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EVIDENCE FOR NEW SINGULARITIES  
IN REGGE PHENOMENOLOGY

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**EVIDENCE FOR NEW SINGULARITIES  
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## I. Introduction

The Regge models, with poles and cuts, allow one to understand qualitatively well the feature of experimental data in the region of middle energy  $5 \text{ GeV}/c < p < 15 \text{ GeV}/c$ . Nevertheless, some unexplained problems remain, and some others are created when new, more accurate, data appear. Let us quote the main ones:

New polarization data in  $\pi N$  charge exchange /1/

Differential cross section for the  $S-U$  crossed reactions with hypercharge exchange:

$$k^- p \rightarrow \pi^- \Sigma^+, \quad \pi^+ p \rightarrow k^+ \Sigma^+$$

Differential cross section in  $NN$  charge exchange:

$$p n \rightarrow n p, \quad p \bar{p} \rightarrow n \bar{n}$$

The point of view which is kept here is not to look for a complicated parametrization or some sophisticated model. Comparing the data with the predictions of the usual models, such as the Reggeized absorption models /2/, we prove to find out the properties of the contribution which is to be added to these models in order to describe the qualitative features of the data. The qualitative character of this study allows us to do so without knowing in detail the model of reference.

In part II, this study leads to the following conclusion: for the three reactions considered the assumed extra contribution has the same properties:

1. Real with negative signature,
2. With no s-channel helicity flip,
3.  $a$  and  $a'$  are small (near zero),
4. Central,

where  $a$  and  $a'$  are the parameters of the effective power law behaviour of the contribution. We describe it very approximately as:

$$C(s, t) = S^{a + a't} f(t).$$

Central means that the residue  $f(t)$  is such that the main contribution comes from impact parameter low with respect to the radius of interaction.

Part III is devoted to a theoretical investigations of such a contribution. We examine successively the kind of singularity in the complex  $l$ -plane which can have the desired properties 1), 2) and 4), the physical meaning of such singularities and their possible interpretation in terms of the absorption model.

As concerns the first point, it appears that only a complex Regge singularity obeys the given properties, excluding real Regge poles and cuts as possible solutions. Two complex conjugate Regge poles at  $Re\ell = 0$  are the simplest realization of the extra-contribution. But the main physical feature of these singularities to be explained is that they appear with different  $t$ -channel quantum numbers. In particular, from conspiracy constraints in NN charge exchange, two contributions of different parity are possible, equal in the forward direction.

The theoretical answer to both problems can be found by summing the Feynman graphs in Lagrangian theory<sup>/3/</sup>. One has found by this method the existence in some cases of a complex cut at  $Re\ell = 0$  in addition to the moving Regge trajectories. This singularity is connected with a scale-invariant behaviour of forces (or potentials) at small distance for renormalizable Lagrangians ( $\gamma^5$ -theory or  $\sigma$ -model), and then it is not associated with definite  $t$ -channel exchange. The properties of such cuts agree quite well with what is expected from the experimental discussion.

In the framework of the usual Reggeized absorption models, these singularities do not appear. In fact, a simple modification in the definition and not in the main properties of these models allows us to find the desired singularities. This modification gives physical cut corrections to the Wrong signature poles (which, as we know, do not give themselves a contribution). In the case of the amplitude of negative signature, we then obtain a contribution of a cut with a branch point at  $\ell = 0$ . This cut is real, but "interacts" with the Wrong signature pole to transfer it into two complex poles near  $\ell = 0$  in a way similar to the one described by Kaus and Zachariasen<sup>/4/</sup>. These Wrong signature poles may be present for any  $t$ -channel quantum number and then the corresponding singularities appear in all the reactions we have studied.

In part IV we discuss some other aspects and consequences of the existence of our singularity. The main of them is the possible manifestation of the singularity in some exotic reactions of the type  $p \bar{p}$  backward scattering, the connection of it with low energy phenomena by duality and the connection with scale invariance.

## II. Experimental Considerations.

### A. $\pi N$ Charge Exchange

The new polarization data<sup>1</sup> for the reaction  $\pi\pi^-p \rightarrow \pi^0 n$  are unexplained by most current models, in particular, by the Reggeized absorption model<sup>2</sup>. Using these data, together with what was already known on  $\pi N$  reactions, one can get the amplitudes and find the necessary correction to the models. Ringland and Roy<sup>5</sup> have shown the role played by the real part of the non-helicity-flip amplitude  $Re M_{\pi\pi^-p \rightarrow \pi^0 n}$  (Fig. 1). In different ways Schrempp-Otto and Schrempp<sup>6</sup>, Martin and Stevens<sup>7</sup> come to the same results: the need of a central negative signature real contribution, mainly with no helicity flip. These are just the properties 1), 2) and 4) listed in the Introduction.

It should be noted that these properties are more or less model independent. For instance, in paper<sup>7</sup>, the model for the amplitude is used only as a parametrization of the data with an absorption depending on the energy<sup>7</sup>. In this paper a comparison is made with the usual Reggeized absorption model<sup>2</sup> where the absorption is determined by the Pomeranchuk pole exchange (Fig. 2).

This comparison leads to the following results: a) the energy increasing the absorption correction goes to the usual one, b) the radius of the corrected absorption in an impact parameter representation remains approximately independent of the energy. Then, if we try to explain the data by a new contribution, we find that its parameters  $a$  and  $a'$  (see the Introduction) have to satisfy  $a < 1/2$  and  $a' < 1$ . The former is a direct consequence of the remark a). The remark b) shows that the correction has the same "radius" as the usual absorption, the effective slope being the same if not lower.

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<sup>x</sup> This parametrization is used to determine the amplitude outside of the region where the polarization data are now available, for instance at  $p_{lab} = 18.2$  in Fig. 2.

In fact the following argument shows that the parameters  $\alpha$  and  $\alpha'$  cannot be very different from zero. In  $\pi N$  it is possible to know quite well the location of the dominant singularities thanks to the finite energy sum rules <sup>/8/</sup>. Dolen, Horn and Schmid have shown that there is a fixed singularity at  $\ell=0$  in amplitude  $B^-$  contributing mainly to the nonflip part in the S-channel. These authors claim for the existence of a Wrong signature pole, but use the conjecture that, besides or instead of this pole, another singularity exists. The discussion of this point will be done later (Sect. IV); we retain it here only to exclude singularities far from  $Re\ell=0$ .

### B. Hypercharge Exchange $S - U$ -Crossed Reactions

$$\pi^+ p \rightarrow k^+ \Sigma^+ \quad \text{and} \quad k^- p \rightarrow \pi^- \Sigma^+$$

One of the most puzzling features of the data, unexplained by current models, is the inequality of the differential cross sections for these reactions at middle energies. As a starting point we shall use the results of Irving et al. <sup>/9/</sup> and of Sadoulet <sup>/10/</sup>. For instance, in Figure 3, from <sup>/9/</sup>, we see both the experimental data of differential cross sections as a function of energy and the feature of the absorption model.

In fact any absorption model gives

$$\frac{d\sigma}{dt}(\pi^+) > \frac{d\sigma}{dt}(k^-).$$

In the contrast with the experimental results they give a larger negative correction to the kaon-induced reaction than for the pion-induced one. Sadoulet <sup>/10/</sup> points out that the correction to the absorption models is the following: The mainly real amplitude  $M$  (for  $k^- p \rightarrow \pi^- \Sigma^+$ ) has to be less absorbed when compared to the usual absorption model with the degenerate  $k^* - k^{**}$  trajectories. On the contrary, the real part of  $M_{\pi}$  (for  $\pi^+ p \rightarrow k^+ \Sigma^+$ ) has to be more absorbed. Following our point of view of looking for an extra-contribution we find that it has to be real, with negative signature and with no helicity flip (properties 1), 2), 3) of the Introduction).

In fact, let us interpret the conclusion of Sadoulet as follows:

$$Re(M_k - [Regge + Abs.]_k) = \epsilon_k$$

$$\operatorname{Re} ( M_{\pi} - [ \text{Regge} + \text{Abs.} ]_{\pi} ) = - \epsilon_{\pi} ,$$

where (Regge + absorption stands for the amplitude, with usual absorption and the  $\epsilon_i$ 's are positive real contributions describing the corrections proposed by Sadoulet. Then

$$\operatorname{Re} ( M_{k+} + M_{\pi} ) = [ \text{Regge} + \text{Abs.} ]_{+} + ( \epsilon_k - \epsilon_{\pi} ) ,$$

$$\operatorname{Re} ( M_{k-} - M_{\pi} ) = [ \text{Regge} + \text{Abs.} ]_{-} + ( \epsilon_k + \epsilon_{\pi} ) ,$$

where the subscripts  $+$  and  $-$  give the signatures of the amplitudes. We find a real contribution mainly in the negative signature amplitude which is nothing else than our statement of a real contribution with negative signature.

As concerns the helicity structure and the parameters  $\alpha$  and  $\alpha'$ ; the first is that hypercharge reactions are dominated by no helicity flip contribution and so is to be the interfering correction. The second is connected to the following two remarks of the authors<sup>/9,10/</sup>. Higher the energy, smaller is the discrepancy from usual absorption model and higher the transfer, bigger it is. This leads to the predictions

$$\alpha_{\text{corr}} < \alpha_{k^* - k^{**}} \sim 0.35 ,$$

$$\alpha'_{\text{corr}} < \alpha'_{k^* - k^{**}} \sim 1 \text{ GeV}^{-2} .$$

Let us now add a few comments. The remark of Sadoulet on the real part can be considered as model independent, though he uses the language of absorption. Very generally, the polarization data show that the real part of  $M_{\pi}$  must be more absorbed than the imaginary part, when  $M_k$ , mainly real, is to be less absorbed to agree with the previous discussion. The extra correction acts mainly as the real.

Sadoulet has also pointed out in his study<sup>/10/</sup>, that most theoretical proposals, if not all, are not able to explain the data. Among them  $k^* - k^{**}$  being complex poles, double Regge exchanges, breaking of exchange degeneracy for  $k^* - k^{**}$  and various absorption models give no natural solution.

### C. NN Charge Exchange Reactions

It is well known that difficulties arise in the explanation of data on NN charge exchange. They are twofold. First,  $p n \rightarrow n p$  being fitted,  $p p \rightarrow n \bar{n}$  is badly described. This appears for the two main classes of models: pion conspiracy<sup>/11/</sup>(Fig. 4) and absorption models<sup>/12/</sup>(Figs. 5 and 6). Secondly, there is a difficulty with the conspiracy mechanism in absorption models<sup>x/</sup>. For instance, conspiracy leads to a need for a strong absorption (more than the usual one) which disagrees with the data<sup>/12/</sup> at  $|t| > 0.3$  (see Fig. 5).

Let us consider the first problem. The data show that we need an extra-contribution, with negative signature interfering with the pion exchange amplitude. The pion exchange amplitude, even corrected by absorption, being real. We need a strong real contribution with negative signature. Let us emphasize that assuming the existence of this extra-contribution with some factorization properties, we predict the good sign of the effect. All contributions of factorizable singularities must be positive in  $p \bar{p} \rightarrow n \bar{n}$  whose t-channel is elastic:  $n \bar{p} \rightarrow n \bar{p}$ . Then we get a constructive interference in  $p \bar{p} \rightarrow n \bar{n}$  and destructive one in  $p n \rightarrow n p$ , where our extra-contribution works in the same way, as the absorption correction - negatively<sup>xx/</sup>.

When studying the proposal  $\alpha \sim 0$ ,  $\alpha' \sim 0$  we can point out that the need of a destructive or constructive interference with pion exchange, if maintained with energy for different transfers (there exist definite experimental indications on this features<sup>/13/</sup>) leads to the following values<sup>xxx/</sup>

$$\alpha \sim \alpha_{\pi} \sim 0 ; \quad \alpha' \sim \alpha'_{\pi}$$

<sup>x/</sup> It is known, that in this case the cut correction to the pion pole is self-conspirative, having an equal contribution in both exchange parities at  $t = 0$ .

<sup>xx/</sup> In fact it should be noticed that the interfering amplitude contributes to the forward peak of the amplitude  $M_{++--}$  (double helicity flip). This means that our calculation verifies the condition of no total helicity flip, better than no helicity flip at a vertex which could be the case from  $\pi N$ . This leads to some difficulties with a possible factorization, as proposed above. We then take the previous result as the sign of the contribution, as an indication for a solution and not a proof.

<sup>xxx/</sup> We get also  $\alpha'_{\pi} \sim 0$  if the pion contribution is reduced to Born term or has a slope smaller than other Regge poles as it is often argued.



As it is summed up in Table I, the analysis of  $\pi N$  and  $NN$  charge exchange and of hypercharge exchange reactions leads to the same set of properties for a contribution to be added:

- real with negative signature;
- with no total helicity flip;
- $\alpha$  and  $\alpha'$  small ( $\approx 0$ );
- central.

### III. Theoretical Considerations

#### A. Some General Features

Let us consider a general singularity in the complex  $l$ -plane. It will be characterized by its trajectory and by strength of its residue (or discontinuity). It appears that the properties 2) and 4) concerning the dependence of spin and transfer of the singularity are the properties of the couplings (residue, discontinuity), especially for a fixed singularity as it has to be. All the problem is then to find singularity in negative signature effective parameters  $\alpha \sim \alpha' \sim 0$  and mostly real in the region of middle energy (i.e. satisfying the properties 1) and 3)).

In fact it is not a simple fact, as it could be stated early. Let us consider Sommerfeld-Watson transformation:

$$M^{-}(s, t) = \frac{1}{2\pi i} \int_C \frac{(2\ell + 1) a^{-}(\ell, t)}{\sin \pi \ell} \pi \frac{(-s)^{\ell} - (s)^{\ell}}{2} d\ell \quad (I)$$

where  $a^{-}(\ell, t)$  is the extrapolated negative signature partial-wave in the  $t$ -channel and  $C$  is the well-known contour around the singularities in the complex  $l$ -plane (Fig. 8). We now show that we need a complex singularity.

Let us assume the singularity to be real, then for the contribution near  $\ell = 0$  will be

$$M^{-} \sim \frac{1}{2\pi i} \int_C a^{-}(\ell, t) d\ell \frac{i}{2}$$

For any real singularity (a pole or a cut near  $\ell = 0$ ),  $M^-$  is mostly imaginary. For instance for a moving pole (the  $\rho'$  in the work of Philipps <sup>111</sup>) we obtain

$$M^-(s, t) \approx S^{\alpha(t)} \frac{i\pi}{2} g(t) \sim \frac{i\pi g}{2}$$

when  $\alpha \sim 0$ . A real contribution with negative signature then corresponds to a complex singularity near  $\text{Re} \ell = 0$ . The main feature of such a singularity is to give rise to two complex poles (Fig. 8). The contribution of such poles in the amplitude is

$$M^-(s, t) \sim h(t) \frac{(-S)^{\alpha(t)} - (S)^{\alpha(t)}}{2} + \text{C.C.}$$

$$h(t) = g(t) \frac{2\alpha + 1}{\sin \pi \alpha}$$

when  $\text{Re} \ell = 0$  it could be written down as

$$M^-(S, t) = |h| \{ \cos(\phi + I \log S)(ch \pi I - 1) + i \sin(\phi + I \log S) sh \pi I \}, \quad (2)$$

where  $h = |h| \exp\{i\phi\}$ ,  $\alpha(t) = iI$ ,  $h$ ,  $\phi$  and  $I$  can depend on  $t$ . The parameters of this oscillating function can easily be fixed in order to obtain an almost completely real contribution at middle energy (Fig. 8). This oscillations are the striking feature of such singularity. Possible consequence in phenomenology at higher or lower energy are discussed in part IV.

In fact, fixed complex Regge poles cannot be a satisfactory answer from the rigorous point of view: they violate the elastic unitarity in  $t$ -channel as all fixed poles on the physical sheet. They have then to be considered as approximation (or effective description) of some other singularities. A useful example is a finite cut along the imaginary axes

$$a^-(\ell, t) = g(\ell, t) (\ell^2 + I^2)^{-\alpha}, \quad \alpha \geq 1/2. \quad (3)$$

The position of the effective poles depends on the residue but in any case they lie within the boundaries of the cut.

Having found singularities obeying our set of properties, the physical problem of the nature of such a singularity remains. The main feature, which completely differs from what we believe about Regge poles is the absence of clear connection with definite t-channel quantum numbers. For instance the quantum numbers for which we have found this phenomena in part II are various as is shown in Table II. In this table we have written the two opposite parities for NN charge exchange. This remark is connected with the conspiracy problem. It was shown in part II that we need a contribution with negative signature interfering with the pion near  $t = 0$ . As it is well-known <sup>/16/</sup> at  $t = 0$  the pion contributes to the amplitude  $A_{++--}(s, t)$  where the signs stand for the helicities of particles ( + or - ) 34, 12 (Fig. 7). For this we need a mechanism of conspiracy (pion conspirator or absorption), in order to fulfill the following equations:

$$A_{++--} = A_{++--}^{(+)} + A_{++--}^{(-)} \neq 0, \quad (4)$$

$$A_{+--+}(t=0) = A_{++--}^{(+)}(t=0) - A_{++--}^{(-)}(t=0) = 0$$

where  $A^{(\pm)}$  are contributions with the definite parity exchange ( + ) or ( - ) to the s-channel helicity amplitudes. Our singularity also could contribute to the different parity exchange amplitudes. However, this contribution should be evasive, by all means, because of the experimental equality <sup>/13/</sup>.

$$\frac{d\sigma}{dt} p n \rightarrow n p = \frac{d\sigma}{dt} p \bar{p} \rightarrow n \bar{n} \quad \text{at} \quad t = 0. \quad (5)$$

So, we have to find a theoretical explanation for such singularities which we know need to have the following general features:

- they are necessary complex,
- they are not connected with a definite set of t-channel quantum numbers.

## B. Field Theory Model .

The argument in favour of the existence of such singularities near  $\ell = 0$  comes from the Lagrangian field theory. The summation of all leading poles of the Feynman diagrams in all orders of the perturbation theory <sup>x/</sup> gives the following expression

$$a^-(\ell, t) = C(t) [v(\ell) - B(t)]^{-2} C^T(t), \quad (6)$$

where  $C$  and  $B$  are some matrices or simple functions known to us as the series in renormalized coupling constant. The main difference in the interaction Lagrangians is in the form of  $v(\ell)$ . For the  $\phi^3$  -

model  $v(\ell) \approx \frac{\ell + 1}{g^2}$ , which corresponds to some moving Regge

poles of  $a^-$ . For the renormalizable theories ( $\gamma^5$  model or  $\sigma$  - model)  $v(\ell)$  have some new fixed singularities (in addition to moving Regge poles due to  $\det(v(\ell) - B(t)) = 0$  the type of which depending on the character of charge renormalization in the theory, i.e., whether the bare charge exists or not. When it exists  $v(\ell)$  has square-root branch points the position of which is determined by this bare coupling constant  $g^2$ .

This fact has a close connection with the scale invariance at small distance which we can illustrate in the case of Schrodinger equation. Any scale invariant at small distance potential must behave like the centrifugal term, i.e.,  $\gamma/r^2$ . The partial wave amplitude has in this case a square root singularity  $[(\ell + 1/2 - \gamma)^2]^{1/2}$ . The same property is supplied by the Yukawa-type potential in the Dirac equation or the Bethe-Salpeter t-channel equation with the scale invariant kernel in the limit of infinite momentum <sup>3/</sup>.

Such singularity automatically guarantees  $a^{\ell=0} = 0$  without contradiction to the unitarity condition. It should be noticed also they are present in any channel with the definite set of t-channel quantum numbers, so they are not connected with some set of quantum numbers, however, the position of this branchpoint is connected with t-channel quantum numbers. It could be estimated as follows.

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<sup>x/</sup> More precisely, the summation of the so-called "end-point" singularities, which contribute to both signatures in distinction with the "pinch"-singularities which contribute to the negative signature only. The experimentally observed exchange degeneracy of the Regge poles is the argument in favor of smallness of the latter contribution.

It was noticed<sup>/3/</sup> that from the identification of the Pomeron with the branchpoint in vacuum channel of highest intercept value  $\alpha'$  it follows the bare coupling constant is small

$$\left(\frac{g'}{4\pi}\right)^2 \approx 0.05.$$

This gives the footing to use the first orders in this parameter and to estimate the position of the branch points in the channels with another exchanged quantum numbers. It appears that in the natural channel with isospin  $I = 1$  exchanged and in the unnatural one with  $I = 0$  exchanged they are two complex conjugated branchpoints at  $\ell \sim \pm 0.55 i$ . So, we are here dealing with a fixed cut of the type  $(\ell^2 + I^2)^{1/2}$  the contribution of which is similar to that of two complex conjugated poles discussed above.

It should be noticed that such fixed cut is proper to both signatures. However it is clear that in the negative signature amplitude it is more noticeable than in positive one because its contribution to the  $S - U$ -crossed reactions is of the opposite sign.

At last, it should be noticed that the above-mentioned notion of centrality, when all the space-time intervals become small (all the relative momenta large), is a-priori different from that of the first part which means only a small impact parameter  $b$  (large  $t$ ). Connection of this two notions seems to exist but is not clear for the moment.

### C. New Singularities in Reggeized Absorption Models

In the Reggeized absorption models we find no singularity satisfying naturally our conditions. In a usual version of the model the amplitude for an inelastic process is written

$$M = R + 2i P * R, \quad (7)$$

where  $R$  is a Regge-pole exchange amplitude,  $P$  is the Pomeron pole exchange and the operator  $*$  is a convolution (the product of  $s$ -channel partial waves). This model can be expressed in terms of singularities of the  $t$ -channel partial wave amplitude  $a^-(\ell, t)$ , or equivalently, of the amplitude  $b^-(\ell, t) = a^-(\ell, t) \sin \pi \ell \Gamma(1-\ell)$ . We have in this case:

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<sup>x/</sup> It is of special interest, that using the same assumption Chiu and Hwa<sup>/16/</sup> could calculate the slopes of Pomeron residues with no free parameter. The agreement with experiment is rather good.

$$M = \int_C b^-(\ell, t) [(-s)^\ell - (s)^\ell] \Gamma(1-\ell) d\ell, \quad (8)$$

where  $C$  is the contour shown in Fig. 8 (dashed line). The model gives the following approximate  $x^{\gamma}$  prescription<sup>7/2/</sup>

$$b^-(\ell, t) = \frac{\beta}{\ell - \alpha(t)} + \gamma \log(\ell - \alpha(0)) \quad (9)$$

$\beta$  and  $\gamma$  are real functions we got from the residues of the Regge and Pomeranchuk poles. Formula (9) can be considered as another definition of the absorption model. In fact, it will be considered not only as an approximation of formula (7) but as an approximation of the "true" definition. We hope that this true definition corresponding to the physical unknown correction, will be precised by the properties of the new singularity.

When considering the properties of  $b^-$  it must be noticed that it can have a singularity at  $\ell = 0$ , a Wrong signature pole ( $\alpha = 0$  in (9)). But such a pole gives no contribution to the physical amplitude  $M$  because the signature factor in (8) has zero at  $\ell = 0$ . Nevertheless, in this case, the two definitions (8) and (9) of absorption model are different. For (8) if  $R$  is zero  $P \neq R$  also and we get no contribution: This is the case of all considered absorption models till now. For (9) with  $\alpha = 0$  on the contrary we get a cut from 0 to  $\infty$  (Fig. 8) which gives a physical contribution

$$M^-(S, t) \sim \frac{i\pi\gamma}{(\log S)^2}$$

We could point out that this cut is obtained in a framework which changes nothing for the usual Regge pole and their corrections.

An important remark is to be made. This result would not be true for the partial wave amplitude  $a^-(\ell, t)$  instead of  $b^-(\ell, t)$ . The Wrong signature pole does not appear in  $a^-$  as a singularity. A coherent definition of our modified model is difficult with  $a^-$ . Our choice of  $b^-$  not  $a^-$  is possible and not so arbitrary, since it corresponds to the Regge formula with no need of ghost-killing factor in the residue. Nevertheless, this choice is to be better understood theoretically.

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<sup>7/2/</sup> This formula is true only at order  $1 / \log S$  and with simple residues.

We have now to refer ourself to a remark which interest in phenomenology has been recently pointed out<sup>/4, 14/</sup>. Two Regge singularities at the same point in the  $l$ -plane "interact" with each other by developing new singularity near the colliding point. In the case of Regge pole colliding with the branch point of its associated cut (in an absorption model, for instance) it was proposed that the pole "extracts" another pole from the cut, with which it forms a complex conjugate pair<sup>/4/</sup>. It was shown also that near the colliding point, the cut and the two poles can be replaced by two effective complex poles for phenomenological use<sup>/5/</sup>.

Here this remark is fundamental. The wrong signature pole is moved out by the interaction and then will give the contribution to the amplitude. If it follows the same procedure it will extract a pole from the cut singularity in order to form a complex conjugated pair of poles which will here play the first role, for it is the dominant singularity (the cut discontinuity goes to zero at  $l = 0$ ).

In this work, we will stay at this point but the matter deserves to be studied in detail. The so-called absorption models are an approximation of physical singularity we do not know. The different changes we make in its definition are not to be considered as some ad hoc addenda to these models. On the contrary the kind of singularity we propose, where the usual dominant singularity, the Regge pole, does not appear, allows us to precise the nature of the cut correction and examine their features and how they are approximated by the starting formula (7).

Let us sum up the main remarks.

Wrong signature poles can be considered as other Regge poles in what concern the origin of cut singularities.

The formula then obtained is also an approximation of the solution for which the cut and the pole interact to give the following singularities at  $a \sim 0$ : a cut and two complex conjugate Regge poles.

On the other side, this points, if checked, can justify a posteriori the absorption model and the conjecture of Zachariasen and others. Now we would notice that the complex pole near  $l = 0$  in negative signature agrees with the main features we have pointed out in part II.

#### IV. Conclusions and Consequences

In this part, after having summed up the main results, we discuss successively how to test the theoretical situation and its possible developments in other reactions or energy regions and the general conclusions to which it leads.

If we had to point out the main features of this work, we would select the following ones:

-the necessity of new complex singularities near  $l = 0$ .

-their existence in field theory with scale invariant "potentials" at small distances,

-their connection with Wrong signature poles in a modified version of Reggeized absorption models.

All these results are for the moment qualitative, as was essentially the whole discussion. We cannot consider the theoretical picture which arises as proved before having looked for stronger tests. For this sake, we would propose two ways: First, an investigation of reactions for which the dominant contributions have  $\alpha \sim 0$ , in particular, the nondiffractive photoproduction reactions, where the trajectories of the dominant Regge poles ( $\rho, A_2$ ) are weakly coupled, and the channels with the exotic exchanges (for instance, backward  $\bar{p} p \rightarrow p \bar{p}$ ). The second way is to perform the more complete quantitative analysis, containing in particular the reactions we have studied. It is worth mentioning the new feature which will have this study when compared with others.

Our work gives a systematic prescription for the second order Regge singularities as compared to usual Regge poles. Knowing that these poles are already tightly connected with low energy results through duality, we expect that this analysis will cover a very large range of energy, if not the whole one. This will give very strong constraints on the solution and provides with good check we need.

As for the consequences and further work, the last remark itself leads to the question to which the new singularities are dual. Without giving an exhausted answer, the very broad oscillations given by two complex poles seem very similar to a "giant resonance effect". For instance in the amplitude  $\nu B^-$  of  $\pi^- p \rightarrow \pi^0 n$  (stands to the signature) a resonance dominance model at low energy shows that the resonances form groups when the imaginary part is strong with alternative sign. As it is shown in Fig.9 the mean effect of the resonances, as a "giant resonance effect" can be well described by the complex poles. It is to be noticed that the amplitude  $B.(-)$  is



just the one for which Dolen, Horn and Schmid<sup>/8/</sup> found a Wrong signature pole at  $\ell = 0$ . This is in agreement with our third result.

This conjecture can have many consequences. Let us quote two of them: The small "wiggles" at low energy, which describe the individual resonances of  $\nu^B$ , as a fine structure of the "giant resonances" we spoke of, could find their dual description in the singularity connected with a Wrong signature pole at  $\ell = -2$ . Equivalently, the resonances in positive signature amplitudes, as appearing upon the Regge pole background, would be related in the same way to the Wrong signature poles at  $\ell = -1$ ,  $\ell = -3$ . The other consequence, which is quite exciting, is that the singularities near  $\ell = 0$  if strongly coupled (as they seem to be in the amplitude  $\nu^B$  for instance) can give appreciable effects at higher energy, and are of interest in explanation of the features of data obtained at Serpukhov. An intensive study will be made in this direction.

Let us now have a more synthetic view on the new singularities and their properties. Their existence, in a field yet intensively studied experimentally as well as theoretically, provides one with some interesting perspectives. First, they seem to show that the building of resonances by duality is not so simple as predicted in Veneziano theory: a diversified structure appears with "giant resonances" and resonances separated in contributions of different level -  $\ell = 0, -1, -2, \dots$  - deserves to be more studied. In the theory already discussed<sup>/3/</sup>, it is striking that the singularity at  $\ell = 1$  (the Pomeranchuk one) is obtained together with the singularities at  $\ell = 0$  in another channels. The singularities at  $\ell = 0$  and  $\ell = 1$  can then be of the same "nature" different from the moving Regge poles with intercept  $1/2$ . This is in contrast with the commonly admitted idea that  $\ell = 0$  singularities to be obtained from second order Regge theory: double Regge exchange.

Let us mention, at last, that the singularities we propose are a good phenomenological laboratory to study the more refined structure of models for cut corrections.

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## References

1. O.Guisan et al. Presented at Ranconte de Moriend, 1971.
2. R.C.Arnold. Phys.Rev., 153, 1523 (1967).  
G.Cohen-Tannoudji, A.Morel, H.Navlet N.C. 48A (1967).
3. V.M.Budnev, A.V.Efremov, I.F.Ginzburg, V.G.Serbo. Preprint JINR E2-5509, Dubna, 1970. V.M.Budnev, I.F.Gunzburg, V.G.Serbo. Journ. of Theor. and Math. Phys., 6, 55 (1971).
4. P. Kaus, F.Zachariasen. Phys.Rev., D1, 2962 (1970).
5. G.A. Ringland and D.P.Roy. P.L. 36B, 110 (1971).
6. B.Schrempp-Otto, F.Schrempp. Preprint DESY 71/46 (August 1971).
7. A.Martin, P.Stevens. Michigan Univ. Preprint MSU-71-19.
8. R.Dolen, D.Horn, C.Schmid,P.R., 166, 1763 (1968).
9. A.C.Irving, A.D.Martin and C.Michael. CERN Preprint TH 1304, 1971.
10. B.Sadoulet. Thesis CERN/D Ph. II Phys. 71-39, 1971.
11. R.I.N.Phillips. N.C. B2, 394 (1967).
12. I.M.Drouffe and H.Navelet, N.C. 2A, 39 (1971).
13. H.Schopper. Proc. of Intern. Symp. on Binary React. JINR Prep. D-6004, 1972.
14. Hung Cheng. Phys.Rev., 130, 1283 (1963).
15. F.Zachariasen. Theory and Practice of Complex Regge Poles, Schladming, 1971.
16. L.Bertocchi. Proc. of Heidelberg Conf. 1968.
17. C.B.Chiu, R.C.Hwa. Phys.Rev., 140, 224 (1971).

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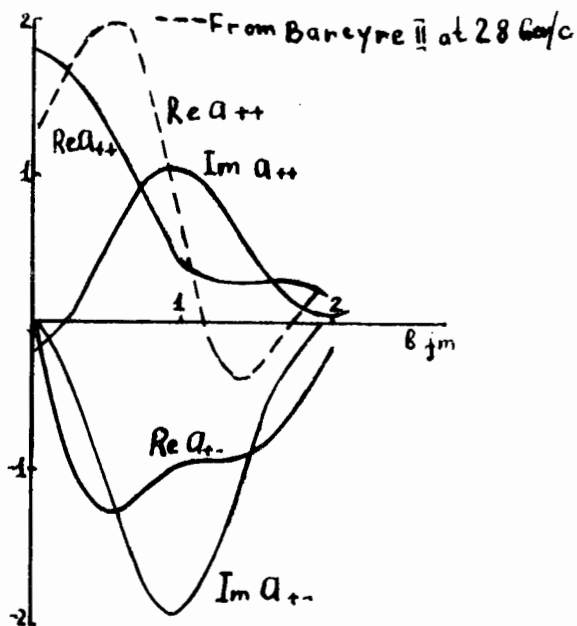


Fig. 1.  $\pi^- p \rightarrow \pi^0 n$ . Impact parameter representation of the s-channel helicity amplitudes (from Ringland, Roy<sup>15</sup>). The continuous lines are computed with use of experimental data at  $p_{lab} = 8 \text{ GeV}/c$ . The dashed line is obtained from phase shift analysis.

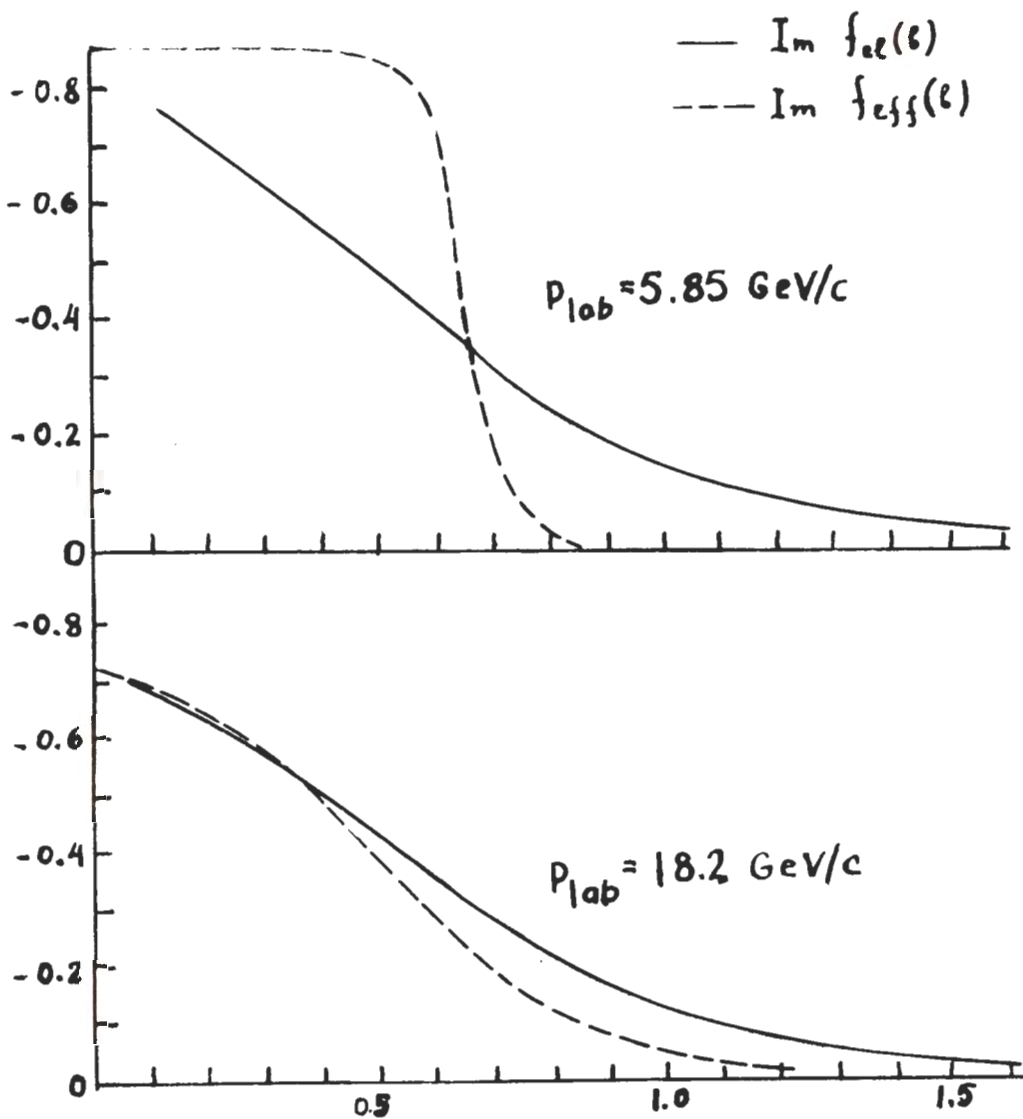


Fig. 2.  $\pi^- p \rightarrow \pi^0 n$ . The effective absorption amplitude compared with the amplitude for elastic scattering (from Martin, Stevens<sup>16</sup>). Elastic scattering amplitude was used here, which gives no big difference with the absorption model of formula (7).

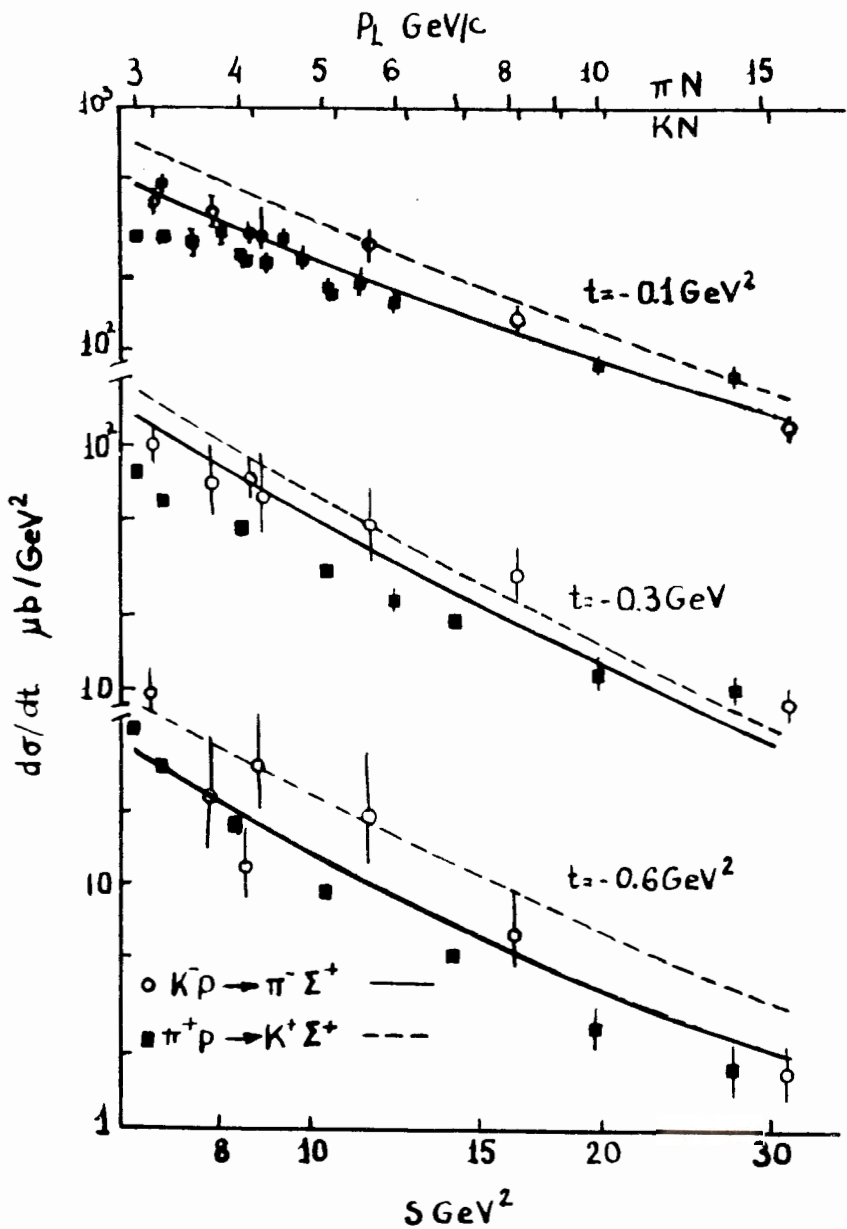


Fig. 3.  $\pi^+ p \rightarrow K^+ \Sigma^+$ ,  $K^- p \rightarrow \pi^- \Sigma^+$  Differential cross sections and prediction of an absorption model at three values of  $t$  (from Irving and others<sup>17</sup>).

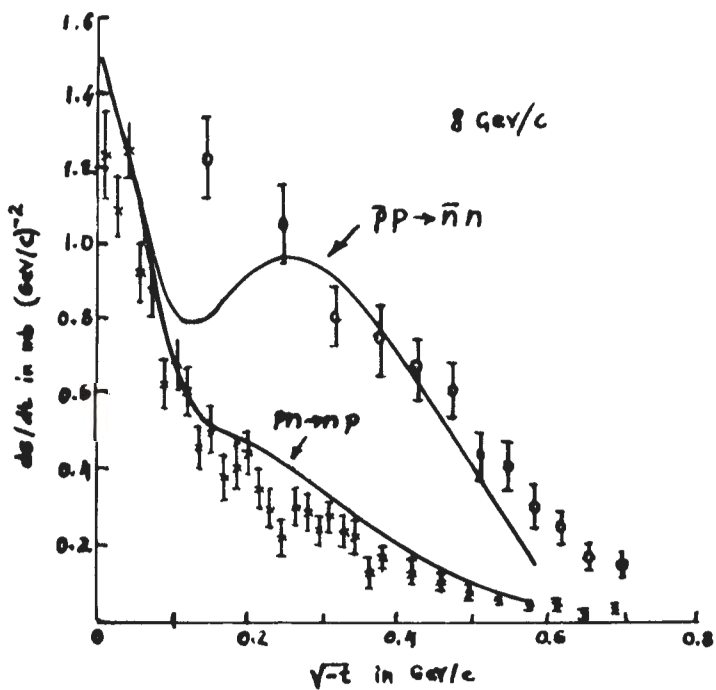


Fig. 4.  $p\bar{n} \rightarrow n\bar{p}$ ,  $p\bar{p} \rightarrow n\bar{n}$ . Pion + Conspirator +  $\rho^-$  model at 8 GeV/c (from Phillips /11/).

Fig. 5.  $pn \rightarrow np$ . Comparison of different absorption models with experiment at  $p_{lab} = 8$  GeV/c (from Drouffe, Navelet<sup>(12)</sup>).  $A_1$ : Absorption model with nondegenerate poles.  $A_2$ : Idem + strong absorption for pion exchange.  $B$ : Absorption model, degenerate poles, strong absorption. (The model  $A_2$  for  $\pi N$ ,  $kN$  and  $NN$  reactions needs 42 parameters, the model  $B$  19).

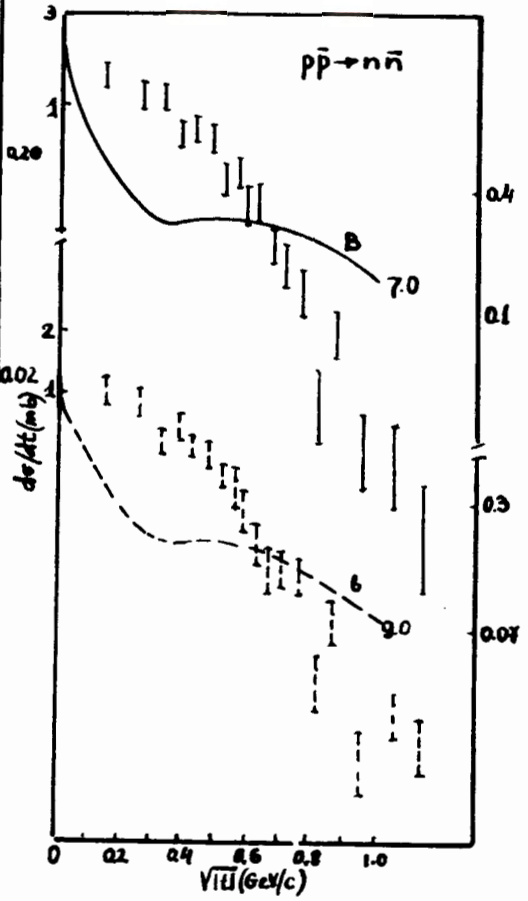
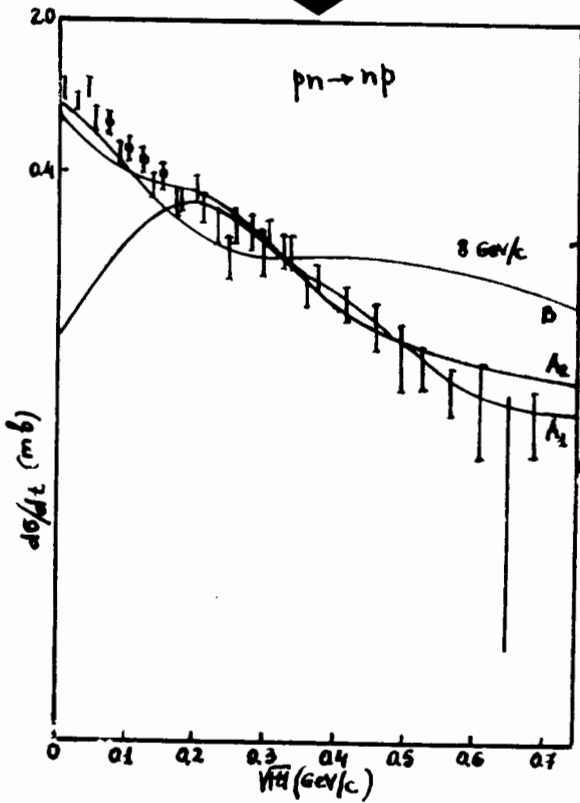


Fig. 6.  $p\bar{p} \rightarrow n\bar{n}$ . Comparison of experimental data with the model  $B$  at 7 and 9 GeV/c (from Drouffe, Navelet<sup>(12)</sup>).

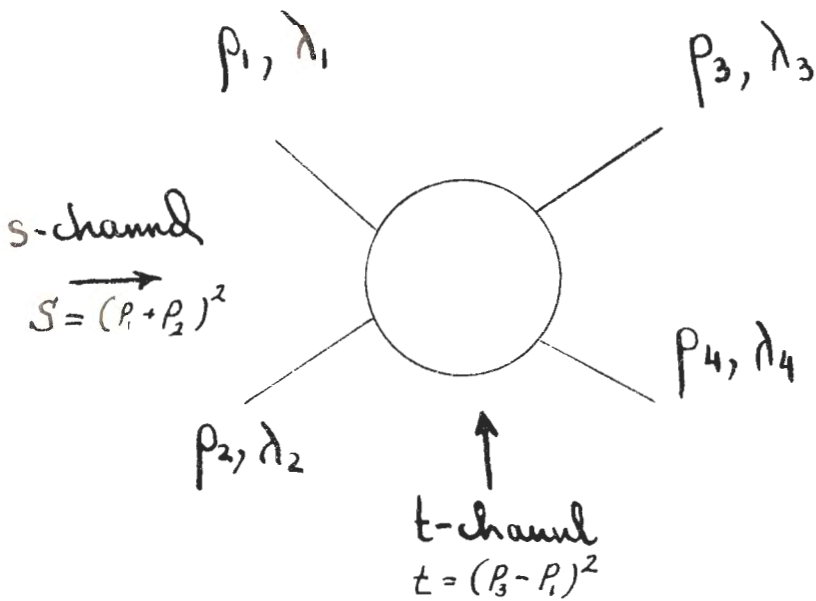


Fig. 7. Kinematics of two body scattering  $p_i$  and  $\lambda_i$  are momentum and s-channel helicity of the particle  $i$ .



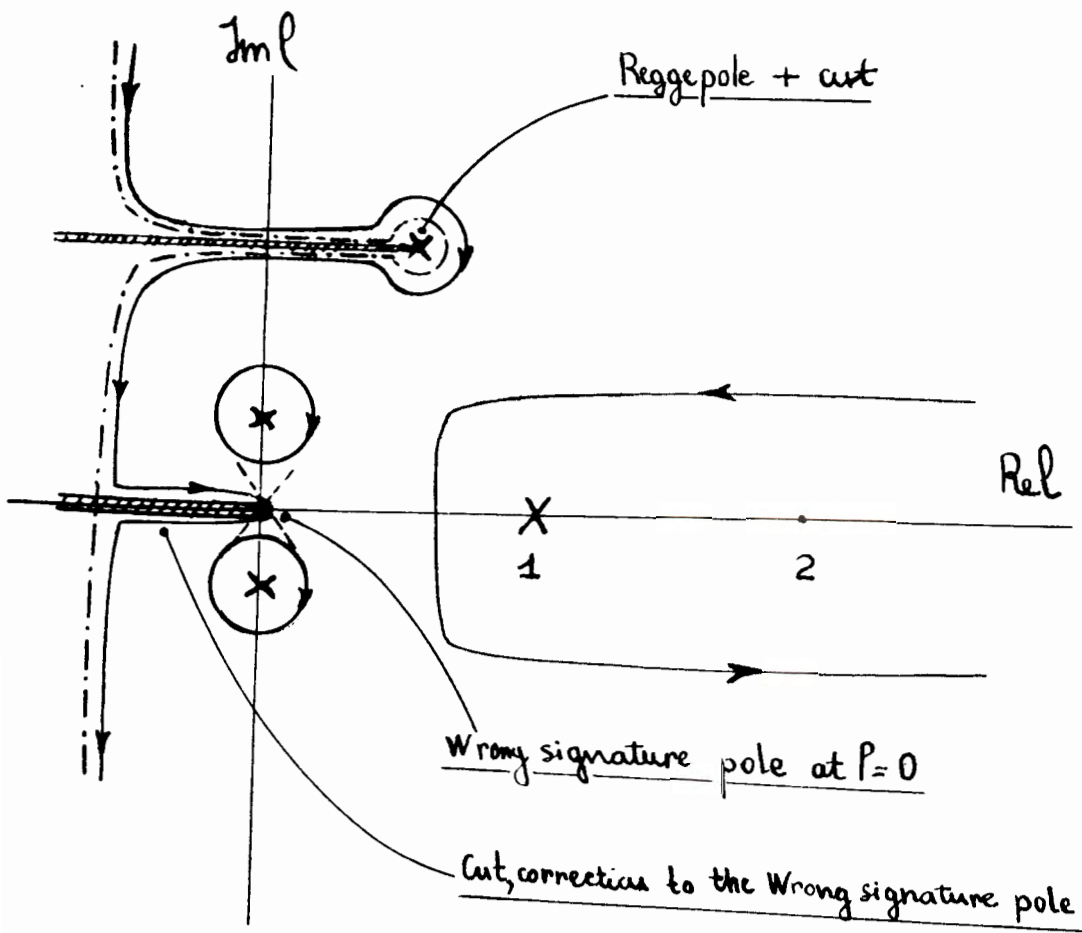


Fig. 8.  $l$ -plane singularities of the amplitude. The crosses are the poles, the continuous line represents the counter of integration of the (modified) Sommerfeld-Watson formula before and after deformation, the dotted-dashed line is the deformed contour: in the case of usual absorption models without the new singularities.

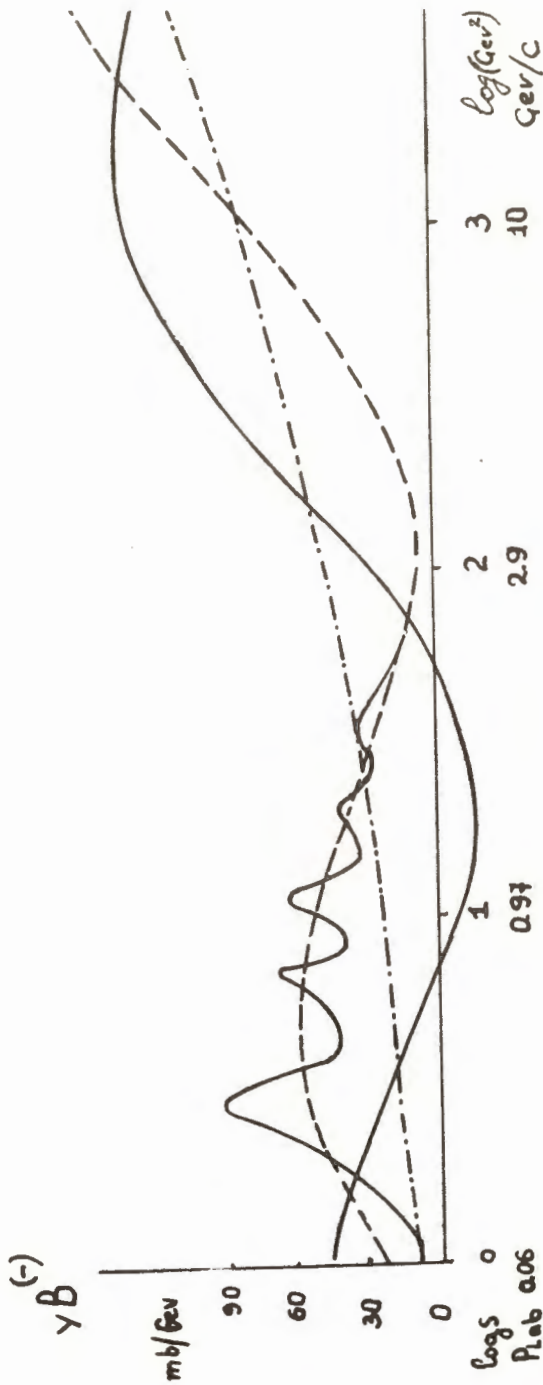


Fig. 9. An example for complex poles contribution in the amplitude  $\nu_B -$  at  $t = 0$ . The continuous thick line is the real part of the total amplitude, the dashed line is the imaginary part, the dotted-dashed line are the Regge contribution, their lines are the calculation of resonances in the imaginary part. The two last curves are taken from Dolen, Horn, Schmid <sup>1/8/</sup>.

Table I,

	$\pi N$	$K N$	$NN$
1) neg. signature strongly real	X	X	X
2) no helicity flip	X	X	X
3) $\alpha$ } small $\alpha'$ }	X	X	X
4) central	X	—	X

x- evidence

Table II. t-channel quantum numbers for which the singularity appears.

Reactions	Y hypercharge	I Isospin	P Parity	G G-parity	G Naturality	Signature
$\pi^+ p \rightarrow \pi^0 n$	0	1	-	+	+	-
$K^+ p \rightarrow \pi^+ \Sigma^+$ $\pi^+ p \rightarrow K^+ \Sigma^+$	1	$\frac{1}{2}$ $\frac{5}{2}$	-		+	-
$p n \rightarrow n p$ and $p \bar{p} \rightarrow n \bar{n}$	0	1	+	+	-	-