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Nguyen Van Hieu

EQUAL-TIME CANONICAL COMMUTATION RELATIONS AND PERTURBATION THEORY

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EQUAL-TIME CANONICAL COMMUTATION RELATIONS AND PERTURBATION THEORY

In current algebra [1] we assume equal-time commutation relations of vector and axial currents and derive the sum rules connecting different physical measurable quantities. Recently Lackiw and Preparata [2], and Adler and Wu Ki Tung [3] have observed the breakdown of several current algebra sum rules in the perturbation theory. The purpose of this note is to study the sum rules obtained from the equal-time canonical commutation relations of the field operators and to prove the breakdown of these sum rules in the perturbation theory.

As an example it is sufficient to consider the theory of a neutral scalar field $\phi(x)$ with the interaction of the form

$$L = g\,\phi(x)^3 \;. \qquad (1)$$

In the conventional theory $\phi(x)$ satisfies following equal-time canonical commutation relation

$$[\,\phi(x)\,,\,\frac{\partial\phi(y)}{\partial y_0}\,]_{x_0=y_0} = i\delta\,(\vec{x}-\vec{y}\,)\;. \qquad (2)$$

Therefore the matrix elements of the equal-time commutator

$$[\,\phi(x)\,,\,\frac{\partial\phi(y)}{\partial y_0}\,]_{x_0=y_0}$$

between orthogonal asymptotic states must vanish. In particular

$$\langle 0|\,[\,\phi(x)\,,\,\frac{\partial\phi(y)}{\partial y_0}\,]\,|\,q\rangle_{x_0=y_0} = 0, \qquad (3)$$

$$\langle q' | [\phi(x) , \frac{\partial \phi(y)}{\partial y_0}] | q \rangle_{x_0 = y_0} = 0 . \tag{4}$$

Now we compute the left-hand sides of Eqs. (3) and (4). We have

$$\begin{aligned} \langle 0 | [\phi(x), \phi(y)] | q \rangle = \sum_n \int \frac{d^3k_1 \dots d^3k_n}{(2\pi)^{3n}} \{ \exp[iqy + i \sum_i k_i (x-y)] \\ - \exp[iqx + i \sum_i k_i (y-x)] \} \langle 0 | \phi(0) | k_1 \dots k_n \rangle \langle k_1 \dots k_n | \phi(0) | q \rangle , \end{aligned} \tag{5}$$

$$\begin{aligned} \langle q' | [\phi(x), \phi(y)] | q \rangle = \sum_n \int \frac{d^3k_1 \dots d^3k_n}{(2\pi)^{3n}} \{ \exp[iqy - iq'x + i \sum_i k_i (x-y)] \\ - \exp[iqx - iq'y + i \sum_i k_i (y-x)] \} \langle q' | \phi(0) | k_1 \dots k_n \rangle \langle k_1 \dots k_n | \phi(0) | q \rangle . \end{aligned} \tag{6}$$

In the lowest order of the perturbation theory only the one-particle intermediate states contribute to the right-hand sides of Eqs. (5) and (6), and we have

$$\begin{aligned} \langle 0 | [\phi(x), \phi(y)] | q \rangle = \int \frac{d^3k}{(2\pi)^3} \{ \exp[iqy + ik(x-y)) - \\ - \exp[iqx + ik(y-x)] \} \langle 0 | \phi(0) | k \rangle \langle k | \phi(0) | q \rangle , \end{aligned} \tag{7}$$

$$\begin{aligned} \lim_{q' \to q} \langle q' | [\phi(x); \phi(y)] | q \rangle = \int \frac{d^3k}{(2\pi)^3} \{ \exp[i(k-q)(x-y)] - \\ - \exp[i(k-q)(y-x)] \} | \langle k | \phi(0) | q \rangle |^2 , \end{aligned} \tag{8}$$

where

$$\langle 0 | \phi(0) | q \rangle = \frac{1}{\sqrt{2q_0}} ,$$

$$\langle k | \phi(0) | q \rangle = \frac{1}{\sqrt{4k_0 q_0}} \, 3g \, \frac{1}{(q-k)^2 + m^2} ,$$

$$k_0 = \sqrt{\vec{k}^2 + m^2} , \qquad q_0 = \sqrt{\vec{q}^2 + m^2} .$$

From these relations we get

$$\langle 0 | [\phi(x), \frac{\partial \phi(y)}{\partial y_0}] | q \rangle_{x_0 = y_0 = 0} = \frac{3}{2} \, i g \, \frac{1}{\sqrt{2q_0}} \int \frac{d^2k}{(2\pi)^3} \{ (1 - \frac{q_0}{k_0}) \times$$

$$\exp [i\vec{q}\vec{y} + i\vec{k}(\vec{x}-\vec{y})] + \exp [i\vec{q}\vec{x} + i\vec{k}(\vec{y}-\vec{x})] \} \times \frac{1}{(q-k)^2 + m^2} \tag{9}$$

$$\lim_{q' \to q} \langle q' | [\phi(x), \frac{\partial \phi(y)}{\partial y_0}] | q \rangle_{x_0 = y_0} = \frac{9}{2} \, i g^2 \frac{1}{2q_0} \int \frac{d^3k}{(2\pi)^3} (1 - \frac{q_0}{k_0}) \times$$

$$\{ \exp [i(\vec{k}-\vec{q})(\vec{x}-\vec{y})] + \exp [i(\vec{k}-\vec{q})(\vec{y}-\vec{x})] \} \times \frac{1}{[(q-k)^2 + m^2]^2} , \tag{10}$$

It is obvious that the right hand sides of Eqs. (9) and (10) are not equal to zero identically, in a contradiction with conditions (3) and (4).

References

1. M.Gell-Mann. Phys.Rev., 125, 1067 (1962).
2. R. Jackiw and T.Preparata. Phys.Rev., 185, 1748 (1969).
3. S.L.Adler and Wu Ki Tung. Phys.Rev., D1, 2846 (1970).

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