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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

TIME-DEPENDENT YUKAWA POTENTIALS OF A CLASS OF WEAK GRAVITATIONAL FIELDS

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1. The Einstein vacuum equations of gravitation $R_i^k - \frac{1}{2}\delta_i^k R = 0$ are derived from Lagrangian density equal to the scalar curvature R. If the Lagrangian density is not R but $F(R)$, where $F(R)$ is three times differentiable but otherwise arbitrary function of R, one obtains the generalized gravitational equation /1/.

$$-\frac{1}{2}\delta_i^k F(R) + R_i^k \frac{dF(R)}{dR} + g^{k\ell}\left(\frac{dF(R)}{dR}\right)_{;i\ell} - \delta_i^k g^{\ell m}\left(\frac{dF(R)}{dR}\right)_{;\ell m} = 0. \quad (1)$$

(1) is written in the usual notation /2/. We shall adhere to this current usage in our notation further.

In the present note are to be found in an explicit form central-symmetric solutions of (1) in the Newtonian limit. The main result obtained here is that in the most general case these solutions contain terms of the Ukawa type depending on the time.

2. In order to obtain (1) in the Newtonian limit, it is enough to replace $F(R)$ with the first three terms of its power series expansion $F(R) = F_0 + F_1 R + F_2 R^2$, where F_0, F_1

and F_2 are constants. Such a representation of the Lagrangian density has proved fruitful in the investigation of other problems /3/.

It is expedient to represent ds^2 in the form:

$$ds^2 = c^2 e^{\nu} dt^2 - r^2 e^{\mu} (d\theta^2 + \sin^2\theta \, d\phi^2) - e^{\lambda} dr^2 ,$$

where λ, μ and ν are small numbers vanishing at $r \to \infty$. This choice of ds^2 is convenient since it reduces to the Euclidean form at infinity.

The equation (1) can be rewritten as a set of four scalar equations corresponding to the non-identically vanishing components of the Hamiltonian derivative H_i^k.

It is known, however, that to describe a central-symmetric field it is enough to make use of two functions, for example, $\lambda = \lambda(r,t)$ and $\nu = \nu(r,t)$ (μ may be considered as equal to zero/2/).

This seeming over-determinacy of the system of equations for λ and ν could be eliminated through a theorem of Edington /4/. According to this theorem the Hamiltonian derivative H_i^k of a large class of Lagrangian densities, which includes also the Lagrangian density $F(R)$, is a conservative tensor. Since the left-hand side of (1) is the Hamiltonian derivative in question, we could use the equation $H^k_{i;k} = 0$ in order to express the extra equation through the remaining ones.

Taking into consideration what has been said above, after the linearization (1) is reduced to the system of equations (here a prime denotes differentiation with respect to r and dot - with respect to ct).

$$-\frac{1}{2}(F_0 + F_1 R) + F_1 R_0^0 + 2F_2 (R'' + \frac{2}{r} R') = 0 \qquad (H_0^0 = 0) \qquad (2)$$

$$-\frac{1}{2}(F_0 + F_1 R) + F_1 (R^1_1) + 4F_2 \frac{R'}{2} - 2F_2 \ddot{R} = 0 \qquad (H^1_1 = 0) \tag{3}$$

$$F_1 R^1_0 - 2F_2 \dot{R}' = 0 \qquad (H^1_0 = 0) \tag{4}$$

The system of three equations (2), (3) and (4) contains the three unknown functions $\lambda(r,t)$, $\nu(r,t)$ and $R(r,t)$.

In (2) and (3) we substitute $G^0_0 + \frac{1}{2}R$ and $G^1_1 + \frac{1}{2}R$ for R^0_0 and R^1_1, respectively, where $G^k_l = R^k_l - \frac{1}{2}\delta^k_l R$ is the Einstein tensor.

For G^0_0, G^1_0 and G^1_1 we have /2/ $G^0_0 = -e^{-\lambda}(\frac{1}{r^2} - \frac{\lambda'}{r})$, $G^1_0 = -\frac{\dot{\lambda} e^{-\lambda}}{r}$ and $G^1_1 = -e^{-\lambda}(\frac{\nu'}{r} + \frac{1}{r^2}) + \frac{1}{r^2}$, respectively. After substituting in (2) and (3) the linearized expressions for G^0_0, G^1_0 and G^1_1 we obtain:

$$a + \frac{\lambda'}{r} + \frac{\lambda}{r^2} + bR'' + \frac{2bR'}{r} = 0, \tag{5}$$

$$a - \frac{\nu'}{r} + \frac{\lambda}{r^2} + \frac{2bR'}{r} - b\ddot{R} = 0, \tag{6}$$

$$-\dot{\lambda} + b\dot{R}'r = 0, \tag{7}$$

where a and b denote the expressions $-\frac{F_0}{2F_1}$ and $\frac{2F_2}{F}$ respectively, a is the cosmological constant /5/, and b characterizes the influence of the derivatives of fourth order in the linearized gravitational equations.

The expressions for G^2_2 and G^3_3 are

$$G^2_2 = G^3_3 = -\frac{e}{2}(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu'\lambda'}{2}) + \frac{e}{2}(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}).$$

After the linearization of these expressions and making use of the linearized expressions for G^0_0 and G^1_1 through the identity $R + G^l_l = 0$ we obtain

$$-R + 2\frac{\lambda' - \nu'}{r} + \frac{2\lambda}{r^2} + \ddot{\lambda} - \nu'' = 0. \tag{8}$$

From (7) we have $\lambda = bR'r + f(r)$, where $f(r)$ is an arbitrary once differentiable function of r. Introducing in (5) the expression for λ we obtain $f(r) = \frac{C_1}{r}$ and $\lambda = \frac{C_1}{\ } - bR'r - \frac{ar^2}{3}$, where C_1 is a constant. The latter expression for λ enables us to determine ν' from (6): $\nu' = \frac{C}{r^2} - b\ddot{R}r + \frac{2ar}{3} + bR'$. The expressions for λ and ν' substituted in (8) give us

$$R'' + \frac{2}{r}R' - \ddot{R} - \frac{1}{\ }R = -\frac{4a}{\ }, \tag{9}$$

from where for the solutions λ and ν we obtain:

$$\lambda = bC_2\left(k + \frac{1}{r}\right)e^{-kr+\omega t} - \frac{ar^2}{3} + \frac{C_1}{r} + bC_4\left(\frac{1}{r} - k\right)e^{kr+\omega t} \tag{10}$$

$$\nu = bC_2\left(\frac{1}{r} + \frac{\omega^2}{k}\right)e^{-kr+\omega t} + \frac{ar^2}{3} - \frac{C_1}{r} + C_3 + bC_4\left(\frac{1}{2} - \frac{\omega^2}{k}\right)e^{kr+\omega t} + \tag{11}$$

$$+4ab,$$

where $k^2 = \pm\left|\omega^2 + \frac{1}{3b}\right|$ and C_2, C_3 and C_4 are constants.

It is noteworthy that the terms, which correspond to the Newtonian potential are not dependent on the time. This is a reflection of the Birkhoff's theorem and is connected with the special form of the Einstein gravitational equation. As is seen from the solutions obtained here for more general gravitational equations the central symmetric solutions contain Yukawa terms which do depend on the time at least in the Newtonian limit. Hence it is likely that the statements of the Birkhoff's theorem are due to the special properties of the Einstein gravitational equation.

3. Fields of that type which we studied above would explain certain difficulties in the models of astrophysical objects ejecting masses for a considerably long time. Here we have in mind for instance the quasar 3C273 with its jet, or the Seifert galaxies in whose nuclei gas is running in certain directions,or other galactic nuclei ejecting gases in certain directions. For quasars with jets

in[6] the hypothesis has been put forward that they have centers around which in certain periods of time occur explosions having the character of the explosions of the supernovas - more poweful in some cases, and less powerful in other ones. It is essential that they occur around a constant center, the phenomenon being observed not only in the quasars but also in the Seifert galaxies as well as in other galaxies whose nuclei are ejecting gas jets, as for instance the object NGG 4486 and the object NGG 3561. Such centers could be created, for instance, through accretion of gas towards a star situated in a given quasar. It could not be explained, however, in the framework of the conventional theory of relativity how could very powerful explosions occur repeatedly around a single center. The mass of this center would have soon become greater than the critical mass, a collapse would have followed and the further explosions around the same center probably would not have been possible. It is not excluded that solutions of the type of (10) and (11) would be more successful in explaining the stability of the above-mentioned centers of explosions or even of the quasar as a whole. This is because such solutions have in addition to the conventional newtonian and postnewtonian terms extraterms which may balance the effects leading to instability.

4. If in (10) and (11) b is considered to be of the order of the inverse radius of the Universe, then this would lead to a possible explanation of the time-dependence of the gravitational constant if any:

5. The theory of elementary particles offers another possibility for applying the solution (10), (11). This is because the last contains the adjustable constant b . The presence of b may be advantageous when the gravitational constant is too small to explain some phenomena.

6. With $a = 0$ equation (9) can be rewritten in the form $\Box R - \frac{1}{3b} R = 0$. The last equation can be derived directly from equation (1) after its linearization without supposing central symmetry. The identity between the equation $\Box R - \frac{1}{3b} R = 0$ and the equation for the meson field $\Box \psi - (2\pi\mu)^2 \psi = 0$; $(h = c = 1)$ strongly suggests that it may be possible to put $\psi = const R$ and $b = \frac{1}{6\pi\mu}$, where μ is the meson mass.

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