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FORWARD PHOTON-PHOTON SCATTERING AND THE ALGEBRA OF BILOCAL OPERATORS

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I. Introduction

Stimulated by the scaling described in connection with the SLAC-MIT experiment [1] on the deep inelastic electroproduction, Fritzsch and Gell-Mann [2] have proposed a possible extension of the algebra of equal time commutators of axial and vector currents to light-like distances. The new ingredients, which appear in the right-hand side of the commutators at light-like distances, are bilocal generalizations of the axial and vector currents. In order to close the new algebraic system guided by the free quark model they proposed an algebra for these bilocal operators. Recently, Gross and Treiman [3] have pointed out that the commutation relations obtained in the free-quark model are preserved in the gluon-quark model with cut-off if the coordinates of the currents involved in these relations are distributed on a light-like ray. In another paper [4], Gross and Treiman have shown that the algebra of bilocal operators (ABO) controls the cross sections of the deep inelastic processes involving two currents in an asymptotic region defined as a special double limit (see Section II).

The disconnected part of the bilocal operators is measured by vacuum processes like $\gamma^* \to$ hadrons. Similarly the c-number part of the commutators of the ABO is measured by vacuum processes like $\gamma^* + \gamma^* \to$ hadrons or $\gamma^* \to \gamma^* +$ hadrons.

It is well known [5,6] that the imaginary part of the forward photon-photon scattering amplitude can be studied by the measurement e.g. of the process

$$e^- + e^- \to e^- + e^- + hadrons \tag{1}$$

If we require the electrons to be scattered into very near to forward directions i.e. the virtual photons are almost real, the process can be approximated by the Feynman diagram given in Figure I. The contributions of this diagram, however can be studied at high photon masses as well. Generally, the process (I) is described by 10 Feynman diagrams, 8 of them refer to the production of C-odd hadronic states and can be calculated. The remaining two correspond to C-even hadronic states and formally describe the forward and backward scattering, respectively. If the electrons scatter into small angles (in c.m. system of the electrons $\theta_i < 20°$) only the diagram of Figure I remains. For the square of the c.m. energy we have the inequality

$$S = 4E^2 \geq \frac{q_i^2}{\sin^2 \theta_i / 2} \quad (i = 1,2). \qquad (2)$$

Therefore if we wish to study off-mass shell photon-photon scattering at high photon masses ($-q^2 \geq 5\ \mathrm{GeV}^2$) we should increase the beam energy ($E_{beam} \geq$ 30 GeV) to compensate the factor $\sin^2\theta/2$

Cabibbo et al. /7/ proposed a parton model for the reaction $e^+ + e^- \to$ hadrons. According to this model the cross section at high photon mass is determined by the parton-antiparton contributions. Therefore, we may make the conjecture that at high photon masses and energies the forward photon-photon scattering can be approximated by the parton-antiparton contributions /8/ . For spin 1/2 partons we would obtain

$$\sigma^a_{tot}(\gamma^* + \gamma^* \to hadrons) \xrightarrow{-q_1^2,\, -q_2^2,\, P^2 \to \infty} (\Sigma Q_p^4)\, \sigma^a_{tot}(\gamma^* + \gamma^* \to \mu^+ + \mu^-), \qquad (3)$$

where Q_p is the charge of the partons and "a" denotes a definite helicity state of the virtual photons. In the quark model

$$\Sigma\ Q_p^4 = (\frac{2}{3})^4 + (\frac{1}{3})^4 + (\frac{1}{3})^4 = \frac{2}{9}. \qquad (4)$$

Using the ABO in a gluon-quark model with cut-off, we can obtain this result only in a special double limit /4/ (see Sec. II), which

shows that we must define carefully how to reach the region of high q_1^2, q_2^2 and P^2 .

In Section II we rewiev the results obtained by the dominance of the light cone singularities (LCS) and by the use of the ABO. In Section III the contributions of spin 0 and spin 1/2 bare particles to the process $\gamma^* + \gamma^* \to$ hadrons are calculated. Various asymptotic regions are discussed with the help of LCS and the ABO to see under which conditions the parton-antiparton contributions may be regarded as a good approximation. In Section IV we investigate the prospects of future experimental tests. In Appendix A the invariant functions for the process $\gamma^* + \gamma^* \to$ hadrons are defined and discussed, in Appendix B we give the complete expressions for the parton-antiparton contributions and in Appendix C the cross section for the process (I) is calculated in a double limit.

II. Light Cone Kinematics

Let us consider the amplitude of the generalized Compton scattering with two incoming hermitian $SU(3) \times SU(3)$ currents, with m incoming and n outgoing hadrons (see Fig. 2).

$$J^{ba}_{\nu\mu} = i \int d^4x\ e^{iqx} < p_A, A \mid T^* \left(J^e_\nu(\frac{x}{2}),\ J^a_\mu(-\frac{x}{2})\right) \mid p_B, B >_c, \qquad (5)$$

where T^* denotes covariant time ordered product together with sea-gull terms, J^a_μ is the corresponding $SU(3) \times SU(3)$ current, $q = \frac{1}{2}(q_1 - q_2)$, index "c" denotes that we study the connected matrix element. We would like to find the kinematical region, where the amplitude is controlled by the LCS of the operator products of the currents.

The integral obtains the main contributions from the LCS if the phase of the exponential is large everywhere but the neighbourhood of the light cone and if the phase of the matrix element cannot cancel it. To avoid large phases in the matrix elements and to fulfil the conditions required by the validity of the Wilson's expansion we must keep the momenta of the hadrons fixed. (The Wilson expansion is assumed to be valid between states of fixed momenta /9/). Due to the translational invariance, however, the non-diagonal matrix elements will have large phases of type $c(q_1+q_2)x$. But this kind of phases cannot cancel the phase of the exponential if we treat the momenta of the currents q_1 and q_2 symmetrically. Therefore, we conjecture that the LCS will dominate the matrix element in the light cone limit given (by definition) as

$$\text{LC} \quad \lim \quad = \lim \; \{ -q_1^2 - q_2^2 \to \infty \;, \qquad q_1^2 / q_2^2 \; \text{and all the hadron momenta are fixed} \} \tag{6}$$

The four-momentum conservation requires that Pq_1, Pq_2, rq_1, $rq_2 \to \infty$ as their ratios to $|q_1^2|$ remain fixed, therefore we can use the same arguments for the dominance of the LCS in that case as those we have for the structure functions of the deep inelastic electroproduction. In conclusion the integral (5) obtains the main contributions in the limit (6) from the region, where

$$x^2 < Max \frac{1}{|q_i^2|} \;, \qquad (i = 1,2). \tag{7}$$

We can rewrite this result using the light cone generalizations of the Wilson's expansion /10,11/. It has been shown in papers/3,4/ that the product of two local operators can be expanded at light-like separations into a series of bilocal operators of different twist, e.g.

$$A(x)\;B(y) = \sum_{\tau=\tau_{min}}^{\infty} C^{\tau}(x-y)\; V^{\tau}\left(x-y, \frac{x+y}{2}\right) \tag{8}$$

where $V^{\tau}(x-y, \frac{x+y}{2})$ are bilocal operators regular at $(x-y)^2 = 0$, τ is the twist of the local operators appearing in the Taylor-expansion at the point $(x-y)^{\mu} = 0$ of V^{τ} x/ and $C^{\tau}(x-y)$ are C-number functions, which contain the LCS of the operator product. If we assume dilatational invariance at the light cone /2/, the strength of the LCS can be determined by investigating the dimensions of the local operators involved. It is easy to see that

$$C^{\tau}(x-y) = [(x-y)^2 - i\epsilon]^{-\frac{d_A + d_B - \tau}{2}} \tag{9}$$

where d_A and d_B are the dimensions of the operators A and B, respectively. It is clear that the operator product near light-like separations can be approximated by the bilocal operators of lowest twist. Therefore the result obtained above can be reformulated as follows: In the LC limit the amplitude (5) is controlled by the lowest twist bilocal operators. (See Fig. 3).

In a gluon-quark model with cut-off /3/, the matrix element of the time order product of the electromagnetic currents at $x^2 \sim 0$ can be written as

$$\langle p_A, A \,|\, T\left(V_{\nu}^{Q}\left(\frac{x}{2}\right) V_{\mu}^{Q}\left(-\frac{x}{2}\right)\right) |\, p_B, B \rangle_c \;\hat{=}\; \langle p_A, A \,|\, \theta_{\nu\mu\alpha}^{Q^2}(x,0) \,|\, p_B, B \rangle\, \partial_x^{\alpha} D_F(x) \,, \tag{10}$$

where $\hat{=}$ denotes that the equation is valid only near the light cone,

$$D_F(x) = \frac{1}{4\pi}\left[\delta(x^2) - \frac{i}{\pi x^2}\right], \tag{10a}$$

$$\theta_{\nu\mu\alpha}^{Q^2}(x,0) = t_{\nu\mu\alpha\beta} V^{\beta,Q^2}(x,0,-) - i\epsilon_{\nu\mu\alpha\beta} A^{\beta,Q^2}(x,0,+) \tag{10b}$$

$$t_{\nu\mu\alpha\beta} = g_{\nu\alpha} g_{\mu\beta} + g_{\nu\beta} g_{\mu\alpha} - g_{\nu\mu} g_{\alpha\beta} \,,$$

$$V_{\mu}^{Q^2}(x,0,-) = \frac{1}{2}\left[\bar{\psi}\left(\frac{x}{2}\right)\gamma_{\mu} Q^2 e^{-ig\int_{-x/2}^{x/2} dz^{\mu} B_{\mu}(z)} \psi(-x/2) - (x \to -x)\right], \tag{10c}$$

$$A_{\mu}^{Q^2}(x,0,+) = \frac{1}{2}\left[\bar{\psi}\left(\frac{x}{2}\right)\gamma_5\gamma_{\mu} Q^2 e^{-ig\int_{-x/2}^{x/2} dz^{\mu} B_{\mu}(z)} \psi(-x/2) + (x \to -x)\right], \tag{10d}$$

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x/ The twist of a local operator is defined as the difference of its dimension (d) and spin (J): $\tau = d - J$, d is measured in mass units (see /4/).

$$Q = \frac{1}{2} (\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8) . \qquad (11)$$

$\psi(x)_a$ is the quark-field, B_μ is the gluon-field, λ_i's are the Gell-Mann's matrices, g is the quark-gluon coupling constant and $V_\mu^{Q^2}$ and $A_\mu^{Q^2}$ are the corresponding bilocal operators [x].

Calculating the cross sections for the corresponding deep-inelastic processes summing up for all the hadronic final states we obtain

$$d\sigma \propto \sum_A T^A_{\nu\mu}(q_1, q_2, \dots) T^A_{\nu'\mu'}(q_1, q_2, \dots) (2\pi)^4 \delta(q_1 + q_2 - p) \xrightarrow{LC\ limit}$$

$$\longrightarrow \int d^4x \; d^4y \; d^4z \; e^{-iq(x-y)+pz}\, \partial^a D_F(x)\, \partial^\beta_F D(y) \times \qquad (12)$$

$$\times \langle P_B, B \,|\, [\, \theta^{+Q^2}_{\nu'\mu'\beta}(y', -z/2),\ \theta^{Q^2}_{\nu\mu a}(x', z/2)] \,|\, p_B, B \rangle_c ,$$

i.e. if we can circumwent the difficulties caused by the disconnected and semi-disconnected pieces (see e.g./12/), these cross sections are controlled by the commutators of the bilocal operators. We note that if the currents interact at the point x_1, x_2 and y_1, y_2, respectively, then $x = x_1 - x_2$, $y = y_1 - y_2$ and $z = x_1 + x_2 - y_1 - y_2$.

The commutators of the bilocal operators have further singularities at the various light-like separations of the coordinates of the currents. Fritzsch and Gell-Mann/2/ conjectured, recently Gross and Treiman have proved /3/ in a gluon-model with cut-off, that if the points x_1, x_2, y_1, y_2 are distributed on a light-like ray, extracting all the relevant LCS, we can obtain a closed algebra of the currents and the lowest twist bilocal operators.

The cross section (12) will be dominated by the contributions of this light-like ray if having taken the LC limit now $P^2 \to \infty$ as q_1^2 / q_2^2 being fixed. To see that, we repeat the arguments given in the paper/4/. If $P^2 \to \infty$ and Pq/q^2 is fixed, then P^μ must be an infinite four-vector. The integral obtains the main contributions where the phases qx, qy, Pz are finite. Introducing the light-

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[x] It may happen that the current conservation requires to add correction terms to the formulae of eqs. (10) (See Ref. /3/).

like vector $n_\mu = q_\mu / q^2$ we get that nx and ny must be equal to zero and the scalar products qx and qy must be finite. Taking into account that P^μ is an infinite four-vector we obtain that x^μ, y^μ, z^μ and n^μ are parallel approximately q.e.d. To the double limit defined above we refer as to GT limit:

$$GT\ limit = \lim \{ P^2 \to \infty \ ; \ q_1^2 / q_2^2 \ fix \} \lim \{ LC \lim \} . \quad (13)$$

Neglecting for a moment the disconnected contributions, in the GT limit we can write for the cross section (12) (See Fig. 4).

$$d\sigma \sim \xrightarrow{GT\ limit} \int d^4x \ d^4y \ d^4z \ e^{iq(x-y)} e^{-iPz} [S_{1}, {}_{\mu\nu\mu'\nu'\alpha\alpha'\rho\sigma} \times$$
$$\times \partial^{\alpha} D_F(x) \ \partial^{\alpha'} D_F^*(y) \ \partial^{\rho} D \left(\frac{x-y}{2} - z \right) \times$$
$$\times \langle p_B, B \mid V^{Q^4, \sigma} \left(\frac{x-y}{2} + z, 0, - \right) \mid p_B, B \rangle + \quad (14)$$
$$+ \ 15 \ similar \ terms ,$$

where $D(x) = \frac{1}{4\pi} \epsilon(x_0) \delta(x^2)$, s_i is constructed by use of the products of the matrix tensor and the additional 15 terms differ from each other only in the sign of x, y and z.

For the process $\gamma^* + \gamma^* \to$ hadrons the question of disconnected contributions does not arise. Therefore the C-number part of the algebra of bilocal operator (ABO) is measured by the GT limit of the imaginary part of the amplitude of the forward photon-photon scattering (see Fig. 5).

Assuming that the vacuum expectation values of the bilocal operators are also dictated by a gluon-quark model with cut-off we get that the parton result given by the eq. (3) can be reproduced in the GT limit with the help of the ABO. (Fig. 5).

In turn, the ABO can be tested by measuring the C-even part of the cross section $d^3\sigma / dq_1^2 \, dq_2^2 \, dP^2$ of the process (1) in the region

$$s \gg | q_i^2 | \gg P^2 \gg m_\pi^2 \quad (15)$$

III. Parton-Antiparton Contributions, Various Limits

The imaginary part of the amplitude of the forward photon-photon scattering is determined by a dimensionless fourth-rank tensor (see Fig. 5).

$$W_{\mu\nu,\mu'\nu'}(q_1,q_2) = 4\pi \sum_N (2\pi)^4 \delta(P_N - q_1 - q_2) \int d^4x \; e^{iqx} \times$$

$$\times \langle N | T^c(J_\nu(\tfrac{x}{2}) J_\mu(-\tfrac{x}{2}) | 0 \rangle \int d^4y \; e^{-iqx} \langle N | T^c(J_{\nu'}(\tfrac{y}{2}) J_{\mu'}(-\tfrac{y}{2})|0\rangle^* \tag{16a}$$

$$= 4\pi \int d^4x \; d^4y \; d^4z \; e^{iq(x-y)} e^{iPz} \langle 0 | [T^c(J_{\nu'}(\tfrac{y-z}{2}), J_{\mu'}(-\tfrac{y+z}{2}))^+,$$

$$T^c(J_\nu(\tfrac{x+z}{2}), J_\mu(-\tfrac{x-z}{2})] | 0 \rangle , \tag{16b}$$

where T^c denotes the covariant time-ordered product together with sea-gull terms, $J_\mu(x)$ is the electromagnetic currents of hadrons. Using Lorentz, gauge, P, T, PT invariance this tensor can be expanded in terms of eight real invariant functions. We choose an orthogonal helicity basis, similar to that proposed in paper [5]

$$W_{\mu\nu,\mu'\nu'}(q_1,q_2) = \sum E^a_{\mu\nu,\mu'\nu'}(q_1,q_2) \; W_a(q_1^2,q_2^2,q_1 q_2), \tag{17}$$

where

$$a = TT, (TT)_\tau, (TT)_a, (TS)_+, (TS)_-, (TS)_\tau, (TS)_{\tau a}, SS .$$

The explicit form of the tensors $E_a^{\mu\nu,\mu'\nu'}$ together with some properties of the functions $W_a(q_1^2,q_2^2,q_1 q_2)$ can be found in Appendix A.

The parton-antiparton contributions, for spin 1/2 and spin 0 massless partons (see Fig. 6) can be expressed as follows

$$W_a(q_1^2,q_2^2,\nu) = \sum_{J=0,\frac{1}{2}} \langle Q_J^4 \rangle \left[W_a^J(\omega) + \rho \, \overline{W_a^J}(\omega,\rho) \right] , \tag{18}$$

where [x]

$$\langle Q_J^4 \rangle = \sum_p \text{partons of spin } J \;\; Q_p^4$$

$\omega = \nu/l$, $\rho = P^2/2l$, $\nu = q_1 q_2$ and $l = \sqrt{\nu^2 - q_1^2 q_2^2}$ being the flux of the incoming photons, the functions $\overline{W_a^J}(\omega,\rho)$ are given in Appendix B. The functions $W_a^J(\omega)$ are as follows:

for spin 0 partons

$$W^0_{TT}(\omega) = \frac{1}{2} W^0_{(TT)_\tau}(\omega) = W^0_{(TT)_a}(\omega) = 1 , \tag{19a}$$

$$W^0_{(TS)_+}(\omega) = W^0_{(TS)_-}(\omega) = 0 , \tag{19b}$$

$$W^0_{(TS)_\tau}(\omega) = W^0_{(TS)_{\tau a}}(\omega) = \frac{1}{2} \int_{-1}^{1} dz \; \frac{\omega\sqrt{\omega^2-1}}{\omega^2 - z^2} , \tag{19c}$$

$$W^0_{SS}(\omega) = \int_{-1}^{1} dz \; \frac{\omega^2(\omega^2-1)}{(\omega^2-z^2)^2} , \tag{19d}$$

for spin 1/2 partons

$$W^{1/2}_{TT}(\omega) = W^{1/2}_{(TT)_a}(\omega) = \int_{-1}^{1} \frac{(1-z^2)(\omega^2+z^2)}{(\omega^2-z^2)^2} \tag{20a}$$

$$1/2 \; W_{(TT)_\tau}(\omega) = -\int_{-1}^{1} \frac{(1-z^2)}{\omega^2 - z^2} dz \tag{20b}$$

$$\text{all the others} = 0 \tag{20c}$$

[x] For $q_i^2 = 0$ $(i = 1,2)$ $\omega = 1$; as $P^2 \to \infty$ and $q_1^2 \to q_2^2$, $\omega \to \infty$

It is obvious from eqs. (18)-(20) that the spin of the partons is important. Since the particles are massless their contributions are scale invariant and diverge logarithmically at (infrared-type divergence).

We would like to see in which asymptotic regions the parton-antiparton contributions (18) may be regarded as a good approximation for the amplitude (16). In eq. (3) we did not specify uniquely how to reach the asymptotic region of q_1^2, q_2^2 and P^2. In the previous section discussing the GT limit (13) we have however seen that it is very important to define the asymptotic regions uniquely.

Let us consider the following limit

i) *LC limit* (see eq. (6)) (6)

$$-q_i^2 \gg m_\pi^2 \;, \quad (i = 1,2) \;,$$

ii) *GT limit* (see eq. (13)) (13)

$$-q_i^2 \gg P^2 \gg m_\pi^2 \;,$$

iii) *A limit* $= \lim \{ -q_1^2, -q_2^2, P^2 \to \infty \;;\; q_1^2/q_2^2,\; P^2/q_1^2 \text{ fixed} \}$

$$-q_i^2, P^2 \gg m_\pi^2 \;, \tag{21}$$

iv) A_{b_i} *limit* $= \lim \{ -q_2^2 \to \infty \;,\; P^2/q_1^2 \text{ fixed} \} \lim \{ P^2, -q_1^2 \to \infty \;;$ $-q_2^2, q_1^2/P^2 \text{ fixed} \}$

$$P^2, -q_1^2 \gg -q_2^2 \gg m_\pi^2 \;, \tag{22}$$

v) A_R *limit* $= \lim \{ \text{LC limit} \} \lim \{ P^2 \to \infty \;;\; q_1^2, q_2^2 \text{ fixed} \}$

$$P^2 \gg -q_i^2 \gg m_\pi^2 \;. \tag{23}$$

To see the relevance of the parton-antiparton contributions we will compare them with the contributions from the LCS.

In section II we have seen that in the LC limit using a gluon-quark model with cut-off, the LCS can be extracted, the LC limit exists and we get for the W_a functions

$$W_a^J(q_1,q_2,\nu) \xrightarrow{LC\ limit} W_a^J(\omega, P^2). \qquad (24)$$

The functions $W_a^J(\omega, P^2)$ are determined by the vacuum expectation values of the commutators of the bilocal operators of twist 2 (see Fig. 5). As to the parton-antiparton contributions, the $W_a^J(\rho,\omega)$ functions in eq. (18) are irrelevant because in this limit $\rho = 0$. On the other hand we do not expect the expression (18) to be a good approximation in the LC limit since at low values of P^2 the resonance contributions are important.

In the GT limit in a gluon model with cut-off the tensor $W_{\mu\nu,\mu'\nu'}(q_1,q_2)$ is completely determined by its free field LCS i.e. by the spin 1/2 parton-antiparton contributions (20), as we have seen in the previous Section. We note that for the parton-antiparton contributions the GT limit gives the same result as the LC limit as a consequence of their scaling property.

Let us investigate now the A limit (see ref. /13/). The main difference to the GT limit is that here the limit $P^2 \to \infty$ is done first. It might happen that the matrix elements $\langle N|T(J_\nu(x/2)\, F_\mu(-x/2)|0\rangle$ of eq. (16a) have large phases cancelling the phase of the exponential. Furthermore, we are in general not allowed to use the Wilson's expansion. Therefore we cannot argue as in the case of the GT limit. On the other hand we can read off from the formula (16b) that the main contributions come from the region, where $|(x-y)^2| < 1/|q^2|$ and $|z^2| < 1/P^2$ but we cannot say that the integral obtains the main contributions from the region, where x^μ, y^μ and z^μ are distributed on a light-like ray. This does not mean, however, that the dominance of this region is ruled out; there are weaker arguments for the dominance of the all LCS. Nevertheless, it will be interesting to compare the parton-antiparton contributions (18) in this limit with the experiment.

In the A_{bj} limit, performing first the bj limit $-q_1^2 P^2 \to \infty$; q_1^2/P^2 and q_2^2 fixed, we can extract the LCS of the operator product $J_\mu(x)\, J_{\mu'}(x')$ (see Fig. 5)

$$W_{\mu\nu,\mu'\nu'}(q_1,q_2) \xrightarrow{bj} \int dx\, dy\, e^{iq_1x}\, e^{iq_2(z-y)} \times \qquad (25)$$

$$\times\, \theta(z_0-y_0)\,\theta(y_0-x_0) \langle 0|[J_{\nu'}(z'),\ [J_\nu(y),\ \theta_{\mu\mu' a}(x,0)]]|0\rangle \partial^a D(x),$$

where $\theta_{\mu\mu'a}$ is the twist 2 bilocal operator defined by eq. (10b). Here again we cannot argue that the integral (25) is dominated only by the LCS of the matrix element since y and z appear in the combination $(z-y)$. Therefore we cannot say that the integral obtaines the main contributions from the region, where x^μ, y^μ and z^μ are parallel and distributed on a light-like ray. If we accept the bj limit to exist we get the usual scaling in q_1^2 and p^2/q_1^2, q_2^2 being fixed (deep-inelastic electroproduction on photon-target/15/); it is not permitted, however, to assume at the same time, that the parton-antiparton contributions determine the A_{bj} limit of $W_{\mu\nu,\mu'\nu'}$ since in the bj limit they diverge $(\omega=1)$.

Finally, in the A_R limit there is no argument for the relevance of the LCS and the parton-antiparton contributions (see e.g. /14/).

In conclusion, among the limits GT, LC, A, A_{bj} and A_R only the GT limit is controlled by the LCS completely. It might be interesting, however, to confront the parton-antiparton contributions with the experiment in the A and the A_{bj} limits as well.

IV. Prospects of Experimental Tests

In the Introduction it was pointed out that the process (1) can be used to study the imaginary part of the forward photon-photon scattering amplitude by measuring the C-even part of its cross section at high beam energies ($\gtrsim$ 30 GeV) and at small scattering angles[x] ($\theta_i < 20$). Since the beam energy should be increased considerably the experiment will be done only remotely. We cannot investigate the high mass behaviour in the near future because of a more serious reason: the value of corresponding cross section is of order 10^{-39} cm^2 as we will see below.

In order to see the significance of the kinematical constant determined by the GT limit (see eq. (15) we think worth calculating the corresponding cross section.

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[x] We note that the process $e^+e^-\to\mu^+\mu^-$+hadron /4/ makes it possible to study the photon-photon scattering at the same high photon masses using colliding beams of lower energies ($E_{beam} \geq 3$GeV). Its cross section, however, is smaller than those for the process (1). The scattering of highly virtual photons can be studied as well, using the process $e^+e^-\to e^+e^-$ + hadrons under the same kinematical constraints which are put on the process (1).

Let us consider the C-even part of the cross section for the process (1). It can be expressed as follows (see Fig. 1)

$$\frac{d^3\sigma^{(+)}}{dq_1^2\, dq_2^2\, dP^2} = \frac{\alpha^4}{\pi}\,\frac{1}{q_1^2\, q_2^2}\int_R \frac{dx\, dy}{\sqrt{\Delta}}\, M\,, \qquad (26)$$

where $x = E_1/\sqrt{s}$, $y = E_2/\sqrt{s}$, Δ is defined by a Gram-determinant as $\Delta = -4s\, \mathcal{G}\begin{pmatrix} k & \ell & k' & \ell' \\ k & \ell & k' & \ell' \end{pmatrix}$ k, ℓ, k', ℓ' denote the four-momenta of the electrons and M is a trace given as

$$\begin{aligned} M &= \frac{1}{S^2}\,\ell^{\nu\nu'}(\ell,\ell')\, k^{\mu\mu'}(k,k')\, W_{\mu\nu,\mu'\nu'}(q_1,q_2) \\ &= \sum_a F_a(x,y,s,q_1^2,q_2^2,\nu)\, W_a(q_1^2,q_2^2,\nu)\,, \end{aligned} \qquad (27)$$

where $\ell^{\nu\nu'}$ and $k^{\mu\mu'}$ are the contributions of the electron lines (e.g. $k_{\mu\mu'} = k_\mu k'_{\mu'} + k'_\mu k_{\mu'} - g_{\mu\mu'}(kk')$) the coefficient functions F_a and the region of the integration R are given in Appendix C.

In the GT limit we can perform the integration and obtain simple asymptotic expression in leading order of s/q^2. As an illustration we give the contribution of the structure function W_{TT} at the points where $q_1^2 \sim q_2^2$

$$\frac{d^3\sigma}{dq_1^2\, dq_2^2\, dP^2} \sim \frac{6\alpha^4}{\pi} \langle Q^4\rangle \frac{\omega^2(\omega^2-1)}{(q_1^2\, q_2^2)^2}\,\ell n\, \frac{s}{2|q_1^2+q_2^2|}\, W_{TT}(\omega) + \qquad (28)$$
$$+ \ldots$$

Note that the cross section increases logarithmically with s. At the point $s = 5.10^3$ GeV^2, $-q_i^2 \sim 10$ GeV^2, $P^2 \sim 3$ GeV^2, assuming resolution of 10% for $\Delta q_i^2/q_i^2$ and $\Delta P^2/P^2$ we obtain in gluon quark model

$$\Delta\sigma \sim 10^{-39}\ \text{cm}^2. \qquad (29)$$

If the sea-gull contributions are important, the cross section can increase up to the value 5.10^{-38} cm^2 .

Finally we give some comments on the prospects of testing the ABO.

The ABO, as we have seen in Section II, controls processes where at least two currents with high mass are involved. Consequently their cross sections will have a damping factor like $\frac{\alpha^4}{(q_1^2 q_2^2)^2}$ or $\frac{G^2 \alpha^2}{(q_1^2)^2}$. It leads us to expect that the cross sections will be smaller than 10^{-39} cm^2 ($|q|^2 \sim 10$ GeV^2 , $\Delta q^2/q^2 \sim 10\%$). It might be the case that the LC dominates the amplitude when only one of the currents has high mass.

This does not test however the validity of the ABO even when we would find the predictions of ABO in agreement with the experiment. Therefore the measurement of the process (I) will give us, although remotly one of the best possibilities to test the ABO.

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Appendix A

In this Appendix we present an expansion of the tensor $W_{\mu\nu,\mu'\nu'}(q_1,q_2)$ proposed (see /5/) and list some of its properties. The invariant functions are defined as

$$W_{\mu\nu,\mu'\nu'}(q_1,q_2) = \sum_a E^a_{\mu\nu,\mu'\nu'}(q_1,q_2)\, W_a(q_1^2, q_1^2, \nu), \quad \text{(A.1)}$$

where

$$E^{\mu\nu,\mu'\nu'}_{TT}(q_1,q_2) = R^{\mu\nu} R^{\mu'\nu'} \quad \text{(A.2)}$$

$$E^{\mu\nu,\mu'\nu'}_{(TT)_\tau}(q_1,q_2) = \frac{1}{2}\left(R^{\mu\nu}R^{\mu'\nu'} + R^{\mu\nu'}R^{\mu'\nu} - R^{\mu\mu'}R^{\nu\nu'}\right) \quad \text{(A.3)}$$

$$E^{\mu\nu,\mu'\nu'}_{(TT)_a}(q_1,q_2) = R^{\mu\nu}R^{\mu'\nu'} - R^{\mu\nu'}R^{\mu'\nu} \quad \text{(A.4)}$$

$$E^{\mu\nu,\mu'\nu'}_{(TS)_+}(q_1,q_2) = \frac{1}{2}\left(\frac{Q_1^\mu Q_1^{\mu'}}{Q_1^2} R^{\nu\nu'} + \frac{Q_2^\nu Q_2^{\nu'}}{Q_2^2} R^{\mu\mu'}\right) \quad \text{(A.5)}$$

$$E^{\mu\nu,\mu'\nu'}_{(TS)_-}(q_1,q_2) = \frac{1}{2}\left(\frac{Q_1^\mu Q_1^{\mu'}}{Q_1^2} R^{\nu\nu'} - \frac{Q_2^\nu Q_2^{\nu'}}{Q_2^2} R^{\mu\mu'}\right) \quad \text{(A.6)}$$

$$E^{\mu\nu,\mu'\nu'}_{(TS)_\tau}(q,q) = \frac{1}{\sqrt{Q_1^2 Q_2^2}}\left(Q_1^{\mu'}Q_2^{\nu'}R^{\mu\nu} + Q_1^\mu Q_2^\nu R^{\mu'\nu'} + Q_1^\mu Q_2^{\nu'} R^{\mu'\nu} + Q_1^{\mu'} Q_2^\nu R^{\mu\nu'}\right) \quad \text{(A.7)}$$

$$E^{\mu\nu,\mu'\nu'}_{(TS)_{\tau_a}}(q_1,q_2) = \frac{1}{\sqrt{Q_1^2 Q_2^2}}\left(Q_1^{\mu'}Q_2^{\nu'}R^{\mu\nu} + Q_1^\mu Q_2^\nu R^{\mu'\nu'} - Q_1^\mu Q_2^{\nu'} R^{\mu'\nu} - Q_1^{\mu'} Q_2^\nu R^{\mu\nu'}\right) \quad \text{(A.8)}$$

$$E^{\mu\nu,\mu'\nu'}_{SS}(q_1,q_2) = \frac{Q_1^\mu Q_1^{\mu'} Q_2^\nu Q_2^{\nu'}}{Q_1^2 \; Q_2^2} \quad \text{(A.9)}$$

and

$$Q_1^\mu = q_2^\mu - \frac{\nu}{q_1^2} q_1^\mu, \quad Q_2^\nu = q_1^\nu - \frac{\nu}{q_2^2} q_2^\nu, \quad q_i Q_i = 0 \;(i=1,2);$$

$$R^{\mu\nu} = -g^{\mu\nu} - \frac{1}{l^2}\left[q_1^\mu q_1^\nu q_2^2 + q_2^\mu q_2^\nu q_1^2 - \nu\left(q_1^\mu q_2^\nu + q_2^\mu q_1^\nu\right)\right]$$

$$\nu = q_1 q_2, \qquad l^2 = \nu^2 - q_1^2 q_2^2$$

the indices T and S refer to transversal and longitudinal photons, respectively. We notice, that the tensors $E_a^{\mu\nu,\ \mu'\nu'}$ have the properties

$$E_a^{\mu\nu,\mu'\nu'}(q_1, q_2) = E_a^{\mu\nu,\mu'\nu'}(q_1, q_2) \ . \qquad \text{(A.10)}$$

By use of crossing symmetry we obtain

$$W^{\mu\nu,\mu'\nu'}(q_1, q_2) = W^{\mu'\nu',\mu\nu'}(-q_1, q_2) \qquad \text{(A.11)}$$

and

$$W_a(q_1^2, q_2^2, \nu) = \pm W_a(q_1^2, q_2^2, -\nu) \ , \qquad \text{(A.12)}$$

where we have sign + for all the W_a but $W_{(TS)Ta}$ and $W_{(TS)Ta}$, which are odd functions under crossing. Taking into account a property of the T-product of the electromagnetic currents

$$\int d^4x\ e^{iqx} < N|\, T(J_\nu(\tfrac{x}{2})\, J_\mu(-\tfrac{x}{2}))\,|0> =$$

$$= \int d^4x\ e^{-iqx} < N|\, T(J_\mu(\tfrac{x}{2})\, J_\nu(-\tfrac{x}{2}))\,|0> \qquad \text{(A.13)}$$

we obtain certain Bose-type symmetry given as

$$W^{\mu\nu,\mu'\nu'}(q_1, q_2) = W^{\nu\mu,\nu'\mu'}(q_2, q_1) \ . \qquad \text{(A.14)}$$

For the invariant functions this leads to symmetry in the photon masses. We can write

$$W_a(q_1^2, q_2^2, \nu) = W_a(q_2^2, q_1^2, \nu) \qquad \text{(A.15)}$$

for all W'_a but $W_{(TS)_-}$ when we obtain

$$W_{(TS)_-}(q_1^2, q_2^2, \nu) = -W_{(TS)_-}(q_2^2, q_1^2, \nu) \qquad \text{(A.16)}$$

that is $W_{(TS)_-}$ must be proportional with the factor $q_1^2 - q_2^2$.

As to the sign of $W'_{a\,s}$ four of them are simply connected with the total cross sections of photon-photon scattering as follows

$$W_{TT} = 16\pi\, I\, \sigma_{TT} \;, \qquad \text{(A.17)}$$

$$W_{(TS)_\pm} = 16\pi\, I\, (\sigma_{TS} \pm \sigma_{ST}) \,, \qquad \text{(A.18)}$$

$$W_{SS} = 16\pi\, I\, \sigma_{SS} \;. \qquad \text{(A.19)}$$

So they are positive functions, the others, however, do not have definite sign.

Finally, measuring the process $e^- + e^+ \to e^- + e^\pm + \text{hadrons}$ with unpolarized electron beams, the structure antisymmetric in $\mu\mu'$ or/and $\nu\nu'$ do not contribute, therefore by these measurements $W_{(TT)_{Ta}}$ and $W_{(TS)_{Ta}}$ cannot be studied.

Appendix B

Here we give the functions $\bar{W}_i(\omega, \rho)$ defined by eq. (18). For spin 0 partons

$$\overline{W}^{0}_{TT}(\omega, \rho) = \int_{-1}^{1} dz \, \frac{(1-z^2)}{(\omega^2 - z^2)^2} \left[\rho\omega (1-z^2) - (\omega^2 - z^2) \right] \qquad \text{(B.1)}$$

$$\overline{W}^{0}_{(TT)_T}(\omega, \rho) = \int_{-1}^{1} dz \, \frac{(1-z^2)}{(\omega^2 - z^2)^2} \left[2\omega (1 - 1/2\omega\rho)\, z^2 - \omega(4\omega^2 - 1) + \rho(3\omega^2 - 1) \right] \qquad \text{(B.2)}$$

$$\overline{W}^{0}_{(TT)_a}(\omega, \rho) = \int_{-1}^{1} dz \, \frac{\omega(1-z^2)}{\omega^2 - z^2} \qquad \text{(B.3)}$$

$$\overline{W}^{0}_{(TS)_+}(\omega,\rho) = \int_{-1}^{1} dz\, \frac{(1-z^2)\,z^2}{(\omega^2-z^2)^2}\,(2\omega^2\rho - \omega - \rho) \tag{B.4}$$

$$\overline{W}^{0}_{(TS)_-}(\omega,\rho) = \frac{q_1^2 - q_2^2}{2l} \int_{-1}^{1} \frac{(1-z^2)z^2}{(\omega^2-z^2)^2} \tag{B.5}$$

$$\overline{W}^{0}_{(TS)_\tau}(\omega,\rho) = \frac{1}{2}\sqrt{\omega^2-1} \int_{-1}^{1} \frac{dz}{(\omega^2-z^2)^2}\,[\,(\omega^2-z^2)\,(1+z^1) + 2\,(1-z^2)\,z^2\,(1-2\omega\rho)\,] \tag{B.6}$$

$$\overline{W}^{0}_{(TS)_{\tau_a}}(\omega,\rho) = \frac{1}{2}\sqrt{\omega^2-1} \int_{-1}^{1} dz\, \frac{1+z^2}{\omega^2-z^2} \tag{B.7}$$

$$\overline{W}^{0}_{SS}(\omega,\rho) = 4\,(\omega^2-1) \int_{-1}^{1} dz\, \frac{1}{(\omega^2-z^2)^2}\, z^2(\rho z^2 - \omega) \tag{B.8}$$

and for spin 1/2 partons

$$\overline{W}^{1/2}_{TT}(\omega,\rho) = 2\omega \int_{-1}^{1} dz\, \frac{1-z^2}{(\omega^2-z^2)^2}\,[\,\rho\omega\,(1+z^2) - (\omega^2+z^2)\,] \tag{B.9}$$

$$\overline{W}^{1/2}_{(TT)_\tau}(\omega,\rho) = 2\omega \int_{-1}^{1} \frac{dz}{(\omega^2-z^2)^2}\,[\,(\omega^2-z^2)(3+z^2-2z^4) - \rho\omega(1-z^2)^2\,] \tag{B.10}$$

$$\overline{W}^{1/2}_{(TT)_a}(\omega,\rho) = -4\omega \int_{-1}^{1} dz\, \frac{1}{(\omega^2-z^2)^2}\,[(\omega^2-z^2)+(1-z^2)z^2(\omega^2-z^2+1)\,] \tag{B.11}$$

$$\overline{W}^{1/2}_{(TS)_+}(\omega,\rho) = -4 \int_{-1}^{1} dz\, \frac{1}{(\omega^2-z^2)^2}\,[\,\frac{1}{2}(1-z^2)(\omega^2-z^2)(\omega-\rho) + z^2(\omega-\rho z^2)(\omega^2-1)\,] \tag{B.12}$$

$$\overline{W}^{1/2}_{(TS)_-}(\omega,\rho) = \frac{q_1^2-q_2^2}{l} \int_{-1}^{1} dz\, \frac{1}{(\omega^2-z^2)^2}\,[(1-z^2)\,(\omega^2-z^2)-2\,(\omega^2-1)\,z^2\,] \tag{B.13}$$

$$\overline{W}^{1/2}_{(TS)_\tau}(\omega,\rho) = 2\sqrt{\omega^2-1} \int_{-1}^{1} dz\, \frac{(1-z^2)\,z^2}{(\omega^2-z^2)^2}\,(\omega^2-z^2-1+2\omega\rho) \tag{B.14}$$

$$\overline{W}^{1/2}_{(TS)_{\tau_a}}(\omega,\rho) = -2\sqrt{\omega^2-1} \int_{-1}^{1} dz\, \frac{(1-z^2)\,z^2}{(\omega^2-z^2)^2}\,(\omega^2-z^2+1) \tag{B.15}$$

$$\overline{W}^{1/2}_{SS}(\omega,\rho) = 8\rho\,(\omega^2-1) \int_{-1}^{1} dz\, \frac{(1-z^2)\,z^2}{(\omega^2-z^2)^2} \tag{B.16}$$

Appendix C

The coefficients in formula (27) are the following

$$F_{TT} = \frac{q_1^2 q_2^2}{s^2}\,[1 + \frac{s^2}{2l^2}\,e(1)\,]\,[\,1 + \frac{s^2}{2l^2}\,e(2)\,] \tag{C.1}$$

$$F_{(TT)_\tau} = F_1 - \frac{\nu s}{4l^2}\,F_2 + \frac{\nu^2}{q_1^2 q_2^2}\,F_{SS} - \frac{1}{2}\,F_{TT} \tag{C.2}$$

$$F_{(TT)_a} = F_{(TS)_{\tau_a}} = 0 \tag{C.3}$$

$$F_{(TS)_+} = -\frac{q^2 q^2}{4l^2}\,[\,e(1) + e(2) + \frac{s^2}{l^2}\,e(1)\,e(2)\,] \tag{C.4}$$

$$F_{(TS)_-} = -\frac{q_1^2 q_2^2}{4l^2}\,[\,e(1) - e(2)\,] \tag{C.5}$$

$$F_{(TS)_\tau} = -\frac{s\sqrt{q_1^2 q_2^2}}{2l^2}\,F_2 + \frac{4\nu}{\sqrt{q_1^2 q_2^2}}\,F_{SS} \tag{C.6}$$

$$F_{SS} = \frac{q_1^2 q_2^2 s^2}{4l^2}\,e(1)\,e(2)\,, \tag{C.7}$$

where

$$e(1) = (1-2x)^2 + (1-2x)\,\frac{q_2^2-p^2-q_1^2}{s} + \frac{p^2 q_2^2}{s^2} \tag{C.8}$$

$$e(2) = e(1,\ x \leftrightarrow y,\ q_1^2 \leftrightarrow q_2^2\,) \tag{C.9}$$

$$F_1 = \frac{1}{s^2} \ell_{\mu\nu} k^{\mu\nu} = \frac{1}{2} (2x + \frac{q_1^2}{s})(2y + \frac{q_2^2}{s}) + (x+y -1/2+ \frac{P^2}{2s}) \qquad \text{(C.10)}$$

$$F_2 = \frac{8}{s^2} \ell_{\mu\nu} q_1^{\nu}, k^{\mu\rho} q_{2\rho} = (1 - \frac{q_2^2}{s} - 2y)(1 - \frac{q_1^2}{s} - 2x) \times$$

$$\times (2x + 2y - 1 - \frac{P^2}{s}) + \textit{similar terms} \qquad \text{(C.11)}$$

$$k_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} (kk')$$

The region of integration is determined by the following conditions

i) $x', y', \geq 0$; $x' = 1-2x$, $y' = 1-2y$

ii) $x' < 1 - \frac{|q_1^2|}{s}$; $y' \leq 1 - \frac{|q_2^2|}{s}$

iii) $\Delta \geq 0$

iv) since $\theta_i < 20°$

$$\eta_i = \frac{2\,|q_i^2|}{s} \ll 1 \quad \text{and} \quad \frac{\eta_1}{x}, \frac{\eta_2}{y} \ll 1 .$$

Taking into account the conditions iv) and that $P^2 \ll |q_i^2|$ the condition iii' can be studied at $P^2 = 0$ in first order of η, s. In this approximation we obtain

$$\Delta = xy(2x+2y-1+\frac{P^2}{s}) + (x+\frac{q_1^2}{s})(y+\frac{q_2^2}{s})(2x+2y-1+\frac{P^2}{s})$$

$$-2xy(x+\frac{q_1^2}{s})(y+\frac{q_2^2}{s}) - y^2(x+\frac{q_1^2}{s})^2 - x^2(y+\frac{q_2^2}{s})^2 - \qquad \text{(C.12)}$$

$$-\frac{1}{4}(2x+2y-1+P^2/s)^2 \sim$$

$$\simeq \frac{1}{4} x'y' [\eta_1 + \eta_2 - (\eta_1 + x')(\eta_2 + y')] \geq 0$$

The boundary of the physical region in x' and y' is a hyperbola As $\eta_i \to 0$ this region goes to zero as

$$T = \int_R dx'\, dy' = \eta \ln \frac{1}{\eta} \qquad (C.12)$$

(since the main contributions come from the region where $x'y' \sim \sim \sqrt{\eta_1 + \eta_2}$ later on we assume that $\eta_1 \sim \eta_2$).

Keeping only the leading terms in $1/\eta$ we obtain, that

$$\int \frac{dx\, dy}{\sqrt{\Delta}} F_a \sim C \ln 1/\eta \qquad (C.13)$$

for all the non-zero coefficients, e.g.

$$C_{TT} = C_{SS} = C_{(ST)_+} = 6\pi \omega^2 (\omega^2 - 1) \qquad (C.14)$$

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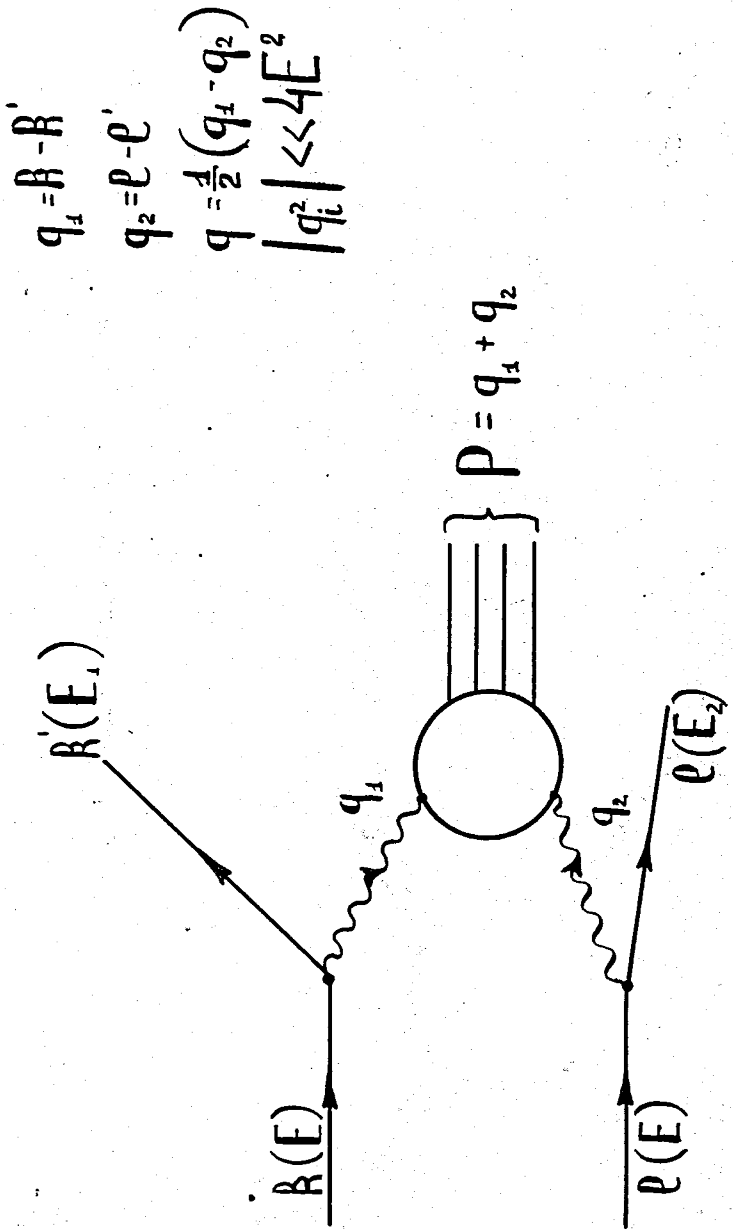

Fig. 1. Inelastic electron-electron scattering.

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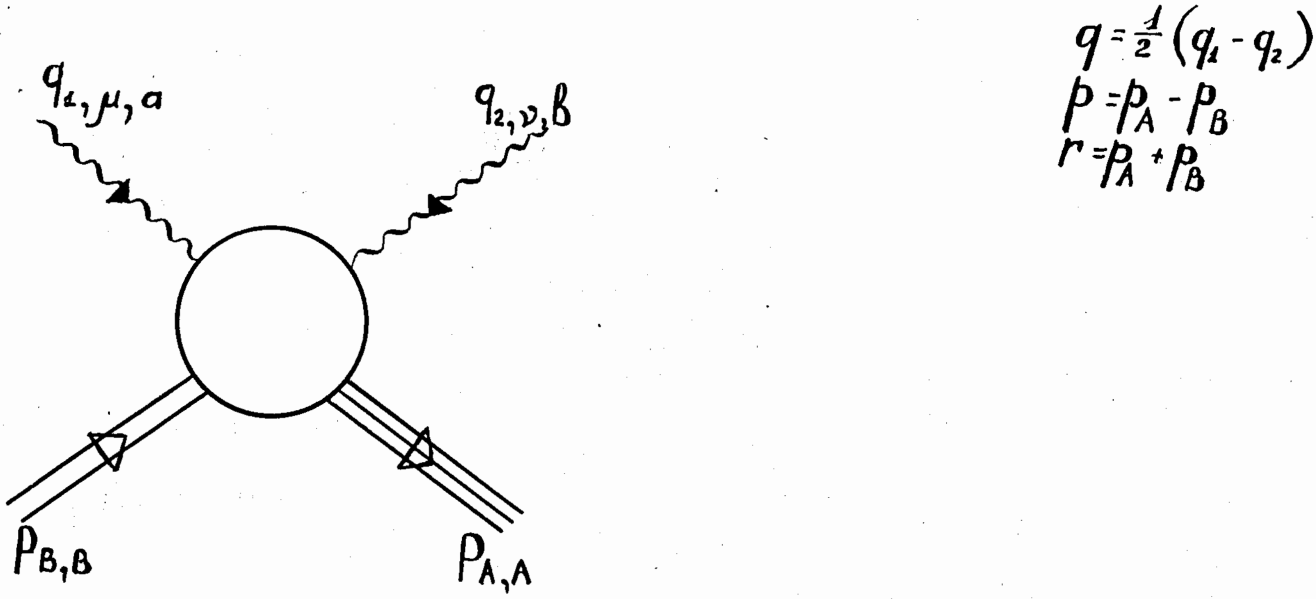

Fig. 2. Generalized Compton amplitude.

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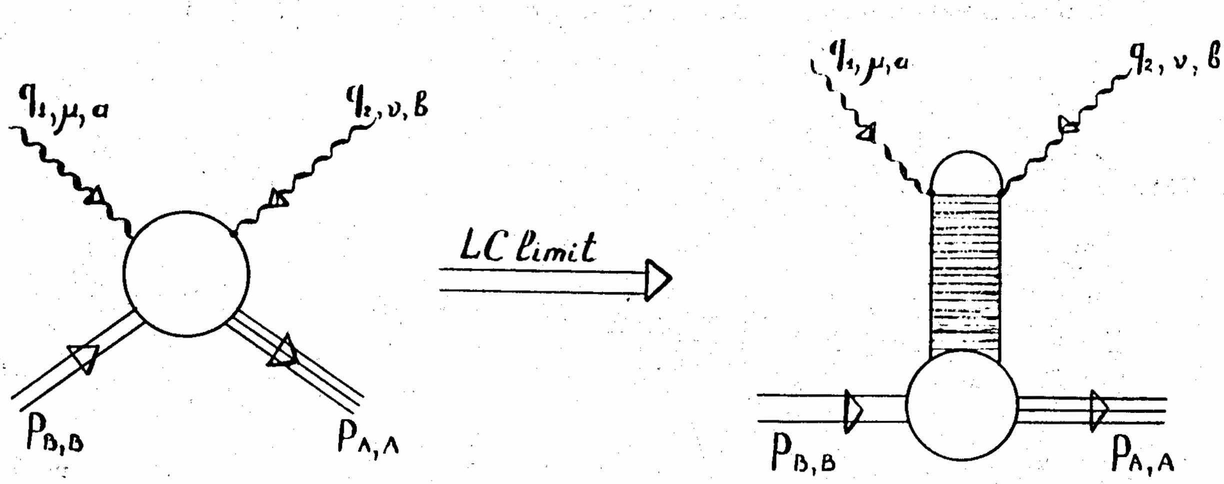

Fig. 3. In the LC limit the generalized Compton amplitude is controlled by the matrix elements of the bilocal operators.

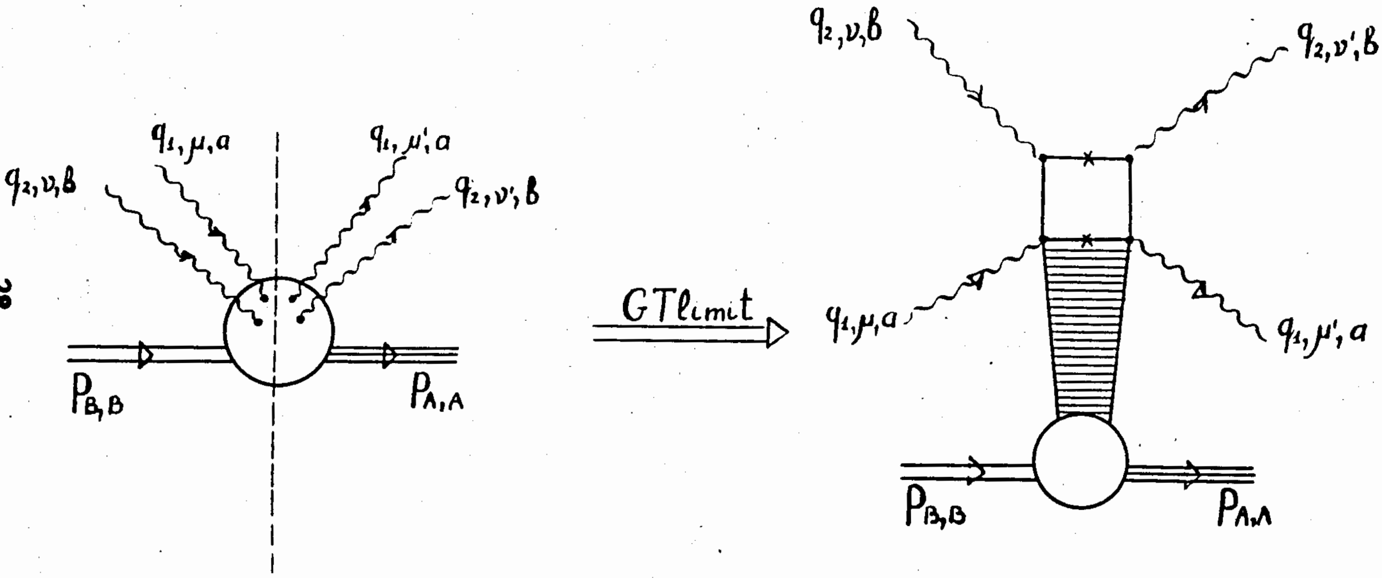

Fig. 4. In the GT limit the cross sections of the relevant deep-inelastic processes are determined by the matrix elements of the bilocal operators (neglecting the disconnected contributions).

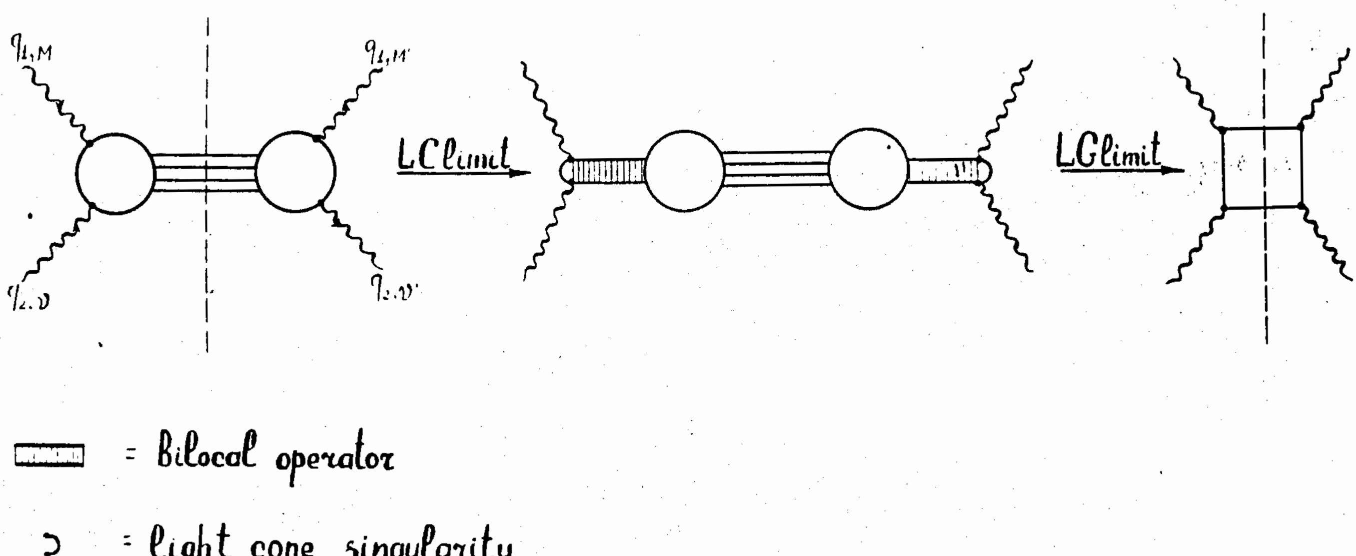

Fig. 5. Forward photon-photon scattering and the light cone singularities.

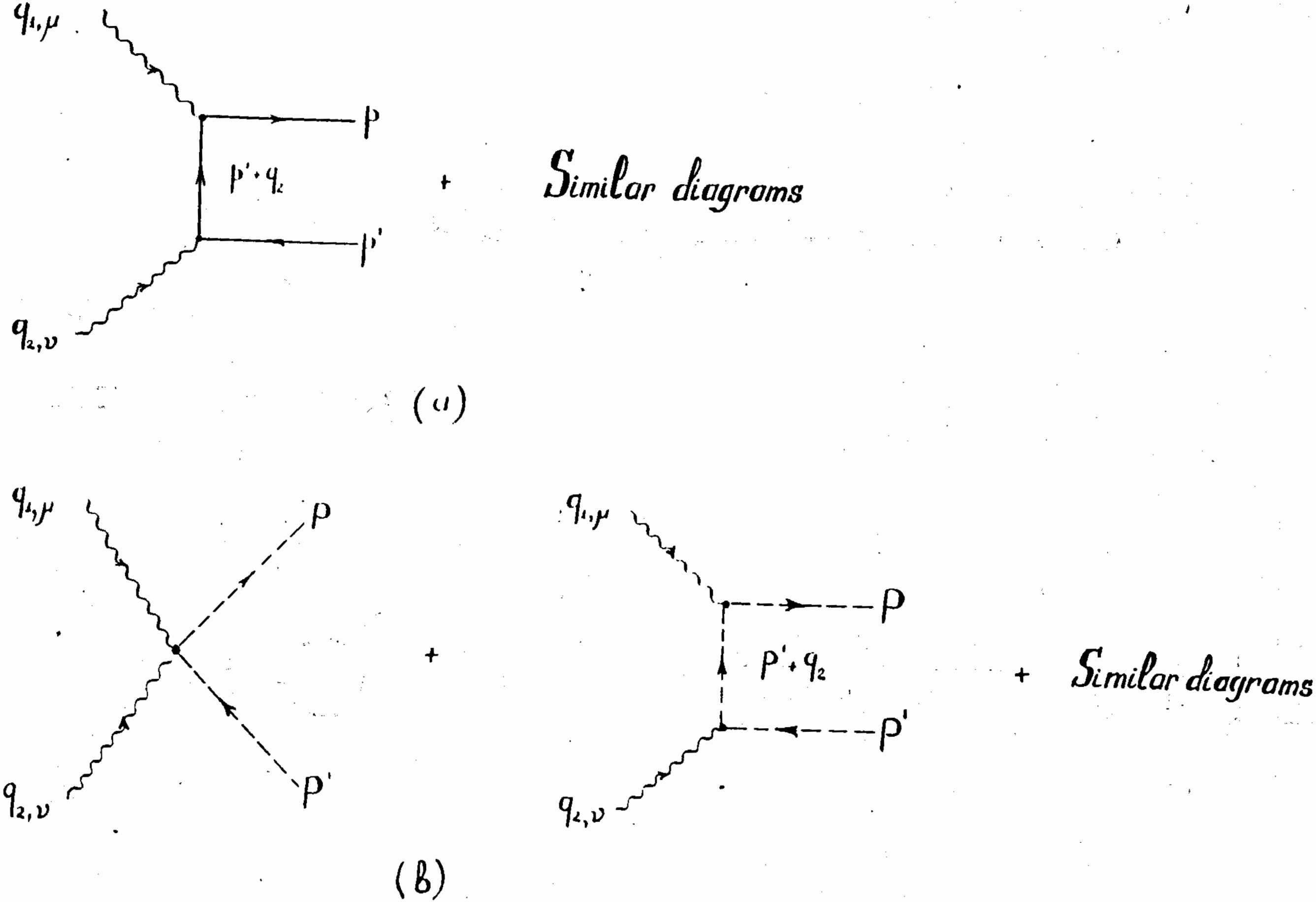

Fig. 6. Parton-antiparton final states. a) spin 1/2 parton; b) spin 0 parton.