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TWO-PION PRODUCTION IN THE INTERACTION OF VIRTUAL PHOTONS

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TWO-PION PRODUCTION IN THE INTERACTION OF VIRTUAL PHOTONS

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Under certain kinematic conditions in inelastic scattering of electrons and positrons in colliding beams the virtual photon interaction is very essential [1-7]. The known connection between cross sections of the reactions $e+e \to e+e+$ hadrons and $\gamma+\gamma \to$ hadrons makes it possible, in principle, to derive an information concerning the light-light interaction process [3,4,7]. Probably, because of this physicists are deeply interested in the processes $\gamma+\gamma \to$ hadrons and, particularly, $\gamma+\gamma \to \pi+\pi$.

In the present paper the process $\gamma+\gamma \to \pi+\pi$ for virtual space-like photons with given four-momentum squared ("massive"photons) is described by dispersion relation method. The description is correct in low-energy region.

In the dispersion equations for the reaction amplitudes the two-particle unitarity condition is used, and the crossing-channel contribution is approximately given by that of Born terms and the nearest ω and ρ resonances.

In this case the partial S waves are analytically expressed by the low-energy phases of $\pi\pi$ scattering. Differential cross sections of the reactions $\gamma+\gamma \to \pi+\pi$ and $e+e \to e+e+\pi+\pi$ are obtained by using the analytic formulae for the $\pi\pi$ phases of scattering agreeing well with experimental points.

I. Kinematics of the Reaction

Let 4-momenta of the initial photons be $K=(K_0,\overline{K})$ and $K'=(K_0',\overline{K}')$; $K^2=K'^2=-m_\gamma^2$; ε_μ and ε'_ν the polarization vectors and $K_1(i)=(\omega_1,\overline{K}_1(i))$ and $K_2(j)=(\omega_2,\overline{K}_2(j))$ are 4-momenta of the final pions (i, j -projections of the isotopic spin).

Let us introduce in the c.m.s. of the process $\gamma+\gamma\to\pi+\pi$ the relativistic amplitudes $T^{i}_{\mu\nu}(t,\cos\varphi_+)$ in the following manner:

$$\langle f|\hat{S}|i\rangle = \langle K_1(i); K_2(j)|\hat{S}|K, \varepsilon; K', \varepsilon'\rangle =$$

$$= i\frac{1}{(2\pi)^2}\frac{1}{4\sqrt{\omega_1\omega_2 K_0 K_0'}}\delta^{(4)}(K+K'-K_1-K_2)T^{i}_{\mu\nu}(t,\cos\varphi_+)\varepsilon^{\mu}\varepsilon'^{\nu}. \quad (1)$$

Here: $t = 4K_0^2 = 4\omega^2$; $\omega_1=\omega_2=\omega$; $K_0=K_0'$;

φ_+ is the scattering angle;

$i=-j$ follows from the charge conservation.

The amplitudes $T^{i}_{\mu\nu}(t,\cos\varphi_+)$ describe the interaction of two virtual photons with definite polarizations (longitudinal, transverse, time).

Mandelstam variables have the form:

$$t = (K+K')^2 = 4K_0^2 = 4\omega^2$$

$$S = (K-K_2)^2 = -m_\gamma^2+\mu^2-\frac{t}{2}-2\cos\varphi_+\sqrt{\left(\frac{t}{4}+m_\gamma^2\right)\left(\frac{t}{4}-\mu^2\right)}$$

$$u = (K-K_1)^2 = -m_\gamma^2+\mu^2-\frac{t}{2}+2\cos\varphi_+\sqrt{\left(\frac{t}{4}+m_\gamma^2\right)\left(\frac{t}{4}-\mu^2\right)},$$

μ is the pion mass.

The cut in the complex t plane of direct channel is related to the two-meson intermediate state:

$$t \geqslant 4\mu^2.$$

The time components of the polarization vectors may be omitted by means of gauge invariance condition

$$T_{\mu\nu}K^{\mu}=0, \quad T_{\mu\nu}K'^{\nu}=0$$

and the relations $(\mathcal{E},K)=(\mathcal{E}',K')=0$ (for electron and positron colliding beams:

$$(\mathcal{E},K)=\bar{u}^{s'}(p_-')(\hat{p}_- - \hat{p}_-')u^{s}(p_-)=(\mathcal{E}',K')=\bar{u}^{r}(p_+)(\hat{p}_+ - \hat{p}_+')u^{r}(p_+')=0$$

here: p_+, p_+', p_-, p_-' are momenta of positrons and electrons, $u^{r'}(p_+'), u^{s}(p_-)$ are the corresponding spinors). Let us direct z - axis of coordinate frame along the vector $\bar{K}$, then the procedure of excluding those components will be such that the longitudinal components of the polarization vectors must be multiplied by $\frac{K^2}{K_0^2}$. In what follows we will suppose that the polarization vectors as well as the polarization density matrices of virtual photons contain only the space components.

The amplitudes $T^{i}_{\alpha\beta}(t,\cos\varphi_+)$ are superpositions of the isotopic amplitudes $T^{(T=0)}_{\alpha\beta}(t,\cos\varphi_+)$ and $T^{(T=2)}_{\alpha\beta}(t,\cos\varphi_+)$:

$$T^{i}_{\alpha\beta}(t,\cos\varphi_+)=\langle 1,i;1,-i|0,0\rangle T^{(T=0)}_{\alpha\beta}(t,\cos\varphi_+)+\langle 1,i;1,-i|2,0\rangle T^{(T=2)}_{\alpha\beta}(t,\cos\varphi_+),$$

here indices α and β take three values x, y, z; $\langle 1,i;1,-i|T,0\rangle$ are Klebsch-Gordon coefficients.

The amplitude with the isospin $T=1$ is absent in the expansion because C-parity of the initial state is positive and this state has even values of ℓ, but the amplitude with $T=1$ contains odd degrees of $\cos\varphi_+$.

2. Dispersion Equations

Diagrams presented in Fig. 1 a) and b) correspond to the contribution of crossing channels to the reaction amplitude. The ω -meson contributes only to the process $\gamma+\gamma\to\pi^0+\pi^0$.

Born term, or the amplitude of diagrams given in Fig. 1a), is as follows:

$$B^i_{\alpha\beta}(t,\cos\varphi_t)=-4e^2\left\{\frac{K_{1\alpha}(i)K_{2\beta}(-i)}{u-\mu^2+i\varepsilon}+\frac{K_{1\beta}(i)K_{2\alpha}(-i)}{s-\mu^2+i\varepsilon}-\frac{1}{2}\delta_{\alpha\beta}\right\}.$$

To calculate the ω and ρ resonance contributions we employ the effective interaction Hamiltonians:

$$H_\omega=g_{\omega\pi\gamma}\frac{\partial\omega^\alpha(x)}{\partial x_\beta}F^{\gamma\delta}(x)\,\varepsilon_{\alpha\beta\gamma\delta}\,\varphi_3(x),$$

$$H_\rho=g_{\rho\pi\gamma}\frac{\partial\rho^j_\alpha(x)}{\partial x_\beta}F_{\gamma\delta}(x)\,\varphi_j(x)\,\varepsilon^{\alpha\beta\gamma\delta};$$

$\varepsilon^{\alpha\beta\gamma\delta}$ is completely antisymmetric tensor,

$F_{\gamma\delta}(x)$ is the electromagnetic field tensor operator,

$\omega^\alpha(x)$, $\rho^j_\alpha(x)$, $\varphi_j(x)$ are the field operators of ω, ρ and π -mesons, respectively; j is the isotopic index.

Taking into account the relation $g_{\rho\pi\gamma}=\frac{1}{3}g_{\omega\pi\gamma}$ the contribution of resonance terms is written in the form:

$$P^i_{\alpha\beta}(t,\cos\varphi_t)=-4g^2_{\omega\pi\gamma}\left(\delta_{i0}+\frac{1}{9}\right)\left\{\frac{\varepsilon_{\alpha\gamma\delta\lambda}\varepsilon_{\beta\gamma'\delta'\lambda}K^\delta K'^{\delta'}K_1^\gamma(i)K_2^{\gamma'}(-i)}{u-M_\omega^2+i\varepsilon}+\right.$$

$$\left.+\frac{\varepsilon_{\alpha\gamma\delta\lambda}\varepsilon_{\beta\gamma'\delta'\lambda}K^\delta K'^{\delta'}K_1^{\gamma'}(i)K_2^\gamma(-i)}{s-M_\omega^2+i\varepsilon}\right\}$$

(γ, δ, λ are four-dimensional indices; α,β are three-dimensional indices).

Now we write the dispersion equation for the amplitude $T^i_{\alpha\beta}(t,\cos\varphi_t)$ for $\cos\varphi_t$ -fixed:

$$T^i_{\alpha\beta}(t,\cos\varphi_t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{\operatorname{Im}T^i_{\alpha\beta}(t',\cos\varphi_t)}{t'-t-i\varepsilon}dt'+\tilde{B}^i_{\alpha\beta}(t,\cos\varphi_t)+\tilde{P}^i_{\alpha\beta}(t,\cos\varphi_t).$$

This equation can be re-written for the amplitudes with given isotopic spin:

$$T^{(T)}_{\alpha\beta}(t,\cos\varphi_t) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{\operatorname{Im} T^{(T)}_{\alpha\beta}(t',\cos\varphi_t)}{t'-t-i\varepsilon}\,dt' + \frac{8e^2}{\sqrt{3}}(\sqrt{2})^{-\frac{T}{2}}\tilde{B}_{\alpha\beta}(t,\cos\varphi_t) +$$

$$+\frac{32 g^2_{\omega\pi\gamma}}{9\sqrt{3}}\left(-\frac{22}{8\sqrt{2}}\right)^{\frac{T}{2}}\tilde{P}_{\alpha\beta}(t,\cos\varphi_t), \qquad (2)$$

here: $T = 0,2$;

$$\tilde{B}_{\alpha\beta}(t,\cos\varphi_t) = \frac{(K_{1\alpha}K_{2\beta})^{u=\mu^2}}{u-\mu^2} + \frac{(K_{1\beta}K_{2\alpha})^{s=\mu^2}}{s-\mu^2}; \qquad u = M_\omega^2)$$

$$\tilde{P}_{\alpha\beta}(t,\cos\varphi_t) = \frac{\{(KK')K_{2\alpha}K_{1\beta} + (K_1K_2)K'_\alpha K_\beta + ((KK')(K_1K_2) - (K_1K)(KK_2))\delta_{\alpha\beta} - K_{1\beta}K'_\alpha(KK_2) - K_{2\alpha}K_\beta(KK')\}}{u - M_\omega^2} +$$

$$+\left\{\begin{matrix} K_1 \leftrightarrow K_2 \\ u \leftrightarrow s \end{matrix}\right\}.$$

Next, we expand the isotopic amplitudes $T^{(T)}_{\alpha\beta}(t,\cos\varphi_t)$ in the partial waves. From charge invariance it follows that the relative orbital moment of pions is even, i.e.

$$T^{(T)}_{\alpha\beta}(t,\cos\varphi_t) = T^{(T)}_{\alpha\beta}(t)_s + 5T^{(T)}_{\alpha\beta}(t)_d P_2(\cos\varphi_t) + 9T^{(T)}_{\alpha\beta}(t)_g P_4(\cos\varphi_t) + \ldots$$

Two-particle unitarity condition for partial amplitudes is written in the form:

$$\operatorname{Im} T^{(T)}_{\alpha\beta}(t)_\ell = e^{-i\delta^{(T)}_\ell(t)} \sin\delta^{(T)}_\ell(t)\, T^{(T)}_{\alpha\beta}(t)_\ell, \qquad (3)$$

where $\delta^{(T)}_\ell(t)$ are phases of the partial waves of the $\pi\pi$ scattering amplitude with isospin T. Our consideration is restricted to the case of low energies when the lowest partial S-wave mainly contributes to the amplitude. To pick out the S-wave in the dispersion relation (2) we put $\cos\varphi_t = \frac{1}{\sqrt{3}}$. In this case $P_2(\cos\varphi_t = \frac{1}{\sqrt{3}}) = 0$ and $T^{(T)}_{\alpha\beta}(t,\cos\varphi_t = \frac{1}{\sqrt{3}}) \simeq T^{(T)}_{\alpha\beta}(t)_s$.

The final equation is:

$$T^{(I)}_{\alpha\beta}(t)_s = \frac{i}{\pi}\int\limits_{4\mu^2}^{\infty} \frac{e^{-i\delta^{(I)}_s(t')}\,\mathrm{Im}\,\delta^{(I)}_s(t')\,T^{(I)}_{\alpha\beta}(t')_s}{t'-t-i\varepsilon}\,dt' + \tilde{B}_{\alpha\beta}(t)_s + \tilde{P}_{\alpha\beta}(t)_s;$$

$$\tilde{B}_{xx}(t)_s = \frac{8e^2}{\sqrt{3}}\left(\sqrt{2}\right)^{-\frac{I}{2}}\left\{\frac{\frac{t^u_\pi}{4}-\mu^2}{u-\mu^2}+\frac{\frac{t^s_\pi}{4}-\mu^2}{s-\mu^2}\right\}\cdot\frac{2}{3};\quad \tilde{B}_{yy}(t)_s = 0;$$

$$u(t^u_\pi) = \mu^2;\quad s(t^s_\pi) = \mu^2;$$

$$\tilde{P}_{xx}(t)_s = \frac{16g^2_{\omega\pi\gamma}}{9\sqrt{3}}\left(-\frac{22}{8\sqrt{2}}\right)^{\frac{I}{2}}\left\{\frac{\frac{1}{3}\left(\frac{t^u_\omega}{2}+m^2_\gamma\right)\left(t^u_\omega-\mu^2\right)-\left(\frac{t^u_\omega}{4}+Q\sqrt{\left(\frac{t^u_\omega}{4}+m^2_\gamma\right)\left(\frac{t^u_\omega}{4}-\mu^2\right)}\right)^2}{u-M^2_\omega}+\right. \tag{4}$$

$$\left.+\left(s\leftrightarrow u,\ t^u_\omega\leftrightarrow t^s_\omega,\ Q\leftrightarrow -Q\right)\right\};$$

$$\tilde{P}_{yy}(t)_s = \frac{16g^2_{\omega\pi\gamma}}{9\sqrt{3}}\left(-\frac{22}{8\sqrt{2}}\right)^{\frac{I}{2}}\left\{\frac{\left(\frac{t^u_\omega}{2}+m^2_\gamma\right)\left(\frac{t^u_\omega}{2}-\mu^2\right)-\left(\frac{t^u_\omega}{4}+Q\sqrt{\left(\frac{t^u_\omega}{4}+m^2_\gamma\right)\left(\frac{t^u_\omega}{4}-\mu^2\right)}\right)^2}{u-M^2_\omega}+\right.$$

$$\left.+\left(s\leftrightarrow u,\ t^u_\omega\leftrightarrow t^s_\omega,\ Q\leftrightarrow -Q\right)\right\};$$

$$u(t^u_\omega) = M^2_\omega;\quad s(t^s_\omega) = M^2_\omega;\quad Q = \frac{1}{\sqrt{3}}.$$

Here the contributions of transverse photons to the process $\gamma+\gamma\rightarrow\pi+\pi$ are written.

If one or both γ -quanta possess the longitudinal polarization, then Eq. (4) includes xz and zz components of matrices $\tilde{B}_{\alpha\beta}(t)_s$ and $\tilde{P}_{\alpha\beta}(t)_s$:

$$\tilde{B}_{zz}(t)_s = \frac{1}{\sqrt{2}}\tilde{B}_{xz}(t)_s = \frac{1}{2}\tilde{B}_{xx}(t)_s;$$

$$\tilde{P}_{zz}(t)_s = \frac{16 g^2_{\omega\pi\gamma}}{9\sqrt{3}}\left(-\frac{22}{8\sqrt{2}}\right)^{\frac{T}{2}}\left\{ \frac{\frac{t^u_\omega}{6}\left(\frac{t^u_\omega}{4}-\mu^2\right)}{u-M_\omega^2} + \frac{\frac{t^s_\omega}{6}\left(\frac{t^s_\omega}{4}-\mu^2\right)}{s-M_\omega^2} \right\};$$

$$\tilde{P}_{xz}(t)_s = \frac{16 g^2_{\omega\pi\gamma}}{9\sqrt{3}}\left(-\frac{22}{8\sqrt{2}}\right)^{\frac{T}{2}}\left\{\left[-\frac{\sqrt{2}}{3}\left(\frac{t^u_\omega}{2}+m_\gamma^2\right)\left(\frac{t^u_\omega}{4}-\mu^2\right)+Q\sqrt{\left(\frac{t^u_\omega}{4}+m_\gamma^2\right)\left(\frac{t^u_\omega}{4}-\mu^2\right)}\times\right.\right.$$

$$\left.\left.\times\sqrt{2}\left(\frac{t^u_\omega}{4}+Q\sqrt{\left(\frac{t^u_\omega}{4}+m_\gamma^2\right)\left(\frac{t^u_\omega}{4}-\mu^2\right)}\right)\right]\Big/(u-M_\omega^2) + \left(\begin{matrix} s\leftrightarrow u;\ t^u_\omega\leftrightarrow t^s_\omega; \\ Q\leftrightarrow -Q\end{matrix}\right)\right\}.$$

Eq. (4) is singular integral equation. It is solved by transforming to the Riemann boundary problem /8/.

If the phases $\delta_s^{(T=0)}(t)$ and $\delta_s^{(T=2)}(t)$ asymptotically approach zero, then the integral equation (4) has the unique solution:

$$T^{(T)}_{\alpha\beta}(t)_s = X^+(t)A^+_{\alpha\beta}(t) + \tilde{B}_{\alpha\beta}(t)_s + \tilde{P}_{\alpha\beta}(t)_s$$

$$X^+(t) = e^{i\delta_s^{(T)}(t)}\exp\left\{\frac{t}{\pi}P\int_{4\mu^2}^{\infty}\frac{\delta_s^{(T)}(x)}{x(x-t)}dx\right\}$$

$$A^+_{\alpha\beta}(t) = \frac{i}{\pi}P\int_{4\mu^2}^{\infty}\frac{e^{i\delta_s^{(T)}(x)}\sin\delta_s^{(T)}(x)\left(\tilde{B}_{\alpha\beta}(x)_s+\tilde{P}_{\alpha\beta}(x)_s\right)}{(x-t)\quad X^+(x)}dx .$$

The region of applicability of this solution is determined by that of the unitarity condition (3) in which the lower intermediate state is two-pion state (2γ ; 2μ intermediate states must give small contribution).

The natural limit of applicability is $t \sim 16\,\mu_\pi^2$.

3. Cross Sections of the Reactions $\gamma+\gamma \to \pi+\pi$ and $e+e \to e+e+\pi+\pi$

The differential cross section of the process $\gamma+\gamma\to\pi+\pi$ relates to the amplitudes $T_{\alpha\beta}(t)_s$ in the following way:

$$\frac{d}{d\Omega}\sigma_{\gamma\gamma\to\pi\pi}(t) = \frac{1}{64\pi^2}\,\frac{|K_1|}{|K|}\cdot\frac{1}{t}\,T^*_{\alpha\beta}(t)_s\,T_{\alpha'\beta'}(t)_s\,e^{\alpha\alpha'}e'^{\beta\beta'},$$

here $e^{\alpha\alpha'}$ and $e'^{\beta\beta'}$ are the polarization density matrices of virtual photons. It represents the cross section of production by two γ quanta of pion pair $\pi\pi(T=0)$, $\pi\pi(T=2)$, $\pi^+\pi^-$ or $\pi^0\pi^0$, in accordance with that either one uses the amplitudes $T^{(T=0)}_{\alpha\beta}(t)_s$, $T^{(T=2)}_{\alpha\beta}(t)_s$ or their combinations:

$$T^{\pi^+\pi^-}_{\alpha\beta}(t)_s = \frac{1}{\sqrt{3}}\,T^{(T=0)}_{\alpha\beta}(t)_s + \frac{1}{\sqrt{6}}\,T^{(T=2)}_{\alpha\beta}(t)_s$$

$$T^{\pi^0\pi^0}_{\alpha\beta}(t)_s = \frac{2}{\sqrt{6}}\,T^{(T=2)}_{\alpha\beta}(t)_s - \frac{1}{\sqrt{3}}\,T^{(T=0)}_{\alpha\beta}(t)_s\,.$$

If in the reaction $e+e\to e+e+\pi+\pi$ electron and positron scatter in the forward direction then the density polarization matrix of virtual photons has the form[7]

$$e^{\alpha\alpha'} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & K(t) \end{pmatrix},$$

where $K(t) = 16\,\frac{m_\gamma^2}{t^2}\left[(2E-\sqrt{t})E + \frac{m_\gamma^2}{2}\right]$,

and E, $E-K_0 \gg m_e$ is supposed (E is the energy of electrons in the colliding beams, K_0 the energy of virtual photons). The 2-pion production cross section by virtual photons is written as:

$$\sigma_{\gamma\gamma\to\pi\pi}(t) = \sigma^{\perp\perp}_{\gamma\gamma\to\pi\pi}(t) + K(t)\,\sigma^{\perp\parallel}_{\gamma\gamma\to\pi\pi}(t) + K^2(t)\,\sigma^{\parallel\parallel}_{\gamma\gamma\to\pi\pi}(t),$$

here there are introduced the notations:

$$\sigma^{\perp\perp}_{\gamma\gamma\to\pi\pi}(t) = \frac{1}{16\pi}\,\frac{|\bar{K}_1|}{|\bar{K}|}\,\frac{1}{t}\left\{\frac{1}{4}|T_{xx}(t)_s|^2 + \frac{1}{4}|T_{yy}(t)_s|^2\right\};$$

$$\sigma^{\perp\parallel}_{\gamma\gamma\to\pi\pi}(t) = \frac{1}{16\pi}\,\frac{|\bar{K}_1|}{|\bar{K}|}\,\frac{1}{t}\,|T_{xz}(t)_s|^2;\quad \sigma^{\parallel\parallel}_{\gamma\gamma\to\pi\pi}(t) = \frac{1}{16\pi}\,\frac{|\bar{K}_1|}{|\bar{K}|}\,\frac{1}{t}\,|T_{zz}(t)_s|^2.$$

For small energy loss by electron and positron (forward scattering) the masses m_γ^2 of virtual photons are small as compared to E^2, t, μ^2 and may be neglected in the formulae. Then it is sufficient in the cross section to take into account only the transverse amplitudes.

The cross section of the process $e+e\to e+e+\pi+\pi$ is expressed via the cross section of the reaction $\gamma+\gamma\to\pi+\pi$ as[3]:

$$\sigma_{ee\to ee\pi\pi}(E) = 2\left(\frac{\alpha}{\pi}\right)^2 \ln^2\frac{E}{m_e}\int_{4\mu^2}^{4E^2}\frac{f\left\{\left(\frac{t}{4E^2}\right)^{\frac{1}{2}}\right\}}{t}\,dt\;\sigma_{\gamma\gamma\to\pi\pi}(t)$$

Here $f(x) = (2+x^2)\ln\left(\frac{1}{x}\right) - (1-x^2)(3+x^2)$ is a function of the number of equivalent photons; E the energy of colliding beams.

4. Numerical Calculations

For calculations the $\pi\pi$ scattering phases were taken in the form:

$$\delta_s^{(T=0)}(t) = 2\,\mathrm{arctg}\left\{\left(\frac{t-4\mu^2}{t_0-4\mu^2}\right)^{1/2}\left(\frac{t}{t_0}\right)^5\right\};$$

$$\delta_s^{(T=2)}(t) = \begin{cases} -\frac{\pi}{10}\left(\frac{t-4\mu^2}{96\mu^2}\right)^{1/2}\exp\left\{0.5\left(1-\frac{t-4\mu^2}{96\mu^2}\right)\right\} & \frac{t}{4\mu^2}\leqslant 25 \\ -\frac{\pi}{10}\,\frac{1}{\left(\frac{t-100\mu^2}{67.2\mu^2}\right)^2+1} & \frac{t}{4\mu^2} > 25\,. \end{cases} \qquad (5)$$

The parameter $t_c = 730^2$ MeV^2 defines a point where the phase $\delta_S^{(T=0)}(t)$ passes through 90°.

In Fig. 2 curves (5) are given in comparison with experimental points taken from works [9].

In accordance with the choice of $\delta_S^{(T=0)}(\infty)$ we have either above-written solution for $\delta_S^{(T=0)}(\infty) = 0$ or this solution multiplied by some first order polynomial for $\delta_S^{(T=0)}(\infty) = \pi$. At present time we are not able to determine the constants of this polynomial, so the case $\delta_S^{(T=0)}(\infty) = \pi$ we shall interpret as the $\delta_S^{(T=0)}(\infty) = 0$ case, assuming that for large energy phase shift goes back to zero. The contribution of the second resonance, which appears in such a case will be small, obviously, because the integrand is small for t large. Thus in both $\delta_S^{(T=2)}(\infty) = 0$ and $\delta_S^{(T=0)}(\infty) = \pi$ we shall use above-described solution.

The constant $g_{\omega\pi\gamma}$ of electromagnetic decay of the meson has been found from the relation [11]

$$\Gamma_{\omega\to\pi+\gamma} = \frac{1}{3}\,\frac{g^2_{\omega\pi\gamma}}{4\pi}\,M_\omega^3\left(\frac{M_\omega^2-\mu^2}{M_\omega^2}\right)^3$$

with the width $\Gamma_{\omega\to\pi+\gamma} = 1.2$ MeV.

The numerical calculations of cross sections have been performed at a computer. Calculation results for the interaction cross section of the transverse photons are presented in Figs 3a) and b). In Figs 4a), b) and c), d) there are plotted the behaviours of photon interaction cross sections with produced pions in those cases when one or both photons have the longitudinal polarization. The curves of cross sections for the processes $e+e\to e+e+\pi+\pi$ are given in Fig. 5.

It is interesting to compare these results with those of work [10] in which this process has been considered for the real photons.

In our case the cross section of $\pi\pi\,(T=0)$ pair production has a resonant behaviour generated by the σ meson (σ meson is the resonance state of $\pi\pi$ system with zero isospin and zero spin). The resonant enhancement is observed in the cross sections for production of $\pi^+\pi^-$ pairs. In paper [10] these peculiarities are absent which obviously is due to a parametrization of phase shifts.

In our work the differential cross section of the process $e+e\to e+e+\pi^0+\pi^0$ proves to be by two orders greater than that of the reaction $e+e\to e+e+\pi^++\pi^-$. In the paper [10] the situation is opposite. This is due to that there is not taken into account the contribution of ω and ρ resonances (ω meson - to the process $\gamma+\gamma\to\pi^0+\pi^0$ and ρ meson - to both processes), of which the constants are linked via the relation

$$g^2_{\omega\pi\gamma} = 9g^2_{\rho\pi\gamma}.$$

The values of cross sections of the reaction $\gamma+\gamma\to\pi^++\pi^-$ at the maximum of the resonance and the approximate widths of this resonance are:

$\sigma^{\perp\perp}_{\gamma\gamma\to\pi^+\pi^-} \simeq 3.6\cdot10^{-30}\ \mathrm{cm}^2$; $\quad\Gamma_\sigma \simeq 450$ MeV;

$\sigma^{\parallel\perp}_{\gamma\gamma\to\pi^+\pi^-} \simeq 7.9\cdot10^{-31}\ \mathrm{cm}^2$; $\quad\Gamma_\sigma \simeq 450$ MeV;

$\sigma^{\parallel\parallel}_{\gamma\gamma\to\pi^+\pi^-} \simeq 3.2\cdot10^{-31}\ \mathrm{cm}^2$; $\quad\Gamma_\sigma \simeq 440$ MeV.

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References

I. N.Arteago-Romero, A.Jaccarini, P.Kessler.Laboratoire de Physique atomique et moleculaire PAM 7002. Arvil 1970.

2. N.Arteago-Romero, A.Jaccarini, J.Parisi and P.Kessler Lett. Nuovo Cimento,4, 933 (1970); 1, 935 (1971).

3. S.J.Brodsky, T.Kinoshita, H.Terazawa. Phys.Rev.Lett.,25, 972 (1970).

4. В.М.Буднев, И.Ф.Гинзбург.ЯФ, I3, В.2 (I97I).

5. A.Bramon, M.Greco. Preprint LNF-71/34. Rome (1971).

6. H.Cheng, T.T.Wu. Preprint DESY 71/23. Hamburg (1971).

7. П.С.Исаев, В.И.Хлесков. Препринт ОИЯИ,Р2-5505, Дубна (I97I).

8. Ф.Д.Гахов. "Краевые задачи", Гостехиздат, Москва (I960).

9. Proceedings of the Conference on $\pi\pi$ and $K\pi$ interactions, Argonne National Laboratory, May 1969.

IO. D.H.Lyth. Preprint University of Lancaster (1971).

II. Л.Д.Соловьев. Phys.Lett., 16, 345 (1965).

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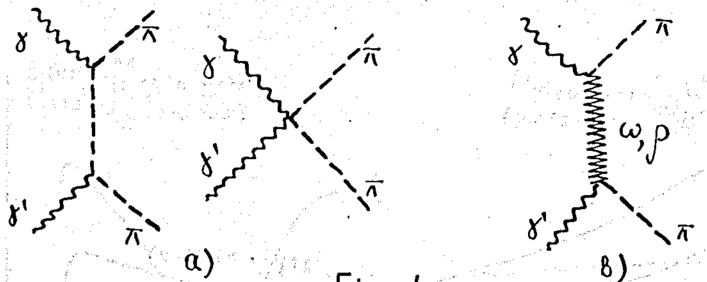

Fig. 1

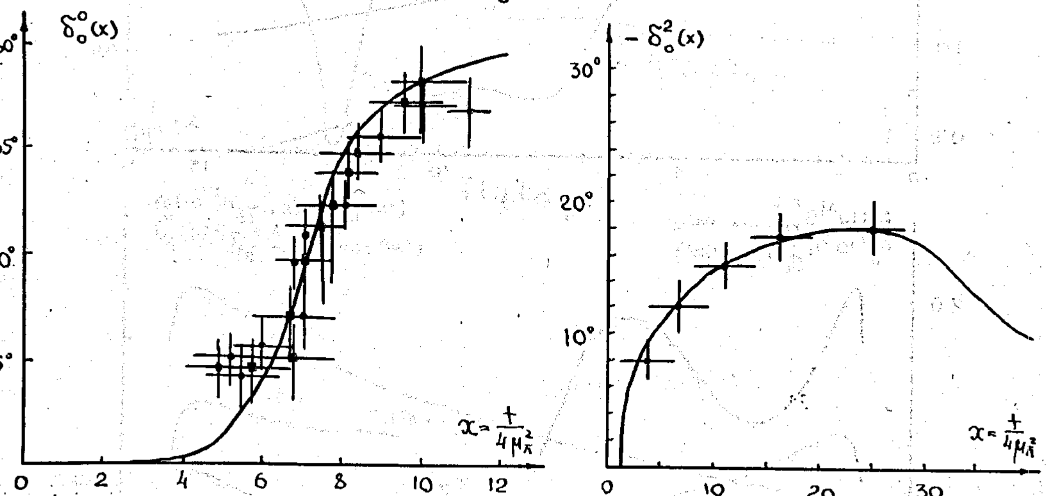

Fig. 2

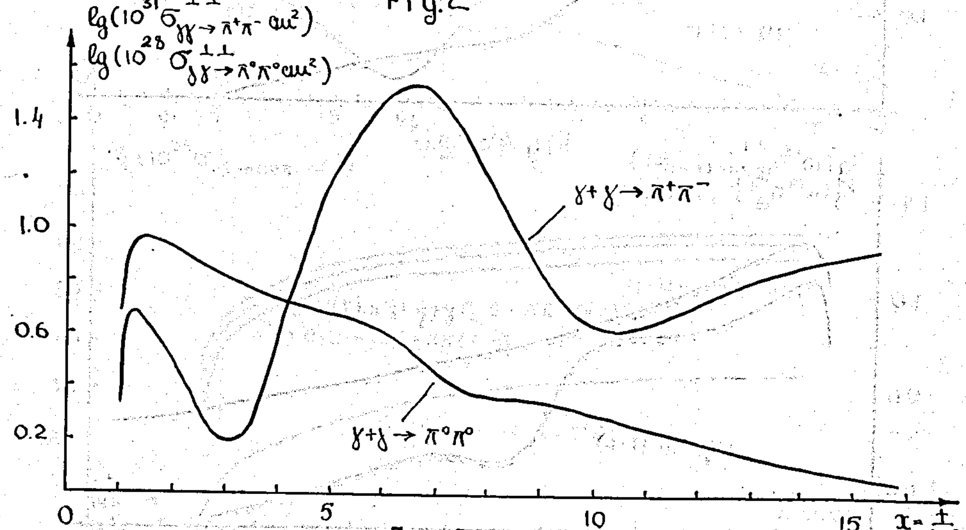

Fig. 3 a)

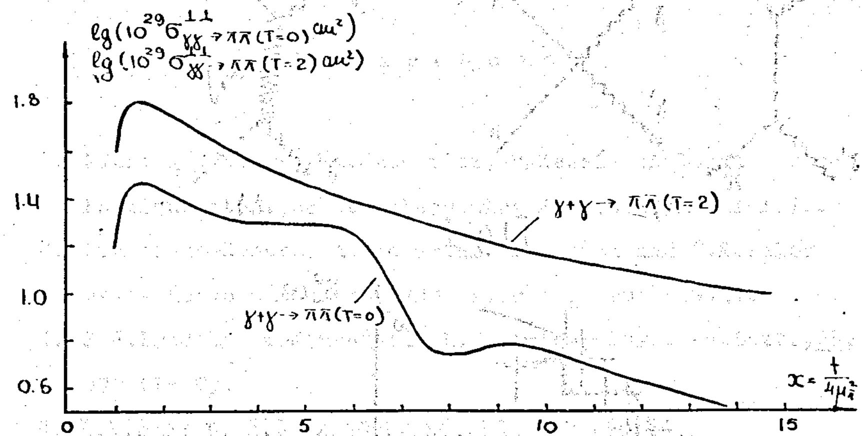

Fig. 3 B)

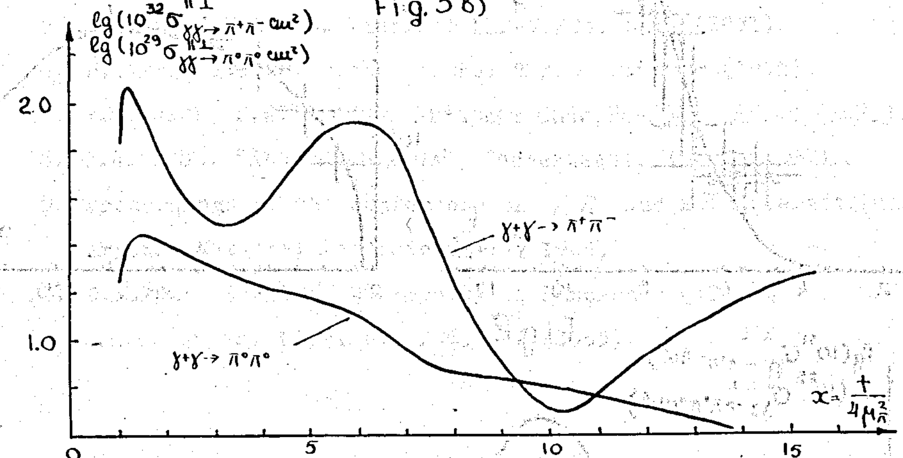

Fig. 4 a)

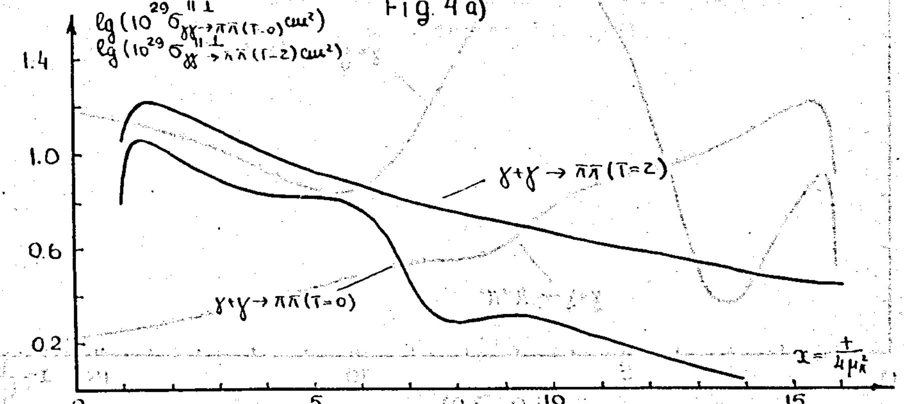

Fig. 4 B)

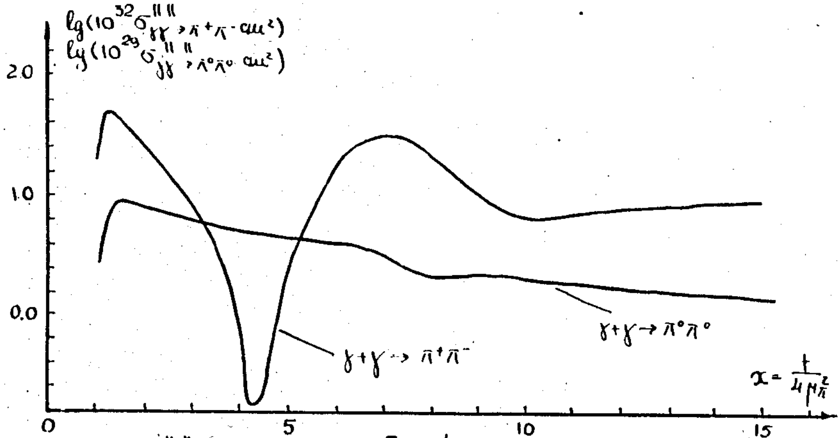

Fig 4 c)

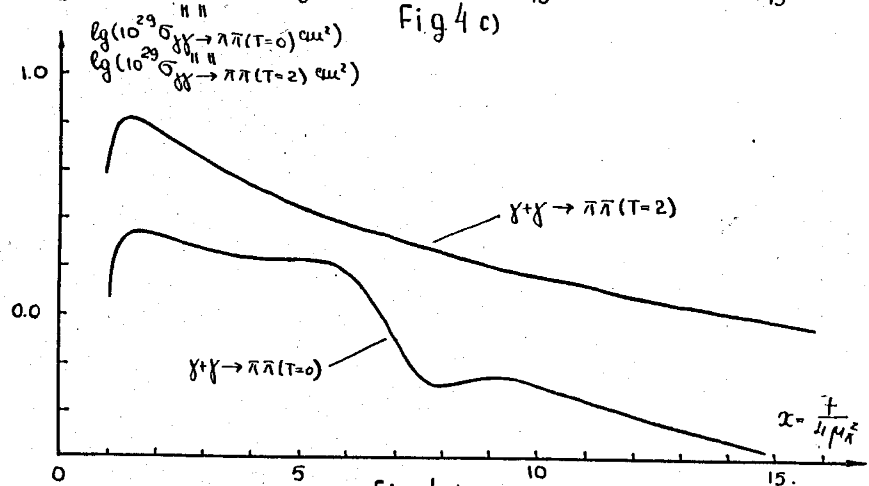

Fig. 4d)

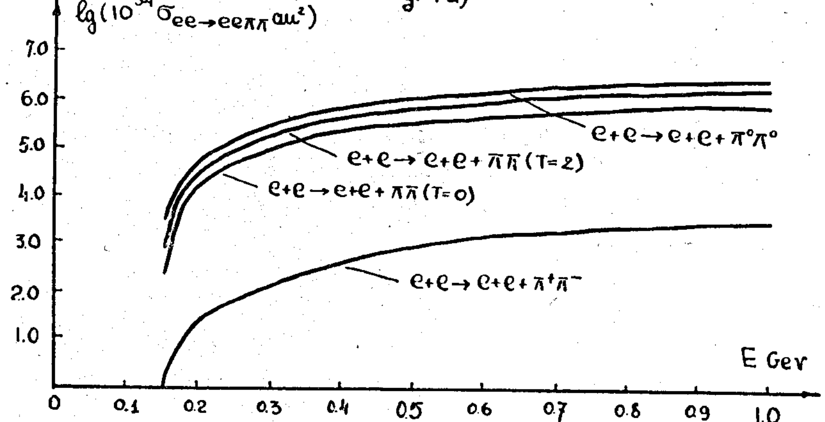

Fig 5