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COMPOSITENESS AND VANISHING OF RENORMALIZATION CONSTANTS

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I. Introduction

Many authors [1] have demonstrated within the framework of model field theories that under vanishing of wavefunction and vertex function renormalization constants an elementary particle can be considered equivalent to a bound state. Similar conclusions [2] have been obtained for the equivalence between an unstable elementary particle and a resonant state. The above conclusions have been obtained for the case in which the bound state or the resonant state in the question consists of two particles. The aim of this paper is to investigate under what conditions an elementary particle can be regarded equivalent to a three particle bound state. To consider this problem we work within the framework of a soluble model field theory which consists of five particles U, V, N, θ and θ_1. U, V and N are fixed in space but θ and θ_1 can have nonrelativistic motion. We demonstrate that the irreducible part of the $N\theta\theta_1 \to N\theta\theta_1$ scattering amplitude develops a pole corresponding to $N\theta\theta_1$ bound state, this pole induces a pole in the reducible part of the scattering amplitude and these two poles do not cancel with each other, i.e., the Jin-MacDowell [3] cancellation does not hold good for a three particle bound state. Conditions are found under which the U-particle can be regarded equivalent to the $N\theta\theta_1$-bound state. It is noticed contrary to two particle case that vanishing of wave function and vertex function renormalization constants are not sufficient conditions for the equivalence between U-particle and $N\theta\theta_1$-bound state.

2. Model Field Theory

The Hamiltonian of the model field theory under consideration is given by

$$H = m_u^{(0)} \int d^3p\, \phi_u^+(p)\, \phi_u(p) + m_v^{(0)} \int d^3p\, \phi_v^+(p)\, \phi_v(p) +$$

$$+ m_N \int d^3q\, \phi_N^+(q)\, \phi_N(q) + \int d^3k \left(\frac{k^2}{2\mu} + \mu\right) \phi^+(k)\, \phi(k) +$$

$$+ \int d^3k_1 \left(\frac{k_1^2}{2\mu_1} + \mu_1\right) \phi_1^+(k_1)\, \phi_1(k_1) +$$

$$+ \left[\frac{g}{(2\pi)^{3/2}} \int d^3p\, d^3k\, \phi_v^+(p)\, \phi_N(p-k)\, \phi(k)\, f(k) + \text{h.c.}\right]$$

$$+ \frac{\lambda}{(2\pi)^6} \int d^3k\, d^3k_1\, d^3k'\, d^3k_1'\, d^3q\, d^3q'\, \delta^{(3)}(q + k + k_1 - q' - k' - k_1') \times$$

$$\times \phi_N^+(q')\, \phi^+(k')\, \phi_1^+(k_1')\, \phi_N(q)\, \phi(k)\, \phi_1(k_1)\, f(k)\, f_1(k_1)\, f(k')\, f_1(k_1')$$

$$+ \left[\frac{g}{(2\pi)^{3/2}} \int d^3p\, d^3k_1\, \phi_u^+(p)\, \phi_v(p - k_1)\, \phi(k_1)\, f_1(k_1) + \text{h.c.}\right]$$

$$+ \left[\frac{\lambda_1}{(2\pi)^3} \int d^3p\, d^3k\, d^3k_1\, \phi_u^+(p)\, \phi_N(p - k - k_1)\, \phi(k)\, \phi_1(k_1)\, f(k)\, f_1(k_1) + \text{h.c.}\right]. \tag{1}$$

The scattering amplitude for the process $N\theta\theta_1 \to N\theta\theta_1$ in the c.m. system can be written as

$$T(E,k) = f^2(k)\, f_1^2(k)\, \Gamma^2(E,k)\, \Delta_u'(E,k) + T_1(E,k), \tag{2}$$

where $T_1(E,k)$ is that part of the $N\theta\theta_1 \to N\theta\theta_1$ scattering amplitude which contains no contribution of U-particle, $\Delta_u'(E,k)$ is the U-particle complete propagator and $\Gamma(E,k)$ is the $UN\theta\theta_1$ vertex function.

$T_1(E,k)$ satisfies an integral equation which is represented diagramatically in Fig. I. The solution of this integral equation is discussed in the Appendix and we find

$$T_1(E,k) = D(E,k) + C(E,k),$$

where $D(E,k)$ and $C(E,k)$ represent contributions from disconnected and connected Feynman diagrams, respectively. The integral equation of the U-particle complete propagator, in terms of the

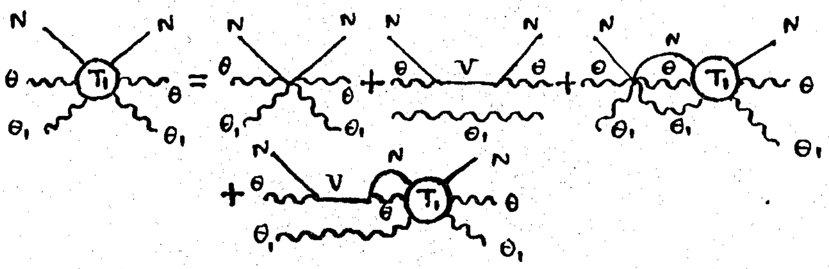

Fig. I. Integral equation for the irreducible part of the $N\theta\theta_1 \to N\theta\theta_1$ scattering amplitude

connected part C , is given in Fig. 2. In these diagrams the thick lines represent complete U -particle propagator and double lines represent complete V -particle propagator.

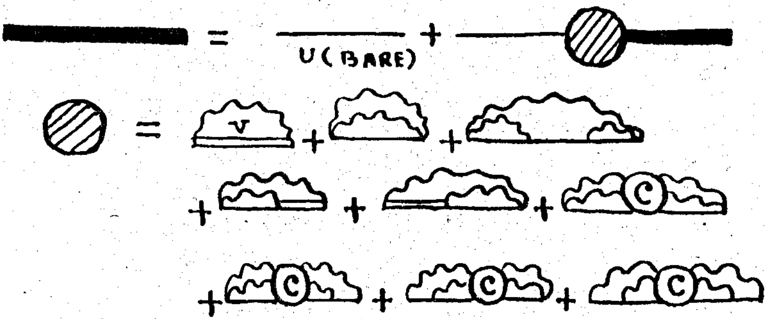

Fig. 2. Integral equation for the U -particle propagator

Therefore the U -particle complete propagator is given by

$$\Delta'_u(E) = [E - m_u^{(0)} - \Sigma(E)]^{-1}$$

$$
\begin{aligned}
\Sigma(E) &= \frac{g_1^2}{(2\pi)^3} \int d^3 q_1\, f_1^2(q_1)\, \Delta'_v (E - \omega_{1_{q_1}}) + \\
&+ \frac{\lambda_1^2}{(2\pi)^6} \int d^3 q\, d^3 q_1 \frac{f^2(q)\, f_1^2(q_1)}{E - \omega_q - \omega_{1_{q_1}} - \omega_N} + \\
&+ \frac{\lambda_1^2 g^2}{(2\pi)^9} \int \frac{d^3 q_1 d^3 \ell\, d^3 \ell'\, f_1^2(q_1)\, f^2(\ell)\, f^2(\ell')\, \Delta'_v (E - \omega_{1_{q_1}})}{(E - \omega_\ell - \omega_{1_{q_1}} - m_N)(E - \omega_{\ell'} - \omega_{1_{q_1}} - m_N)} + \\
&+ \frac{2\lambda_1 g g_1}{(2\pi)^6} \int d^3 q\, d^3 q_1 \frac{f^2(q)\, f_1^2(q_1)}{E - \omega_q - \omega_{1_{q_1}} - m_N} + \frac{\lambda_1^2}{(2\pi)^6} \int d^3 q\, d^3 q_1\, d^3 q' d^3 q'_1 \times \\
&\times \frac{f(q)\, f(q')\, f_1(q_1)\, f_1(q'_1)\, C(q', q'_1 ; q, q_1)}{(E - \omega_q - \omega_{1_{q_1}} - m_N)(E - \omega_{q'} - \omega_{1_{q'_1}} - m_N)} + \frac{\lambda_1 g g_1}{(2\pi)^6} \int d^3 q\, d^3 q_1\, d^3 q' d^3 q'_1 \times \\
&\times \frac{f(q)\, f(q')\, f_1(q\ )\, f_1(q'_1)\, C(q', q'_1 ; q, q_1)}{(E - \omega_q - \omega_{1_{q_1}} - m_N)(E - \omega_{q'} - \omega_{1_{q'}} - m_N)} \times \qquad (3) \\
&\times \left[\frac{1}{E - \omega_{1_{q'_1}} - m_v^{(0)}} + \frac{1}{E - \omega_{1_{q_1}} - m_v^{(0)}} \right] + \\
&+ \frac{g^2 g_1^2}{(2\pi)^6} \int \frac{d^3 q\, d^3 q_1\, d^3 q' d^3 q'_1\, f(q)\, f(q')\, f_1(q_1)\, f_1(q'_1)\, C(q', q'_1 ; q, q_1)}{(E - \omega_q - \omega_{1_{q_1}} - m_N)(E - \omega_{q'} - \omega_{1_{q_1'}} - m_N)(E - \omega_{1_{q_1}} - m_v^{(0)})(E - \omega_{1_{q_1'}} - m_v^{(0)})},
\end{aligned}
$$

where

$$\omega_i(k) = \frac{k^2}{2\mu_i} + \mu_i .$$

$\Lambda'_v(E)$ is complete v-particle propagator.

$$\Lambda'_v(E) = [E - m_v^{(0)} - \frac{g^2}{(2\pi)^3} \int \frac{d^3k f^2(k)}{E-\omega_k - m_N}]^{-1} .$$

The renormalized mass, m_u and the wave function renormalization constant, z_u of the u-particle are defined by

$$m_u = m_u^{(0)} + \Sigma(m_u) , \quad (4a)$$

$$z_u^{-1} = \frac{\partial \Sigma(E)}{\partial E} /_{E = m_u} . \quad (4b)$$

The $UN\theta\theta_1$ - vertex function is represented in Fig. 3.

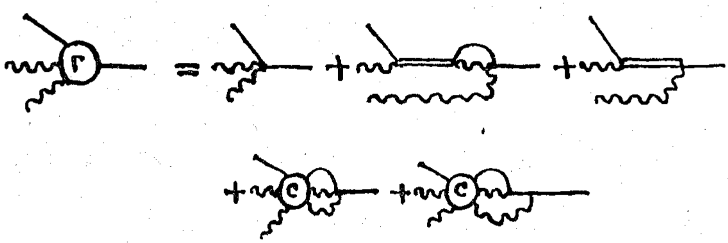

Fig. 3. $UN\theta\theta_1$ vertex function expressed in terms of the connected part of the irreducible part of the $N\theta\theta_1 \rightarrow N\theta\theta_1$ scattering amplitude

Therefore

$$\Gamma(E,k) = \frac{\lambda_1}{(2\pi)^3} \Gamma_1(E,k) + \frac{g_1}{(2\pi)^{3/2}} \Gamma_2(E,k) , \quad (5)$$

where

$$\Gamma_1(E,k) = 1 + \frac{g^2}{(2\pi)^3} \Delta'_v(E-\omega_{1_k}) \int \frac{d^3q f^2(q)}{E-\omega_q-\omega_{1_k}-m_N} +$$

$$\int d^3q\, d^3q_1 \frac{f(q)\, f_1(q_1)\, f^{-1}(k)\, f_1^{-1}(k_1)\, C(q,q_1;k)}{E-\omega_q-\omega_{1_{q_1}}-m}$$

and

$$\Gamma_2(E,k) = \frac{g}{(2\pi)^{3/2}} \Delta'_v(E-\omega_{1_k}) +$$

$$+ \frac{g}{(2\pi)^{3/2}} \int \frac{d^3q\, d^3q_1\, f(q)\, f_1(q_1)\, f^{-1}(k)\, f_1^{-1}(k_1)\, C(q,q_1;k)}{(E-\omega_q-\omega_{1_{q_1}}-m_N)(E-\omega_{1_{q_1}}-m_v^{(0)})}$$

The $UN\theta\theta$ and $UV\theta_1$ vertex renormalization constants are definedby

$$z_1^{-1} = \Gamma_1(E,k)/_{E=m_u}, \qquad (6a)$$

$$z_2^{-1} = \Gamma_2(E,k)/_{E=m_u}. \qquad (6b)$$

The renormalized $UN\theta\theta_1$and $UV\theta_1$ coupling constants are given by

$$\lambda_{1r} = z_u^{1/2}\, z_1^{-1} \lambda_1, \qquad (7)$$

$$g_{1r} = z_u^{1/2}\, z_2^{-1}\, g_1.$$

3. Poles of the T(E,k)

$T(E,k)$ the total scattering amplitude for $N\theta\theta_1 \to N\theta\theta_1$ is given by

$$T(E,k) = f^2(k)\, f_1^2(k)\, \Gamma^2(E,k)\, \Delta'_u(E,k) + D(E,k) + C(E,k). \qquad (8)$$

The expression (A5) indicates that C can develop a pole for sufficiently large g and λ corresponding to the $N\theta\theta_1$ bound state, i.e.,

$$d(E = m_B) = 0,$$

where m_B is the mass of the $N\theta\theta_1$ bound state.

Eqs. (3), (5), and (2) indicate that this pole induces a pole in the reducible part of the scattering amplitude at $E = m_B$. It can easily be checked that x/

$$R_1 \neq - R_2, \qquad (9)$$

where R_1 is the residue of the pole of the irreducible part at $E = m_B$ and R_2 is the residue of the induced pole of the reducible part at $E = m_B$. It implies that the pole of T_1, the irreducible part of the scattering amplitude, does contribute to the total scattering amplitude contrary to the assertion made by Jin and MacDowell /3/ for 2 particle → 2 particle scattering amplitude. The pole structure of $T(E,k)$ is given by

$$T(E,k) = \frac{\left[\frac{\lambda_{1r}}{(2\pi)^3} + \frac{g_{1r}}{(2\pi)^{3/2}}\right]^2 f^2(k) f_1^2(k)}{E - m_v} + \frac{R_1}{E - m_B} + \frac{R_2}{E - m_B} + \dots \qquad (10)$$

4. Equivalence of U -Particle and $N\theta\theta_1$ Bound State

The conditions under which the U particle pole disappears and the $N\theta\theta_1$ bound state pole replaces U -particle pole are

a. $m_B \to m_u$

b. $R_1 \to + \left[\frac{\Lambda_{1r}}{(2\pi)^3} + \frac{g_{1r}}{(2\pi)^{3/2}}\right]^2 f^2(k)\, f_1^2(k_1)$

---

x/ The explicit expression for R_1 can be obtained from Eq. (A1) and for R_2 from Eqs. (3), (5), and (2).

c. $R_2 \to -\left[\frac{\lambda_{1r}}{(2\pi)^3} + \frac{g_{1r}}{(2\pi)^{3/2}}\right]^2 f^2(k)\, f_1^2(k_1)$.

Eqs. (6a) and (6b) indicate that the condition (a) implies $z_1 = 0$ and $z_2 = 0$. A little algebra indicates that the condition (c) is equivalent to $\lim_{\substack{z_1 \to 0 \\ z_2 \to 0}} z_u = 0$. Condition (b) is completely independent of $z_1 = 0$, $z_2 = 0$ and $z_u = 0$ and is not satisfied under $z_1 = 0$, $z_2 = 0$ and $z_u = 0$. We, therefore, conclude that $z_1 = 0$, $z_2 = 0$ and $z_u = 0$ are not sufficient conditions for the equivalence of the U -particle and the $N\theta\theta_1$ bound state.

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Appendix

The integral equation for $T(E,k)$ represented diagramatically in Fig. I is

$$T_1(k', k_1'; k, k_1) = \frac{\lambda}{(2\pi)^6} f(k') f_1(k_1') f(k) f_1(k_1) +$$

$$+ \frac{g^2}{(2\pi)^3} \frac{\delta^{(3)}(k_1 - k_1') f(k) f(k')}{E - \omega_{1_{k_1}} - m_v^{(0)}}$$

$$+ \frac{\lambda}{(2\pi)^6} \int d^3q\, d^3q_1 \frac{f(q) f_1(q_1) f(k) f_1(k_1) T_1(k', k_1'; q, q_1)}{E - \omega_q - \omega_{1_{q_1}} - m_N} \qquad \text{(AI)}$$

$$+ \frac{g^2}{(2\pi)^3} \int d^3q \frac{f(k) f(q) T_1(k', k_1'; q, k_1)}{(E - \omega_{1_{k_1}} - m_v^{(0)})(E - \omega_q - \omega_{1_{k_1}} - m_N)},$$

where k, k_1 and k', k_1' represent momenta of incoming and outgoing θ, θ_1. Here momentum dependence is written explicitly for the sake of keeping track of different momenta while solving the integral equation. To solve this integral equation we make use of Weinberg's method of decomposing T_1 the irreducible part of $N\theta\theta_1 \to N\theta\theta_1$ scattering amplitude, into disconnected part and connected part, and then solving the integral equation for the connected part.

$$T_1(k', k_1'; k, k_1) = D(k', k_1'; k, k_1) + C(k', k_1'; k, k_1), \quad \text{(A2)}$$

where the disconnected part

$$D(k', k_1'; k, k_1) = \delta^{(3)}(k_1 - k_1')\Delta_v'(E) \quad \text{(A3)}$$

and a little manipulation shows that the connected part satisfies an integral equation as represented diagramatically in Fig. 4.

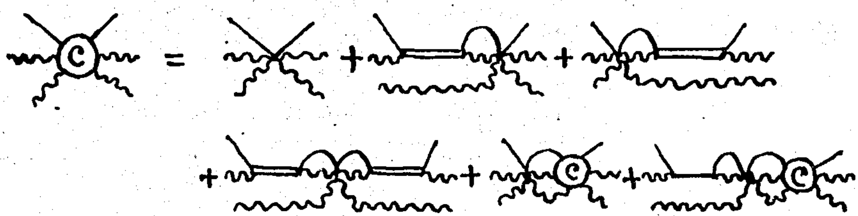

Fig. 4. Integral equation for the connected part of the irreducible part of the $N\theta\theta_1 \to N\theta\theta_1$ scattering amplitude

Therefore we have

$$C(k', k_1'; k, k_1) = A(k', k_1'; k, k_1) +$$

$$+ \frac{\lambda}{(2\pi)^6} \int d^3q\, d^3q_1 \frac{f(k) f_1(k_1) f(q) f_1(q_1) C(k', k_1'; q, q_1)}{E - \omega_q - \omega_{1q_1} - m_N} +$$

$$+\frac{\lambda}{(2\pi)^6}\int\frac{d^3\ell\, t(\ell,k)}{E-\omega_\ell-\omega_{1k_1}-m_N}\int d^3q\, d^3q_1\frac{f(\ell)\, f(q)\, f_1(k_1)\, f_1(q_1)\, C(k',k_1';q,q_1)}{E-\omega_q-\omega_{1q_1}-m_N} \tag{A4}$$

where

$$A(k',k_1';k,k_1) = \frac{\lambda}{(2\pi)^6}\, f(k')\, f_1(k_1')\, f(k)\, f_1(k_1) +$$

$$+\frac{\lambda}{(2\pi)^6}\int d^3q\; d^3q_1\, f(k)\; f_1(k_1)\; f(q)\; f_1(k_1') \times \frac{t(q,k)}{E-\omega_q-\omega_{1k_1}-m_N} +$$

$$+\frac{\lambda}{(2\pi)^6}\int d^3q\; \frac{f(k)\, f_1(k_1)\, f(q)\, f_1(k_1')\, t(k',q)}{E-\omega_q-\omega_{1k_1'}-m_N}$$

$$+\frac{\lambda}{(2\pi)^6}\int d^3q\; d^3q_1 \frac{f(q)\, f_1(k_1)\, f(q')\, f_1(k_1')\, t(q,k)\, t(k',q')}{(E-\omega_q-\omega_{1k_1}-m_N)\,(E-\omega_{q'}-\omega_{1k_1'}-m_N)},$$

and

$$t(\ell,k) = \frac{g^2}{(2\pi)^3}\, f(\ell)\, f(k)\, \Delta_v' .$$

The solution of the integral equation can be obtained by method of iteration, the final result obtained is

$$C(k',k_1';k,k_1) = A(k',k_1';k,k_1) +$$

$$+\Big[\frac{\lambda}{(2\pi)^6}\int d^3q\; d^3q_1 \frac{f(k)\, f_1(k_1)\, f(q)\, f_1(q_1)}{E-\omega_q-\omega_{1q_1}-m_N}\, A(k',k_1';q,q_1) +$$

$$
+ \frac{\lambda}{(2\pi)^6} \int \frac{d^3\ell \; t(\ell, k)}{E - \omega_\ell - \omega_{1k_1} - m_N} \iint d^3q \; d^3q_1 \times
$$

$$
\times \frac{f(\ell)\, f_1(k_1)\, f(q)\, f_1(q_1)}{E - \omega_q - \omega_{1q_1} - m_N} A(k', k_1'; q, q_1)] \times d^{-1}(E) \tag{A5}
$$

with

$$
d(E) = [\, 1 - \frac{\lambda}{(2\pi)^6} \int d^3q \; d^3q_1 \frac{f^2(q)\, f_1^2(q_1)}{E - \omega_q - \omega_{1q_1} - m_N} -
$$

$$
- \frac{\lambda}{(2\pi)^6} \int d^3q \; d^3q_1 \frac{f(q)\, f_1^2(q_1)}{E - \omega_q - \omega_{1q_1} - m_N} \int d^3\ell \frac{T(\ell, q)\, f(\ell)}{E - \omega_\ell - \omega_{1q_1} - m_N}]
$$

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