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LIGHT CONE BEHAVIOUR OF DUAL MODELS

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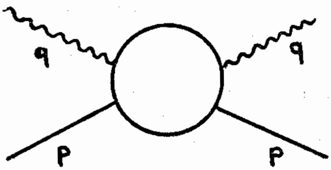
1. Introduction

Recently in many papers [1-6] the connections between light cone singularities of current products on the one side and the scaling or Regge limit on the other have been investigated. The starting point in most cases is the given light cone singularity. We restrict ourselves to deep inelastic scattering on one-particle states (for simplicity we use scalar currents).



The corresponding structure functions W, i.e., the absorptive part, are connected with the matrix element of the current commutator by

$$\langle p|[j(x), j(0)]|p\rangle = \frac{1}{4\pi^2}\int e^{-iqx}\, W(q^2, qp)\, dq \, . \tag{1}$$

The experimental knowledge about W originates from deep inelastic e-p scattering, that means from space-like values of q. However, the support properties of such a matrix element show, that a very important part of the integrations runs over time-like values of q. In the following it is shown, that the light cone singularities derived from the space-like part of the support may be modified by taking into account the full time-like support, too. Of course, if the electromagnetic current is causal, then the structure functions given for space-like q are sufficient for the determination of the leading light cone singularity. In opposite to other works (H.Leutwyler and J.Stern /2/, R.Jackiw et al. /3/) we make no assumptions on the x-space behaviour of the theory (e.g. causality and type of the singularity) from the beginning. Therefore in general it is not allowed to use their support restriction. In section 2 we consider methods for the determination of the leading light cone singular part of the matrix element from the given momentum behaviour of $W(q^2,qp)$. Apart from regular cases we study the conditions under which the time-like support implies modifications. In section 3 we investigate the x-space behaviour of some popular physical models which are formulated in the q-space. They may have connections to deep inelastic scattering or not. In any case they lead to acausal or even non localizable field theories. The model of Nambu shows the expected x^2 singularity apart from a singular factor

in x^0, whereas the Veneziano amplitude and the inclusion of Veneziano terms into loop diagrams lead to non-localizable singularities. With respect to the Veneziano amplitude we have used the simplest off-shell extrapolation which is given by means of the kinematical variables. Of course there are many possible extrapolations, however this is a very natural one and has been used e.g. by Lovelace in his π-π model. On the other side it is this extrapolation which underlies the construction of dual loop amplitudes. For our finally discussed loop amplitude we have the same explicitly given off-shell dependence as for ordinary Feynman diagrams.

2. Determination of the Light Cone Singularities

a) Support Properties and Scaling Variables

Instead of the full current commutator we study at first the Wightman like function

$$M(x^2,x^0) = \langle p|j(x)\,j(0)|p\rangle = \frac{1}{4\pi^2}\int_{q^0>0} dq\, e^{-iqx}\, W(q^2,qp) \tag{2}$$

only. Its support is determined from the representation

$$W(q^2,q_0) = 4\pi^2 \sum_n \delta(p+q-p_n)\,\langle p|j(0)|n\rangle\langle n|j(0)|p\rangle \tag{3}$$

It follows that $W \geq 0$ for $(p+q)^2 = s \geq s_0$ where s_0 denotes the threshold in the s channel. The support includes regions of space-like and time-like q. In the rest system of p $[p=(m,0,0,0)]$ we have

$$q^2 \geqslant s_0 - 2mq_0 - m^2 \quad , \tag{4}$$

Performing the angle integration we may write

$$M(x^2,x^0) = -\frac{i}{2\pi r}\int_0^\infty dq_0 \int dq\, q\, e^{-iq_0x^0}\left(e^{iqr} - e^{-iqr}\right) W(q^2,q_0) \tag{5}$$

with $q = |\vec{q}|$, $r = |\vec{x}|$.
The support properties allow the regularization of this expression by continuation into the lower x_0 half plane $x_0 \to x_0 - i\delta$. It seems to be useful to introduce the variable w, $w = q_0 - q$ which allows a regularization of both integrations

$$M(x^2,x^0) = -\frac{i}{2\pi r}\int_{-m}^{\infty} dw\, e^{-\lambda w}\int_{C(w)}^{\infty} dq\, q\left(e^{-\mu q} - e^{-\nu q}\right) W(q^2,q_0) \tag{6}$$

where $\lambda = \delta + ix^0$, $\mu = \delta + i(x^0 + r)$, $\nu = \delta + i(x_0 - r)$.
The support properties determine the integration boundaries

$$-m \leq w < \infty \,, \quad \mathrm{Max}\left(\frac{s_0 - (m+w)^2}{2(m+w)}, 0\right) \leq q < \infty \quad . \tag{7}$$

We mention that the intervals $-m \leq w < 0$ and $0 \leq w < \infty$ correspond to space-like and time-like regions of q , respectively. Asymptotically w approaches the usual scaling variable $\omega = -\frac{q^2}{2mq_0}$ up to a factor $-m$.

b) Determination of the singularity at $x^2 = 0$

From an inspection of the expression (6) we learn that the q-integral defines

$$G(x^2,x_0,w) = \frac{1}{r}\int_{C(w)}^{\infty} dq\, q\left(e^{-\mu q} - e^{-\nu q}\right) W(q^2,q_0) \tag{8}$$

which is a holomorphic function of the variables x^2 and x^0 for $\mathrm{Im}\, x^0 < 0$ and $\mathrm{Im}\, x^2 \neq 0$ (take into account that $G(x^2,x_0,w)$ appears as an even function of r). Dividing the q - integral into a finite and an infinite part for the corresponding expressions of the function M , $M = M_1 + M_2$ it becomes obvious, that M_1 is an entire function of x^2 , provided that the following w -integration exists in the conventional regularization (i.e. an exponential growing in w has to be excluded). Therefore M_2 contains all of the singularities in x^2 which are independent of the lower boundary of the q -integration. For $\delta > 0$ both integrations are absolutely convergent, so that we can perform the q -integration at first. This procedure implies that it is sufficient for determining the x^2 singularity to take the q -asymptotics of W at every fixed w .

$$W(q^2,q_0) \approx q^{\sigma} g(w) + \cdots \qquad \text{for } q \to \infty,\ w \text{ fix.} \tag{9}$$

The strongest growing term in q corresponds to the leading x^2 singularity. Insertion of the leading part into eq. (6) gives

$$M(x^2,x_0) \sim \frac{x_0^{\sigma+1}}{(x^2)^{\sigma+2}} \int_{-m}^{\infty} dw\, e^{-iw(x^0-i\delta)}\, g(w) \tag{10}$$

which is valid for all fixed $x_0 \neq 0$ near the light cone. From a physical point of view it is interesting that the limit $q \to \infty$, w fix, is equivalent to the limit $q^2 \to \infty$, w fix. That means the x^2 singularity is determined by the large mass behaviour as it is known from the two-point function. However for the four point function this rule may be modified by the w-integration in irregular cases. More complicated is the case $W(q^2,q_0) \approx q^{\sigma(w)} g(w)$. If $\sigma(w)$ arrives the maximum at $w = w_0$ then the contribution from the region $w \approx w_0$ produces the leading singular part

$$M(x^2,x_0) \sim \frac{x_0^{\sigma(w_0)+1}}{(x^2)^{\sigma(w_0)+2}\,[\log(-x^2)]^{1/2}} . \tag{11}$$

Continuating our investigations of the general properties of expression (10) we have to note the possibility that the function

$$f(x^0) = x_0^{\sigma+1} \int_{-m}^{\infty} dw\, e^{-iw(x^0-i\delta)}\, g(w) \tag{12}$$

may be singular for $x_0 \to 0$. The origin of such a singularity would be the behaviour of $g(w)$ for $w \to \infty$, i.e., the properties of $W(q^2,q_0)$ for time-like q. The knowledge of $W(q^2,q_0)$ for $q^2 < 0$ only is not sufficient to determine the function $f(x^0)$. We remark that a singularity of $f(x_0)$ does not imply a singularity in x_0 of the matrix element $M(x^2,x_0)$ in general. Such fictious singularities of the matrix element may appear by the way of exhibiting the leading singularity in x^2. As an instructive example we take $W = q_0^{-1}\,\Theta(q_0)\,\Theta(q^2)$. Formula (10) gives the leading singularity

$$M \sim \frac{1}{x_0}\frac{1}{x^2} \qquad \text{for} \quad x^2 \approx 0 \tag{13}$$

which is to be compared with the exact result

$$M = \text{const}\left\{ \frac{1}{r^2}\frac{x_0}{x^2} - \frac{1}{2r^3}\log\frac{\delta + i(x_0 - r)}{\delta + i(x_0 + r)} \right\} . \tag{14}$$

Taking $x_0 = 0$ in the latter expression we have

$$M \sim \frac{1}{r^3} \tag{15}$$

This example demonstrates another peculiar property of the singular case, namely the light cone singularity approached along $x_0 = 0$ is stronger as in the case $x^2 \to 0$, $x_0 \neq 0$.

Considering $f(x_0)$ for $x_0 \to \infty$ the position $w = 0$ in the remaining integral becomes important. Assuming $g(w) \sim w^{\tau}$ $(w \to 0)$ we have $f(x_0) \sim x_0^{\sigma-\tau}$ $(x_0 \to \infty)$. This result is valid for regularizable w integration. There is a general connection between the behaviour of $f(x_0)$ for $x_0 \to \infty$ on the one side and the usual Regge limit of $W(q^2, q_0)$ on the other (compare also ref. /4/). In the Regge limit $[q \to \infty, qw \text{ fix}]$ we get

$$W \approx q^{\sigma} g(w) \approx q^{\sigma-\tau} \approx q^{\alpha(0)} \tag{16}$$

by taking into account the zero w behaviour of $g(w), [g(w) \sim w^{\tau}]$ Therefore from eq. (12) we obtain

$$f(x^0) \sim x_0^{\alpha(0)} \quad \text{for } x_0 \to \infty \; , \tag{17}$$

However, it may be that the next non-leading term in the limit $q \to \infty, w$ fix becomes dominant for the Regge limit. This mechanism which allows a different q-behaviour in the Regge and in the scaling limit has been discussed by Rühl /6/ starting from the x-space properties.

The foregoing method breaks down if the remaining w integration is not finite within the usual regularization. Examples are $g(w) \sim w^{-1}$, $(w \to 0)$ or $g(w) \sim e^{w}$, $(w \to \infty)$. Such cases may occur for quite acceptable functions $W(q^2, q_0)$ which allow a usual Fourier transform. Examples of this type are the following

$$W = \frac{\theta(w)}{1+wq} \xrightarrow[w \text{ fix}]{q \to \infty} \frac{1}{w}\frac{1}{q} \; , \quad M \sim \frac{\log(-x^2)}{x^2}$$

$$W = \frac{\theta(w)\, q^{n-1}}{1+wq^n} \xrightarrow[w \text{ fix}]{q \to \infty} \frac{1}{w}\frac{1}{q} \; , \quad M \sim \frac{n \log(-x^2)}{x^2}$$

$$W = \frac{\theta(w)\, e^{q}}{q(1+we^{q})} \xrightarrow[w \text{ fix}]{q \to \infty} \frac{1}{w}\frac{1}{q} \; , \quad M \sim \frac{x_0}{(x^2)^2} \tag{18}$$

$$W = \frac{\theta(w)\, e^{w}}{q^{-\sigma}+e^{w}} \xrightarrow[w \text{ fix}]{q \to \infty} q^{\sigma} e^{w} \; , \quad M \sim \frac{\Gamma(2+ix_0\sigma)}{\sin i x_0 \pi}(-x^2)^{-2+ix_0\sigma} + \frac{\Gamma(2+\sigma)}{ix^0-1}(-x^2)^{-2-\sigma} + \cdots$$

Obviously the divergence of the w integration transforms into a more complicated x^2 singularity than it is expected from the q- asymptotics itself. To overcome this difficulty we have to study the origin of this divergence. In the first case our function has the structure

$$W = \frac{B(q)}{A(q)+w} \qquad \text{with } A(q) \to 0 \text{ for } q \to \infty \; . \tag{19}$$

It is needed to perform the w integration over the irregular region $w \approx 0$ at first

$$\int_0^{\eta} dw\, e^{-iwx_0} \frac{B(q)}{A(q)+w} \sim B(q) \log A(q) \tag{20}$$

and afterwards

$$M \sim \int dq\, q \left(e^{-\mu q} - e^{-\nu q}\right) \frac{1}{q} B(q) \log A(q) . \tag{21}$$

A typical peculiarity of the irregular cases is the possibility that different leading light cone singularities give identical scaling limits. This should be compared with the result of Rühl/6/ for the Regge limit.

c) Discussion of Current Commutators

The one-particle matrix element of hermitian currents is given by

$$C(x) = \langle p | [j(x), j(0)] | p \rangle = \langle p | j(x) j(0) | p \rangle - \langle p | j(-x) j(0) | p \rangle \tag{22}$$

where translation invariance has been used. In terms of the Wightman-like functions the commutator is determined by

$$C(x) = M(x^2 - i\varepsilon(x_0)\delta, x_0 - i\delta) - M(x^2 + i\varepsilon(x_0)\delta, -x_0 - i\delta) \tag{23}$$

From this representation we conclude that the fulfillment of both conditions

1. $\lim_{\delta \to 0} M(x^2, x_0 - i\delta)$ exists for all real x_0
2. $M(x^2, x_0) = M(x^2, -x_0)$ for real x_0, $x^2 \neq 0$

leads to a causal commutator. The importance of the first condition is demonstrated by the simple example $M = x_0^{-n} (x^2)^{-1}$. The evaluation of the corresponding $C(x)$ gives

$$\begin{aligned} C(x) = {} & \left(1 + (-1)^n\right) \left[i\pi\, \varepsilon(x_0)\, \delta(x^2) \frac{1}{(x_0)^n} - \frac{i\pi}{(n-1)!} \delta^{(n-1)}(x_0) \frac{1}{x^2} \right] \\ & + \left(1 - (-1)^n\right) \left[\frac{1}{x_0^n} \frac{1}{x^2} - \frac{\pi^2}{(n-1)!} \delta^{(n-1)}(x_0)\, \varepsilon(x_0)\, \delta(x^2) \right] . \end{aligned} \tag{24}$$

(The singular terms $\frac{1}{x_0^n}$ and $\frac{1}{x^2}$ are defined as principal values).

A careful consideration using test functions shows that the upper two conditions are also necessary. If one starts from the assumption that the electromagnetic current behaves causally (as did H.Leutwyler, J.Stern/2/ and R.Jackiw et al./3/) the skew symmetry of the commutator requires an ansatz of the following type

$$C(x) = \varepsilon(x_0)\, \delta^{(n)}(x^2)\, \tilde{f}(x_0) \tag{25}$$

with a symmetric function $\tilde{f}(x_0)$. Consequently the scaling function

$$g(w) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dx_0\, e^{i w x_0} x_0^{-n} \tilde{f}(x_0) \tag{26}$$

is also symmetric or skew symmetric, respectively and the experimental data given for $-m < w < 0$ determine $g(w)$ for $0 < w < m$ too. If in the contrary case one would like to test causality, the knowledge of $g(w)$ in the full support $-m < w < \infty$ is needed in order to prove the fulfillment of the two conditions. We remark the surprising fact, that in the case of a causal commutator with a leading light cone singularity of non-integer type the support of $g(w)$ is not compact. This follows from the symmetry of $M(x^2; x_0)$ and consequently of $\rho(x_0)$ (compare eq. (12)).

From investigations of the two-point function we know that non-localizable behaviour (e.g. $W \sim e^{aq^2}$, $q^2>0$.) leads to an acausal commutator. The same should be true for the four-point function. Therefore the symmetry and regularity condition should be completed by the property of localizability. Whatever the result may be, - a causal or an acausal commutator - the function $M(x^2; x_0)$ is always well defined. There are examples for the validity of the reduction formulae in acausal theories, too. So it seems to be justified to take eq. (1) as basic relation connecting x and p space functions in every case.

3. Consideration of Special Models for W

In this section we shall study the light cone singularities corresponding to some physical models or special Feynman graphs.

a) Nambu's Model for Deep Inelastic Scattering

In the spirit of our simplifications we take for $W(q^2, q_0)$ his function W_2 which is an absorptive part for virtual Compton scattering/7/

$$W = \frac{\Gamma(N-2kq^2)}{\Gamma(N+1)\Gamma(-2kq^2)} \left\{ \frac{\Gamma(\frac{N}{2}+\beta)\,\Gamma(\gamma-kq^2)}{\Gamma(\frac{N}{2}+\beta-kq^2)\,\Gamma(\gamma)} \right\}^2 \qquad (27)$$

$N = s-m^2$, β , γ , k parameters.

Nambu's model is a resonance model containing poles only. The interpolated residua are taken as the absorptive part For suitable values of the parameters β, γ, k the usual scaling behaviour for deep inelastic scattering is obtained. According to our previous method we investigate the foregoing expression in the limit $q \to \infty$, w fix . The usual variables are expressible as

$$\begin{aligned} s &= [(m+w)^2 + 2(m+w)q] \\ q^2 &= 2wq + w^2 \\ q_0 &= q + w \end{aligned} \qquad (28)$$

The result is

$$W(q^2, q_0) \underset{\substack{q\to\infty \\ w \text{ fix}}}{\sim} q^{2\gamma-\frac{3}{2}} |w|^{2\gamma-\frac{1}{2}} (m+w)^{2\beta-\frac{3}{2}} (m+w-2kw)^{\frac{1}{2}-2\beta} . A$$

$$A = \begin{cases} \text{const} & \text{for } -m<w<0 \\ \dfrac{\sin 4\pi kwq}{\sin^2(2kwq-\gamma)\pi} & \text{for } w>0 \end{cases} \qquad (29)$$

We choose $0<k\leq\frac{1}{2}$ to avoid additional singularities of $g(w)$ in the physical region. The parameter β is connected with the asymptotic behaviour of the formfactors $((q^2)^{-\ell}$, $\beta = \ell + \gamma$. This formfactor behaviour is also contained in

the expression (29) by means of the limiting procedure $q \to \infty$, $(m+w)q$ fix . In this sense one formula allows different behaviours for the Bjorken, Regge and form-factor limits, respectively. For $q^2 > 0$ the expression (27) contains poles in the variable q^2 . To overcome this difficulty we take $q \to \infty$ along a ray with a small angle to the real axis in the same manner as it is originally used by Veneziano and replace the quotient of the sinus functions by a constant. The x-space behaviour may be read off from the following table.

	$\gamma \neq \frac{1}{4}, k \neq \frac{1}{2}$	$\gamma \neq \frac{1}{4}, k = \frac{1}{2}$	$\gamma = \frac{1}{4}, k \neq \frac{1}{2}$	$\gamma = \frac{1}{4}, k = \frac{1}{2}$
W $q \to \infty, w$ fix	$q^{2\gamma-\frac{3}{2}} g(w)$		$q^{-1} g(w)$	
M $x^2 \to 0$	$\frac{f(x^0)}{(x^2)^{2\gamma+\frac{1}{2}}}$		$\frac{f(x^0)}{x^2}$	
$g(w)$ $w \to 0$	$w^{2\gamma-\frac{1}{2}}$		const	
$f(x^0)$ $x^0 \to \infty$	x^{0-1}		x_0^{-1}	
$g(w)$ $w \to \infty$	$w^{2\gamma-\frac{3}{2}}$	$w^{2\beta+2\gamma-2}$	w^{-1}	$w^{2\beta-\frac{3}{2}}$
$f(x_0)$ $x_0 \to 0$	const	$x_0^{\frac{1}{2}-2\beta}$	$\log x_0$	$x_0^{\frac{1}{2}-2\beta}$
W q fix $w \to \infty$	$w^{4\gamma-3} k(q)$	$w^{4\gamma+2\beta-\frac{5}{2}} k(q)$	$w^{-2} k(q)$	$w^{2\beta-\frac{5}{2}} k(q)$

The last line gives a hint that the complete matrix element is regular at $x_0 = 0$. However the symmetry property is not satisfied so that we expect an acausal commutator. A suitable choosen parameter γ reproduces the experimental realized scaling behaviour but the corresponding light cone singularity is not the canonical one.

b) Veneziano Model

Here we investigate the crossing symmetric model

$$V(s,t) = B(-\alpha_s, -\alpha_t) + B(-\alpha_s, -\alpha_u) + B(-\alpha_u, -\alpha_t) \tag{30}$$

We want to study the x-space behaviour of this well known model. For our considerations we use the simplest off-shell extrapolation which is given by means of the kinematical variables

$$\begin{array}{lll} \alpha_s = a + bs & \alpha_t = a & s = m^2 + q^2 + 2mq_0 \\ \alpha_u = a + bu & & u = m^2 + q^2 - 2mq_0 \end{array} \tag{31}$$

This extrapolation has been used in other connection and underlies the construction of dual loop amplitudes. To get the absorptive part along the right hand cut we take the interpolated residua of the first and the second term

$$W = \frac{\sin \pi a}{a} \Gamma(-a) \frac{\Gamma(1+a+\alpha_s)}{\Gamma(1+\alpha_s)} + (-1)^{\alpha_s} \frac{\Gamma(-\alpha_u)}{\Gamma(1+\alpha_s)\Gamma(-\alpha_s-\alpha_u)} . \tag{32}$$

The first (s,t)-term leads to

$$W^{(s,t)} \sim q^{a}(m+w)^{a} \qquad \text{for } q\to\infty,\ w \text{ fix.} \tag{33}$$

There is no difference between the Regge and the Bjorken limit. The corresponding light cone singularity is $f(x_0)\ (x^2)^{-a-2}$ with regular $f(x_0)$. For the second (s,u)-term a detailed consideration is required.

$$W^{(s,u)}_{\substack{q\to\infty\\ w\ \text{fix}}} \sim q^{-\frac{1}{2}} A(q,w) \begin{cases} (-1)^{a+bm^2+2b(m+w)q} & -m<w<0 \\ \sin\pi(2a+2bm^2-4bwq) & 0<w<m \\ \text{const} & m<w<\infty \end{cases} \tag{34}$$

where

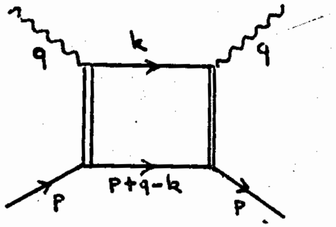
$$A(q,w) = (m+w)^{-2b(m+w)q}\,|m-w|^{2b(m-w)q}\,|-2w|^{4bwq} \tag{35}$$

This term shows some strange features. At first it should be remarked, that the positive definiteness of the forward absorptive part is violated in the (s,u)-term for $-m<w<m$. This may be viewed as a consequence of the well known ghost difficulties of this model. Furthermore for $-m<w<0$ and $w>0$ $A(q,w)$ growth exponentially for $q\to\infty$. If we take

this (s,u)-term seriously, we should conclude, that the model behaves non-locally and consequently the current commutator is acausal.

c) Diagrams with One Loop

We expect essential modifications if dual amplitudes are incorporated into more complicated diagrams by means of a loop integration. Especially we want to build in the s-t part $B(-\alpha_s,-\alpha_t)$ of the Veneziano model into a box diagram. This amplitude gains further interest from the fact that it is a part of a dual loop amplitude, which itself is too difficult for an investigation in this kinematical region for the moment



The corresponding amplitude

$$T(s,q^2) = \int d^4k\ \frac{B(-\alpha_s,-\alpha_{t'})\,B(-\alpha_s,-\alpha_{t'})}{(k^2-m^2)\left((p+q-k)^2-m^2\right)} \tag{36}$$

$$t' = (q-k)^2$$

has according to the Cutkowsky rules the absorptive part

$$W \sim \int d^4k\ \delta_+(k^2-m^2)\,\delta_+\left((p+q-k)^2-m^2\right)\,|B(-\alpha_s,-\alpha_{t'})|^2 . \tag{37}$$

Using the θ and δ functions we perform some integrations and obtain

$$W \sim \frac{\theta(s-4m^2)}{2q} \int_{\mathrm{Max}\{m,k_0^-\}}^{\mathrm{Min}\{m+q_0,k_0^+\}} dk_0 \; |B(-\alpha_s,-\alpha_{\bar{t}})|^2 \tag{38}$$

where

$$k_0^{\pm} = \frac{m+q_0}{2} \pm \frac{q}{2}\sqrt{1-\frac{4m^2}{s}}\,, \quad \bar{t} = 2m(k_0-q_0)\,. \tag{39}$$

Introducing the variable w we obtain for large q at W fixed

$$k_0^- = \frac{m+w}{2} + \frac{m^2}{2(m+w)}\,, \quad k_0^+ = \frac{m+w}{2} - \frac{m^2}{2(m+w)} + q \tag{40}$$

which are the proper boundaries of the k_0 integration. If the t' threshold satisfies $t_0' \geq m^2$ (which will be assumes in the following) the absorptive part given by eq. (38) is positive definite for all values of q^2.

As the simplest example we take for B the simple Feynman propagator $(t'-M^2)^{-1}$ with the result

$$W(q,w) \sim \frac{\mathrm{const}}{q}\left\{\frac{1}{m-w-\frac{m^2}{m+w}-\frac{M^2}{m}} - \frac{1}{m-w+\frac{m^2}{m+w}-\frac{M^2}{m}-2q}\right\}$$
$$\sim \frac{1}{q}\,\frac{1}{m-\frac{M^2}{m}-w-\frac{m^2}{m+w}} \qquad (q\to\infty,\ w\ \text{fix})\,. \tag{41}$$

The corresponding leading singularity is $f(x_0)\cdot\frac{1}{x^2}$ where $f(x_0)$ develops a logarithmic singularity for $x_0\to 0$. Of course this is not the complete result in the fourth order of perturbation theory, especially the disconnected Feynman graphs contribute to $W(q^2,q_0)$. In x-space they may change the symmetry properties but cannot weaken the light cone singularity.

Turning to the loop diagrams with Veneziano terms we convince ourselves that the important contributions to the integral (38) come from the upper and lower boundaries. At the upper boundary the integrand behaves like

$$|B|^2 \sim \frac{\sin^2\pi(\alpha_s+\alpha_{\bar{t}})}{\sin^2\pi\alpha_s}\,\Gamma^2_{(-\alpha_{\bar{t}})}\,[2(m+w)q]^{2\alpha_{\bar{t}}} \tag{42}$$

If we replace the first quotient by a constant then the k_0 integral from the region $k_0 \approx k_0^+$ gives finally

$$W \sim \frac{q^{\sigma(w)}}{\log q}\,\Gamma\!\left(-\frac{\sigma(w)}{2}\right)(m+w)^{\sigma(w)} \tag{43}$$

$$\sigma(w) = 2\alpha_{(\bar{t})} = a - 2mbw + mb\left(m+w-\frac{m^2}{m+w}\right)$$

which is valid for all w. For $w>0$ the contribution of the lower boundary is negligible, whereas for $-m<w<0$ it becomes dominant. In this region we have

$$|B|^2 \sim \frac{1}{\sin^2\pi\alpha_s}\exp 4bq\left\{m\log\frac{m}{m+w} + w\log\frac{-w}{m+w}\right\} \tag{44}$$

which growths exponentially. This property carries over to the integrated expression (38). The conventional rule of dropping terms multiplied by $\sin^{-2}\pi\alpha_s$ seems to be not applicable because of the exponential growth of the remaining part. Therefore we conclude that this loop diagram leads to non-local matrix elements M and consequently to an acausal commutator. Possible contributions of disconnected paths - which are not clear in a dual theory - do not change this conclusions. This result is quite surprising because the s-t term in the usual Veneziano model shows a potential behaviour. On the other hand it is already known /8/ that this amplitude has an essential singularity in the variable s.

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