflections in the leptospace and under a_5 -transformation, coincides with eq. (1) we conclude that these divergences may be factorized and eliminated by renormalizing the coupling constant G . Then the main corrections will be given by next to the highest divergences $\approx G^n L^{2n-4}$ (up to a power of $\ln L$) which for $L \approx G^{-1/2}$ are of order G (up to a power of $\ln G$). These corrections may be different for different processes as the mass terms $m_\mu \bar{\mu} \mu$ and $m_e \bar{e} e$ break down the four-dimensional symmetry of the leptospace, the mass matrix having the form

$$m_L = \frac{1}{2} Q_L [a_{54} (m_\mu / m_e - 1) - (m_\mu / m_e + 1)] .$$

It is worth noting that a T-violation may be easily incorporated in our theory by choosing the new representation for a_4 and a_5

$$a_4^1 = a_4 \cos\phi + a_5 \sin\phi , \quad a_5^1 = -a_4 \sin\phi + a_5 \cos\phi$$

which does not change a_1, a_2, a_3 and $a_4 a_5$. The Lagrangian (1) with these matrices violates T-invariance if $\phi \neq 0, \pi/2$.

The theory also may be formulated in terms of the coupling of the currents J^{μ}_{ij} with six vector bosons W^{μ}_{ij}, the muonic parity being +1 for W_{12}, W_{13}, W_{23} and -1 for W_{14}, W_{24}, W_{34}. The semileptonic interaction can be introduced if we couple the hadronic currents only with the charge leptonic currents J_{23}, J_{13} or with W_{23}, W_{31}.

The weak hadronic currents may be constructed from baryonic fields by using the method of ref. /11/. With this aim we group all the baryons into two four-component spinors

$$\Psi_+ = (p, n\cos\theta + Y\sin\theta, \Sigma^+, Z\cos\theta + \Xi^0 \sin\theta) = (\Psi_{++}, \Psi_{+-})$$

$$\Psi_- = (-n\sin\theta + Y\cos\theta, \Sigma^-, -Z\sin\theta + \Xi^0\cos\theta, \Xi^-) = (\Psi_{-+}, \Psi_{--})$$

where $Y = \frac{\Lambda - \Sigma^\circ}{\sqrt{2}}$, $Z = \frac{\Lambda + \Sigma^\circ}{\sqrt{2}}$ and Θ is a real parameter. These spinors correspond to antileptonic and leptonic spinors, respectively. The charge matrix is $Q_H = \frac{1}{2}(a_{12} + L_H)$, $L_H \Psi_\pm = \pm \Psi_\pm$ and the analog of the muonic parity is represented by the matrix $F_H = i a_4 a_5$, $F_H \Psi_{+(\pm)} = \Psi_{+(\pm)}$, $F_H \Psi_{-(\pm)} = -\Psi_{-(\pm)}$. The matrices Q_H, L_H, F_H give us the complete set of quantum numbers defining the baryonic states.

It is not hard to verify that from the conservation of $L_H Q_H$ and F_H (independently of L_L and F_L) the usual selection rules $|\Delta S| \leq 1$ and $\Delta S = \Delta Q$ follow. By defining the hadronic currents $J_{\pm}^{ij} = \bar{\Psi}_{\pm} a_{ij} \Psi_{\pm}$ which transform under rotations and reflections just like the corresponding leptonic currents we write the semileptonic interaction in the form

$$\mathfrak{L}_{HL} = G' [J_{13} (J_{+}^{13} + J_{-}^{13}) + J_{23} (J_{+}^{23} + J_{-}^{23})] . \qquad (5)$$

One may verify that the semileptonic $|\Delta T| = \frac{1}{2}$ rule for the processes with $|\Delta S| = 1$ is fulfilled for this Lagrangian. The most essential difference between this Lagrangian and that of Cabibbo[12] is that eq. (5) predicts a non-vanishing vector coupling constant for the transition $\Sigma^- \to \Lambda e^- \bar{\nu}_e$. This prediction does not contradict available experimental data[13]. Purely hadronic weak interactions will be considered in a subsequent paper together with a more detailed discussion of semileptonic interaction. Here we only mention that by using neutral currents the $|\Delta T| = \frac{1}{2}$ rule may be easily incorporated in the theory (cf. ref.[14]).

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Footnotes and References

1. E.Fermi. Nuovo Cimento, 11, 1 (1934); R.Feynman, M.Gell-Mann. Phys.Rev., 109, 193 (1958).

2. See e.g. R.Marshak, Riazuddin, C.Ryan. Theory of Weak Interactions in Particle Physics (New York, 1969).

3. For purely leptonic interactions there is no other dimensional parameter which may be related to the unitary cut-off. In the hadronic weak processes the strong interactions play an essential role and there may exist some much lower cut-off parameter related, say, to the slope of the Regge trajectories.

4. The calculations of simplest diagrams were performed in collaboration with V.Gogokhia, who observed that in a theory with multiplicative muonic charge the coefficients C_n may be equal for both processes.

5. See e.g. G.Feinberg, A.Pais. Phys.Rev., 131, 2724 (1963); B.Arbuzov, A.Filippov. Nuovo Cimento, 38, 796 (1965); B.Getmanov, A.Filippov. Teor. i Math. Phys. (USSR) 8, 3 (1971).

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7. We stress that it is the generators of the rotations represented by a_{ij} which are used for constructing the weak currents. This requirement may be considered as a generalization of the universality principle.

8. In what follows we suppress the matrices O_μ and corresponding indices.

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