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INVESTIGATION OF THE HIGH ENERGY BEHAVIOUR OF THE "MESON-NUCLEON" SCATTERING AMPLITUDE IN SCALAR MODEL BY FUNCTIONAL METHOD

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INVESTIGATION OF THE HIGH ENERGY BEHAVIOUR OF THE "MESON-NUCLEON" SCATTERING AMPLITUDE IN SCALAR MODEL BY FUNCTIONAL METHOD

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Исследование функциональным методом высокоэнергетического поведения амплитуды "мезон-нуклонного" рассеяния в скалярной модели

В рамках метода функционального интегрирования изучается амплитуда "мезон-нуклонного" рассеяния в скалярной модели $\mathcal{L}_{int} = g\psi^2\phi$ при больших энергиях. Сумма бесконечного класса диаграмм, в которых "мезон" взаимодействует с "нуклоном", переходя в состояние "нуклон-антинуклонной" пары, приводит к модифицированной эйкональной форме амплитуды рассеяния.

Препринт Объединенного института ядерных исследований.
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Investigation of the High Energy Behaviour of the "Meson-Nucleon" Scattering Amplitude in Scalar Model by Functional Method

The high energy "meson-nucleon" scattering amplitude is investigated in the scalar model $\mathcal{L}_{int} = g\psi^2\phi$ by the functional method. The sum of the infinite class of diagrams in which the "meson" interacts with the "nucleon" turning into a virtual "nucleon-antinucleon" pair can be presented in a modified eikonal form.

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I

Recently the covariant perturbation theory has successfully been used in high energy physics. Although the problem of the convergence of the renormalized perturbation series is open the summation of certain classes of the Feynman graphs describing the high energy particle scattering is carried out. The choice of some or other infinite series of diagrams is based on a preliminary investigation of the asymptotics of the characteristic Feynman graphs of this series or on the comparison of the final result with experiment. The investigation along this lines has a claim on obtaining a physical picture of the high energy interaction in the framework of the notions of the usual field theory. In this field there are various methods for investigating the asymptotic behaviour of some Feynman graphs and for summing them. The following methods appear to be the most known ones.

The approach based on the Feynman α-parametrization and the Mellin integral transformation was used by Efremov /1/, Polkinghorne /2/ et al. to obtain the universal rules for determining the asymptotics of the Feynman graphs in various filed theory models.

This method was found to be convenient for summing the contributions of some graphs aimed at the study of the analytic properties of the scattering amplitude in the complex plane of the angular momenta[3] and, consequently, in the high-energy asymptotic domain.

In some cases, it is possible to simplify the calculation of the asymptotics of the Feynman integrals by using the Sudakov's technique based on the expansion of the virtual momenta of integration over the high-energy particle momenta. This method has been used to consider many processes of quantum electrodynamics[5].

Many studies are performed by means of the infinite momentum frame[6]. This approach may be viewed as the choice of a more convenient Lorentz co-ordinate system.

On the basis of the study of the scattering amplitude, in the framework of perturbation theory, Cheng and Wu[7] have proposed an "impact" picture of high-energy processes and, hence, modified Feynman rules.

The method of functional integration is also extensively used in the investigation of high-energy scattering amplitudes[8]. To estimate the functional integrals a special approach has been suggested which is well demonstrated from the physical point of view in the region of high energies and small momentum transfer and named the straight-line path approximation[9]. The important advantage of this approach compared with the above-mentioned methods is the possibility of obtaining immediately, in a closed form, the sum of the considered class of diagrams.

It should be noted that so far all the enumerated methods, but the first one, have no fundamental mathematical grounds and in these lines only fragmentary results have been obtained[10].

These methods have been used to sum several infinite classes of perturbation diagrams. On the basis of these results the high-energy interaction of "nucleons" can be represented as follows. In the first approximation the scattered "nucleons" behave like point ones and the interaction between them is described by the Yukawa potential. In perturbation theory, this is due to the ladder and cross-ladder diagrams in the s-channel $(s \to \infty)$. The summation of these graphs results in a quantal eikonal formula[8]. If we digress from the nucleon spin and will consider the exchange of only scalar mesons (model $\mathcal{L}_{int} = g\psi^2\phi$), then the scattering amplitude will behave like a constant at high energies. The total cross section decreases as $\frac{1}{s}$ and in the entire eikonal formula only the Born term is predominant. In these respects, the model with vector exchange $\mathcal{L}_{int} = ig :A_\mu \psi^* \overleftrightarrow{\partial}_\mu \psi + g^2 : A_\mu^2 \psi^* \psi:$ is more satisfactory. In this model the total cross section tends to a constant if we restrict ourselves to the consideration of various ladder diagrams[11].

It was found that the eikonal picture of the interaction holds for a more complicated exchange mechanism too[12] (exchange of quanta of two types). This leads to a change in the effective potential, it becomes smoother at small distances.

In the framework of the scalar model, further investigations have given results which make it possible to consider of importance not the exchange of virtual mesons but the exchange of more complicated systems having the Regge behaviour ("multitower" exchange, exchange of reggeons). It is characteristic that this type of exchange may be dominant only for large values of the interaction constant. The sum of the diagrams with such an exchange leads to the Regge-eikonal representation for the scattering amplitude. /13/ .The importance of this result is as follows. Firstly, it confirms, in the framework of the field theory, the hypothesis on the presence of cuts in the Regge plane. Secondly, the reggeon exchange, even in the case of scalar particles, leads to the total cross section which increases with increasing energy: $\sigma^t \sim (\ln s)^2$ and does not fall as a negative power of s contrary to the case of the simple meson exchange. The remarkable feature of such a complex exchange is that Froissart bound is just saturated.

The scattering of "nucleons" may be considered in this case as diffractional, each individual nucleon behaves like a black ball the radius of which increases logarithmically with energy.

The account of the radiative corrections to the ladder diagrams with both the simple meson exchange and the Regge exchange leads to an additional space distribution of the nucleons within a distance of order $\frac{1}{m}$ (m is the nucleon mass). In the scattering amplitude there appears in this case a factor e^{at} /14/ .

The explanation of the experimental data by the models obtained from the above-mentioned methods is given in refs /15/ .

II

The study of the "meson-nucleon" scattering in perturbation theory shows that in these processes of importance must be diagrams in which the "meson" turns into a virtual "nucleon-antinucleon" pair, as is shown in Fig. I, ref. /16/ .

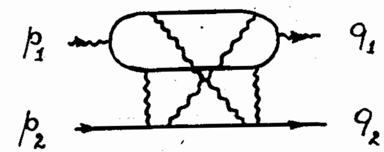

Fig. I

Next, in the framework of the scalar model $\mathcal{L}_{int.} = g\psi^2\phi$ we consider the high-energy "meson-nucleon" scattering using the functional integration and the straight-line path approximation /9/. We restrict ourselves to the summation of the diagrams of the type presented in Fig. I x/.

The general method of constructing the "meson-nucleon" amplitude in the framework of the functional approach is given in ref. /17/. In the present case it is more convenient to perform calculations by another scheme. We first construct the "meson-nucleon" Green function corresponding to the class of diagrams considered

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x/ In the paper by Cheng and Wu, the importance of the diagrams of this type in the sixth order in the interaction constant was shown for the case of the vector particle playing the role of "meson" (γ -quantum, ρ -meson). Considering the same diagrams in the framework of the scalar model we are interested in the structure of the scattering amplitude as a whole, at $s = (p_1 + p_2)^2 \to \infty$.

$$G(q_1,q_2|p_1,p_2) = g^2 \int \prod_{k=1}^{4} d^4x_k \, e^{-iq_1x_1 - iq_2x_2 + ip_1x_3 + ip_2x_4}$$

$$\exp\left\{ -\frac{i}{2} \iint d^4x\, d^4y \frac{\delta}{\delta\phi(x)} D^c(x-y) \frac{\delta}{\delta\phi(y)} \right\} G(x_3, x_1|\phi) \times \qquad (1)$$

$$\times G(x_1,x_3|\phi)\, G(x_2,x_4|\phi) \Big|_{\phi=0},$$

where $G(z_1,z_2|\phi)$ is the "nucleon" Green function in the external classic field ϕ. By means of the functional integral this function can be represented as follows /18/:

$$G(z_1,z_2|\phi) = i\int_0^\infty d\tau\, e^{-i\tau m^2} \int [\delta^4\nu]_0^\tau \exp\left\{ ig\int_0^\tau d\xi\, \phi\left(z_1 + 2\int_0^\xi \nu(\eta)\,d\eta\right)\right\} \delta^{(4)}\left(z_1 - z_2 + 2\int_0^\tau \nu(\eta)\,d\eta\right), \qquad (2)$$

where

$$[\delta^4\nu]_a^b \equiv \frac{\exp\{-i\int_a^b \nu^2(\eta)\,d\eta\}\delta^4\nu}{\int \exp\{-i\int_a^b \nu^2(\eta)\,d\eta\}\delta^4\nu}.$$

After substituting (2) in (1) it is easy to carry out the variational differentiation

$$G(q_1,q_2|p_1,p_2) = g^2 \int \prod_{k=1}^{4} d^4x_k \, e^{-iq_1x_1 - iq_2x_2 + ip_1x_3 + ip_2x_4} \qquad (3)$$

$$\prod_{i=1}^{3} \left(i\int_0^\infty d\tau_i\, e^{-i\tau_i m^2} \int [\delta^4\nu_i]_0^{\tau_i}\right) \exp\left\{ ig^2 \int d^4x\, d^4y\, [i_1(x) + i_2(x)]\right.$$

$$\left. D^c(x-y)\, i_3(y) \right\} \delta^{(4)}\left(x_3 - x_1 - 2\int_0^{\tau_1} \nu_1(\eta)\,d\eta\right) \delta^{(4)}\left(x_1 - x_3 - 2\int_0^{\tau_2} \nu_2(\eta)\,d\eta\right).$$

$$\times \delta^{(4)}\left(x_2 - x_4 - 2\int_0^{\tau_3} \nu_3(\eta)\,d\eta\right),$$

where the currents of three "nucleons" are

$$i_1(z) = \int_0^{\tau_1} d\xi_1\, \delta^{(4)}\left(z - x_3 + 2\int_{\xi_1}^{\tau_1} \nu_1(\eta)\,d\eta\right),$$

$$i_2(z) = \int_0^{\tau_2} d\xi_2\, \delta^{(4)}\left(z - x_1 + 2\int_{\xi_2}^{\tau_2} \nu_2(\eta)\,d\eta\right),$$

$$i_3(z) = \int_0^{\tau_3} d\xi_3\, \delta^{(4)}\left(z - x_2 + 2\int_{\xi_3}^{\tau_3} \nu_3(\eta)\,d\eta\right).$$

For subsequent formulas to be symmetrical with respect to both the virtual "nucleons" into which the "meson" turns it is convenient to make the following replacement of the variables:

$$x_1 = y_1 + y_2,$$

$$x_3 = y_1 - y_2.$$

It is easy to perform the integration over d^4y_2 and d^4x_4 using the δ-functions in (3):

$$G(q_1,q_2|p_1,p_2) = g^2 \int d^4y_1 \int d^4x_2\, e^{iy_1(p_1-q_1) + ix_2(p_2-q_2)}$$

$$\prod_{i=1}^{3}\left(i\int_0^\infty d\tau_i\, e^{-i\tau_i m^2} \int [\delta^4\nu_i]_0^{\tau_i}\right) \int \frac{d^4k}{(2\pi)^4} \exp\left\{ 2i\left[k + \frac{1}{4}(p_1+q_1)\right]\int_0^{\tau_1}\nu_1(\eta)\,d\eta + \right. \qquad (4)$$

$$\left. + 2i\left[k - \frac{1}{4}(p_1+q_1)\right]\int_0^{\tau_2}\nu_2(\eta)\,d\eta - 2ip_2\int_0^{\tau_3}\nu_3(\eta)\,d\eta \right\} \times$$

$$\times \exp\left\{ ig^2 \iint d^4x\, d^4y\, [i_1(x) + i_2(x)]\, D^c(x-y)\, i_3(y) \right\},$$

where now

$$i_i(z) = \int_0^{\tau_i} d\xi_i\, \delta^{(4)}(z - y_i + \int_0^{\tau_i} \nu_i(\eta)[\theta(\eta-\xi_i) - \theta(\xi_i-\eta)]\, d\eta), \quad (i=1,2),$$

$$i_3(z) = \int_0^{\tau_3} d\xi_3\, \delta\, (z - x_2 + 2\int_{\xi_3}^{\tau_3} \nu_3(\eta)\, d\eta).$$

The following replacement of the functional variables

$$\nu_1(\eta) = \nu_1'(\eta) + \frac{1}{4}(p_1 + q_1) + k,$$

$$\nu_2(\eta) = \nu_2'(\eta) - \frac{1}{4}(p_1 + q_1) + k,$$

$$\nu_3(\eta) = \nu_3'(\eta) - p_2.$$

makes it possible to eliminate the terms linear in ν_i in the exponent of the exponential in (4).

Further it is necessary to subtract from the considered "meson-nucleon" Green function the contribution of the disconnected diagram of Fig. 2.

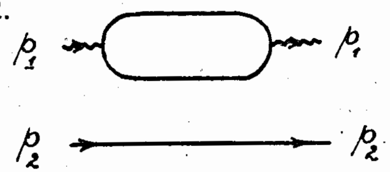

Fig. 2

The resulting Green function is then denoted as $\bar{G}(q_1, q_2 | p_1, p_2)$:

$$\bar{G}(q_1,q_2|p_1,p_2) = g^2\int d^4y_1\, d^4x_2\, e^{iy_1(p_1-q_1) + ix_2(p_2-q_2)}$$

$$\prod_{l=1}^{3}(i\int_0^\infty d\tau_l\, e^{-im^2\tau_l}\int[\delta^4\nu_l]_0^{\tau_l})\int\frac{d^4k}{(2\pi)^4}\exp\{i[k+\frac{1}{4}(p_1+q_1)]^2\tau_1 + \tag{5}$$

$$+ i[k-\frac{1}{4}(p_1+q_1)]^2\tau_2 + ip_2^2\tau_3\}\, ig^2\int\int[i_1(x)+i_2(x)]\, D^c(x-y)\, i_3(y)\, dx\, dy \times$$

$$\times\int_0^1 d\lambda\, \exp\{ig^2\lambda\int d^4z_1\int d^4z_2\, [i_1(z_1)+i_2(z_1)]\, D^c(z_1-z_2)\, i_3(z_2)\},$$

where

$$i_1(z) = \int_0^{\tau_1} d\xi_1\, \delta^{(4)}(z - y_1 + \int_0^{\tau_1}[\nu_1(\eta) + \frac{1}{4}(p_1+q_1)+k][\theta(\eta-\xi_1) - \theta(\xi_1-\eta)]\, d\eta),$$

$$i_2(z) = \int_0^{\tau_2} d\xi_2\, \delta^{(4)}(z - y_1 + \int_0^{\tau_2}[\nu_2(\eta) - \frac{1}{4}(p_1+q_1)+k][\theta(\eta-\xi_2)-\theta(\xi_2-\eta)]\, d\eta),$$

$$i_3(z) = \int_0^{\tau_3} d\xi_3\, \delta^{(4)}(z - x_2 + 2\int_0^{\tau_3}[\nu_3(\eta) - p_2]\, d\eta).$$

To obtain the "meson-nucleon" scattering amplitude it is necessary to go over in eq. (5) to the mass shell along the external adges of the "nucleon" line (with respect to p_2 and q_2) and extract the δ-function of the law of conservation of four momentum.

$$(2\pi)^4\,\delta^{(4)}(p_1+p_2-q_1-q_2)\, f(q_1,q_2|p_1,p_2) =$$
$$= \lim_{p_2^2,\, q_2^2 \to m^2}(p_2^2-m^2)(q_2^2-m^2)\, i\,\bar{G}(q_1,q_2|p_1,p_2). \tag{6}$$

Using for this purpose the technique developed in ref. /19/ we get:

$$f(q_1,q_2|p_1,p_2) = (-i)\,g^4\int e^{ib(p_1-q_1)}\, d^4b\int\frac{d^4k}{(2\pi)^4}$$

$$(\prod_{i=1}^{2}\int_0^\infty d\tau_i\, e^{-i\tau_i m^2}\int[\delta^4\nu_i]_0^{\tau_i})\int[\delta^4\nu_3]_{-\infty}^{+\infty}$$

$$\exp\{i\tau_1[k+\frac{1}{4}(p_1+q_1)]^2 + i\tau_2[k-\frac{1}{4}(p_1+q_1)]^2\}. \tag{7}$$

$$\cdot\int d^4x\, [i_1(x)+i_2(x)]\, D^c(x - \frac{z-b}{2})\int_0^1 d\lambda\, \exp\{ig^2\lambda\int\int[i_1(x)+i_2(x)]$$

$$D^c(x-y)\, i_3(y)\, d^4x\, d^4y\},$$

with the currents

$$i_1(x) = \int_0^{\tau_1} d\xi_1\, \delta^{(4)}\Big(x - \frac{z+b}{2} + \int_0^{\tau_1} [\nu_1(\eta) + \frac{1}{4}(p_1+q_1) + k\,][\theta(\eta-\xi_1) - \theta(\xi_1-\eta)]\,d\eta\Big),$$

$$i_2(x) = \int_0^{\tau_2} d\xi_2\, \delta^{(4)}\Big(x - \frac{z+b}{2} + \int_0^{\tau_2}[\nu_2(\eta) - \frac{1}{4}(p_1+q_1) + k\,][\theta(\eta-\xi_2) - \theta(\xi_2-\eta)]\,d\eta\Big),$$

$$i_3(x) = \int_{-\infty}^{+\infty} d\xi_3\, \delta^{(4)}\Big(x - \frac{z-b}{2} - 2\int_0^{\xi_3} \nu_3(\eta)\,d\eta + 2\xi_3\,(p_2\,\theta(-\xi_3) + q_2\,\theta(\xi_3))\Big).$$

The formula (7) gives the exact value of the contribution to the scattering amplitude of the sum of the diagrams given in Fig. I.

Further consideration will be carried out in the c.m.s. of colliding particles at high energies ($s \to \infty$)and small momentum transfer $t = (p_1 - q_1)^2 \ll s$ (forward scattering). The "nucleon" mass is put to be m , and the "meson" mass μ . The functional integrals over $\delta\nu_i$ $(i = 1,2,3)$ are estimated in the following manner (the straight-line path approximation). All the expressions containing ν_1 and ν_2 are replaced by their averages. $F(\nu_i) \to \bar F$ where $\bar F = \int[\delta\nu_i]_0^{\tau_i} F(\nu_i), (i=1,2)$ and the functional variable ν_3 is omitted. In the diagram technique this corresponds to the dropping in all the "nucleon" propagators of the terms which are quadratic with respect to the virtual momenta of the "mesons", but in the propagators of the "nucleon" loop the squares of these momenta will be retained. Now the formula (7) is (for detailed calculations see Appendix):

$$f(q_1, q_2\,|\,p_1, p_2) = -g^2 s \int e^{-ib_\perp (p-q)_\perp} d^2 b_\perp \int \frac{d^2 k_\perp}{(2\pi)^2} \int_0^\infty d\tau_1\, d\tau_2\, I(k_\perp^2; \tau_1, \tau_2)$$
$$\Big(\exp\Big\{ i\,\frac{g^2}{s} \int \frac{d^2\ell_\perp}{(2\pi)^2} \frac{e^{-i b_\perp \ell_\perp}}{\ell_\perp^2 + \mu^2 - i\epsilon}\Big[e^{-i\frac{\ell_\perp^2 \tau_1}{4}} + e^{-i\frac{\ell_\perp^2 \tau_2}{4}}\Big]\Big\} - 1\Big). \tag{8}$$

The function

$$\int_0^\infty d\tau_1\, d\tau_2\, I(k_\perp^2; \tau_1, \tau_2) = \frac{1}{4\pi} \int_0^\infty \frac{d\tau_1\, d\tau_2}{\tau_1 + \tau_2} \exp\Big\{ -i(k_\perp^2 + m^2)\tau_1 - i(k_\perp^2 + m^2)\tau_2 + i m^2 \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}\Big\}$$

may be interpreted as a factor describing the internal structure of the "meson" consisting of a "nucleon-antinucleon" loop. It is just on these constituents of the "meson" that proceeds the scattering of the high energy "nucleon", this process being described by the amplitude of the eikonal form. A subsidiary integration over $d^2 k_\perp$ in (8) with factor $I(k_\perp^2)$ is responsible for the summation over all the states of the virtual loop.

This picture of the "meson-nucleon" interaction is in agreement with the notions of the high energy interaction of elementary particles in the framework of the Feynmann parton model /20/ and the "impact" picture by Cheng and Wu /7/.

One may expect here, as in the case of the "nucleon-nucleon" interaction, that the Reggion exchange would also lead to a noticeable change of the high energy behaviour of the scattering amplitude (8). (In the considered case $f(s,t) \to const$ at $s \to \infty$ and, consequently, the cross section decreases as $\frac{1}{s}$). The question as to whether the similar interaction mechanism is important for "nucleon-nucleon" scattering remains still unclear /21/ .

In conclusion the authors consider it their pleasant duty to thank D.I. Blokhintsev, A.N. Tavkhelidze, S.P. Kuleshov, V.N. Pervushin and A.N. Sissakian for the interest in the work and stimulating discussions.

Appendix

We consider one of the terms in the exponent of the last exponential in eq. (7):

$$A_1(\nu_1,\nu_3) = g^2 \int d^4x\, d^4y\, i_1(x|\nu_1)\, D^c(x-y)\, i_3(y|\nu_3). \quad \text{(A.1)}$$

We omit the functional variable ν_3 and perform an "averaging" over ν_1 :

$$\bar{A}_1 = \int [\delta^4 \nu_1]_0^{\tau_1} A_1(\nu_1,\nu_3=0) = \frac{-g^2}{(2\pi)^3} \int \frac{d^4\ell \exp(ib\ell + \frac{\ell^2\tau_1}{4})}{\ell^2 - m^2 + i\epsilon}\, \delta(2p_2\ell) \times \frac{\sin[\tau_1 \ell(\frac{p_1}{2}+k)]}{\ell(\frac{p_1}{2}+k)+i\epsilon}. \quad \text{(A.2)}$$

The vectors p_1 and p_2 in the c.m.s. of colliding particles have the following coordinates $p_1=(p_0,0,0,p)$; $p_2=(p_0,0,0,-p)$; $m \simeq \mu$. Taking this into account and using the δ function we integrate over $d\ell_0$. After this the integral over $d\ell_z'$ takes the form ($\ell_z' = \frac{m}{p_0}\ell_z$):

$$\frac{1}{s}\int \frac{d\ell_z'}{(2\pi)} \frac{\exp\{-i\ell_z' \frac{p_0 p}{m}(\frac{b_0}{p_0}+\frac{b_z}{p})\}}{\ell_z'^2 + \ell_\perp^2 + m^2 - i\epsilon} \frac{\sin\{\tau_1[\ell_z'(1+\frac{k_0}{p_0}+\frac{k_z}{p})\frac{p_0 p}{m} + k_\perp \ell_\perp]\}}{\ell_z'(1+\frac{k_0}{p_0}+\frac{k_z}{p}) + m\frac{\ell_\perp k_\perp}{p_0 p}} \quad \text{(A.3)}$$

For $s = 4p_0^2 \to \infty$ the main contribution comes from the integration region at small ℓ_z' and the b_0, b_z, k_0, k_z dependence vanishes. Therefore, at $s \to \infty$ the formula (A.3) can be approximated by the expression: $\frac{1}{2s}(\ell_\perp^2 + m^2 - i\epsilon)^{-1}$

As a result we have the first term in the eikonal phase in eq. (8). The second term is obtained by a simple replacement of τ_1 by τ_2. In just the same way we perform the calculation of the expression standing in front of the integral over d_λ in (7). After this the formula (7) takes a modified eikonal form (8).

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