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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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EIKONAL REPRESENTATION OF THE SCATTERING AMPLITUDE CONTAINING THE VENEZIANO VIRTUAL BLOCKS

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For studying the asymptotic behaviour of the scattering amplitude and analytic properties in the complex angular momentum plane Regge diagram technique has been proposed /1 /. The method consists in the construction of diagrams by combining the usual Feynman propagators with virtual Regge amplitudes. Developing this approach Polkinghorne /2 / has suggested to describe the reggeon contribution by the Veneziano amplitudes. And here, as it has been shown /2 /, it is not necessary to limit beforehand the integration region over virtual momenta which must be done if using the reggeon propagators. A consistent application of this technique and first of all the unitarity condition certainly requires the summation of the diagrams constructed in such a way.

In the present note we would like to show that for this aim in high-energy domain one may employ the functional integration method, used successively in the standard field theory for summing diagrams of various types /3-6/.

To this end, consider the diagrams describing a scattering of two high-energy neutral scalar particles with masses m ("nucleons") exchanging virtual blocks of "meson-meson" interaction (see fig.1)

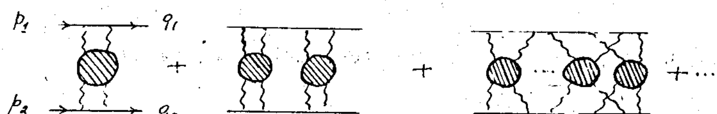

Fig.1

The "mesons" in our model are also the neutral scalar particles differing from nucleons by their masses μ only. An analogous problem has been considered in the framework of the functional method /5,6/ as well as perturbation theory /7/, the virtual "meson-meson" amplitude being described by a sum of the ladder graphs. As a result the Regge eikonal representation has been obtained for the whole amplitude, but here we wish to represent the virtual blocks of "meson-meson" interaction through the Venaziano amplitude, and sum up all the diagrams of the type given in fig.1.

Now we briefly indicate the main steps in constructing the sum of these diagrams by the functional method. (For details, see refs. /6,9/).

The two-particle scattering amplitude relates to the corresponding Green function as:

$$(2\pi)^4 \delta^{(4)}(p_1+p_2-q_1-q_2)\, f(q_1,q_2|p_1,p_2) = \tag{1}$$
$$= i \lim_{p_j^2, q_j^2 \to m^2} \prod_{j=1}^{2} (q_j^2-m^2)(p_j^2-m^2)\; G(q_1,q_2|p_1,p_2) .$$

Two-particle Green function $G(q_1,q_2|p_1,p_2)$ can be found by the functional averaging of one-particle Green functions of the "nucleons" over the external classic field

$$G(x_1,x_2|x_3,x_4) = C_\varphi \int \delta\varphi \exp\{-\tfrac{i}{2}\int \varphi(q)\, \mathcal{D}_c^{-1}(q)\, \varphi(-q)\, d^4q\} \cdot$$
$$\cdot [G(x_1,x_2|\varphi)G(x_2,x_4|\varphi) + G(x_1,x_4|\varphi)G(x_2,x_3|\varphi)]\, S_0(\varphi) \tag{2}$$
$$S_0(\varphi) = \exp\{i\Pi(\varphi)\}$$

$G(x,y|\varphi)$ can be represented by the functional integral

$$G(x,y|\varphi) = i\int_0^\infty ds\, e^{-ism^2}\, C_v \int \delta^4 v \exp\{-i\int_0^s d\xi\, [v_\mu^2(\xi) - \tag{3}$$
$$- g\,\varphi(x - 2\int_\xi^s v(\eta)d\eta)]\}\; \delta^{(4)}(x-y-2\int_0^s v(\eta)d\eta).$$

Inserting (3) into (2) and keeping in the vacuum-polarization operator $\Pi(\varphi)$ only the terms up to the forth order in φ, we get the Green function for two "nucleons" corresponding to the diagrams in fig.1 (see refs./6,9/). Taking the Fourier transform of $G(x_1,x_2/x_3,x_4)$ and going over in (1) on the mass shell /3,10/ , we find the scattering amplitude:

$$
\begin{aligned}
&(2\pi)^4\delta^{(4)}(p_1+p_2-q_1-q_2)\, f(q_1,q_2/p_1,p_2) = \\
&= \prod_{j=1}^{2}\Big(\int[\delta^4 \nu_j]\int_{-\infty}^{+\infty}\frac{d^4y_j}{(2\pi)^4}\, e^{i(p_j-q_j)y_j}\Big) g^4 \prod_{K=1}^{2}\Big(\int d^4x_K\Big)\int_{-\infty}^{+\infty} d\eta_1\, d\eta_2 \qquad (4) \\
&D^c(y_1-x_1)\, D^c(y_1+a_1-x_2)\, D^c(y_2-x_3)\, D^c(y_2+a_2-x_4)\, M(x_1,x_2,x_3,x_4) \\
&\int_0^1 d\lambda\, \exp\Big\{ig^4\lambda \prod_{\sigma=1}^{4}\Big(\int d^4x_\sigma \int_{-\infty}^{+\infty} d\xi_\sigma\Big) D^c(y_1+b_1-x_1)\, D^c(y_1+b_2-x_2) \\
&\qquad D^c(y_2+b_3-x_3)\, D^c(y_2+b_4-x_4)\, M(x_1,x_2,x_3,x_4),
\end{aligned}
$$

where $M(x_1,x_2,x_3,x_4)$ is the virtual block of "meson-meson" interaction, and the notations

$$a_i = 2\int_0^{\eta_i}\nu_i(\eta)\,d\eta - 2\eta_i p_i \qquad (i=1,2)$$

$$b_j = 2\int_0^{\xi_j}\nu_r(\eta)\,d\eta - 2\xi_j p_r \qquad (j=1,2;\ r=1;\quad j=3,4;\ r=2)$$

are introduced.

The main point in our method is the calculation of functional integrals over $\delta\nu$ in the straight-line path approximation 11 that corresponds to the neglecting of functional variables in the D^c-function arguments. In the language of the Feynman diagrams this means a linearization of the "nucleon" propagators in the virtual "meson" momenta. Then the expression (4) in c.m.s. is transformed to the eikonal form / 6/ :

$$f(q_1,q_2/p_1,p_2) = -is\int d^2b_\perp\, e^{ib_\perp(p_1-q_1)_\perp}\left(e^{i\chi}-1\right), \qquad (5)$$

where

$$
\begin{aligned}
\chi = g^4\frac{(2\pi)^2}{s}\int d^2q_\perp\, e^{ib_\perp q_\perp}\int d^4r_1\, d^4r_2\int_{-\infty}^{+\infty} d\xi_1\, d\xi_3\, \exp\{-2i\xi_1 r_1 p_1 - 2i\xi_3 r_2 p_2\} \qquad (6) \\
(r_1^2-\mu^2)^{-1}[(-r_1+q_\perp)^2-\mu^2]^{-1}(r_2^2-\mu^2)^{-1}[(r_2+q_\perp)^2-\mu^2]^{-1} \\
M(r_1,\, q_\perp-r_1,\, r_2,\, -q_\perp-r_2)
\end{aligned}
$$

and $q_\perp$ stands for the four-vector with components $(0, q_1, q_2, 0)$, $q_\perp^2 = -q_1^2 - q_2^2 = -\vec{q}_\perp^2$; the block $M(r_1, q_\perp - r_1, r_2, -q_\perp - r_2)$ graphically can be represented by the following diagram (see fig.2)

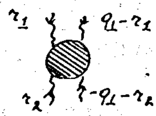

Fig.2

Following paper /2/ we represent the contribution of this block to the eikonal phase (6) by the Veneziano amplitude. With this purpose we put

$$M(r_1, q_\perp - r_1, r_2, -q_\perp - r_2) = G\,\mathcal{B}\left(-\alpha_0 - \alpha'(r_1 + r_2)^2 - i\varepsilon,\; -\alpha_0 + \alpha'\vec{q}_\perp^{\,2} - i\varepsilon\right), \qquad (7)$$

where G is some arbitrary constant, α_0 and α' are parameters of the Regge trajectory: $\alpha(s) = \alpha_0 + \alpha' s$. As usual, we assume $\alpha_0 < 0$, $\alpha' > 0$.

Next, for the Euler B-function it is convenient to make use of the integral representation:

$$\mathcal{B}(v, w) = \int_0^1 x^{v-1}(1-x)^{w-1}\,dx\,. \qquad (8)$$

This representation is valid if the real parts of v and w are positive. In our case, however, the first argument of the B-function in (7) can take the negative value, as r_1 and r_2 are the virtual momenta. Nevertheless as will be clear later, in the limit $s \to \infty$ the basic contribution to the eikonal phase will be made by the integral over dx in (8) only near the upper limit: $x \sim 1$. But here v may be negative, since this causes no divergences. Remembering it we insert (7) and

(8) into (6). Replacing then the variable x by $e^{-i\tau(1-i\varepsilon)}$ we integrate over the virtual momenta $d^4 r_1, d^4 r_2$:

$$\chi = G \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i \vec{b}_\perp \vec{q}_\perp} g^4 \int_0^\infty \frac{d\alpha\, d\tau\, d\xi_1\, d\xi_3}{C^2(\alpha)} (1-e^{i\tau})^{\alpha(q_\perp^2)-1} e^{-i\tau} \tag{9}$$

$$\left[\exp\left\{i \frac{\tau \xi_1 \xi_3}{C(\alpha)} s\right\} + \exp\left\{-i \frac{\tau \xi_1 \xi_3}{C(\alpha)} s\right\}\right] \exp\left\{i \frac{d(\alpha)}{C(\alpha)}\right\}$$

by standard methods / 12 / . An asymptotics of this integral can be found using the technique developed by Efremov /13/ , Polkinghorne /12/ et al. The main contribution to (9) proceeds from the region of small τ, ξ_1, $\xi_3 \sim \frac{1}{s}$ which corresponds to $x \sim 1$ in (8), thereby the use of the integral representation for the B-function is justified in our case. We put $\tau = \xi_1 = \xi_3 = 0$ in $d(\alpha, \xi, \tau)$ and $C(\alpha, \xi, \tau)$, and in the remaining part of the integrand we keep only the term linear in τ . Following the general rules for separating the asymptotics of the Feynman integral we must take $\tau = \frac{1}{s}$.

As a result the eikonal phase takes on the form

$$\chi(b_\perp, s) = \frac{G}{s} \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i \vec{b}_\perp \vec{q}_\perp} (f(q_\perp^2))^2 \cdot \left[(\ln s)^2 s^{\alpha(q_\perp^2)} + (-s)^{\alpha(q_\perp^2)} (\ln(-s))^2\right], \tag{10}$$

where

$$f(q_\perp^2) = \frac{g^2}{16\pi^3} \int \frac{d^2 k}{(k^2+m^2)[(k-q_\perp)^2+m^2]} .$$

Thus, the amplitude (5) becomes of the Regge eikonal form which differs only slightly from the analogous expressions in the earlier models /6,7/. Firstly, (10) has the additional factors $(f(q_\perp^2) \ln s)^2$, the appearance of which can be easily explained graphically (see fig.3)

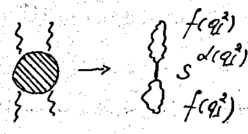

Fig.3

As $s \to \infty$ the block through which the exchange proceeds, contracts as is shown in fig.3. The Veneziano amplitude gives the factor $s^{\alpha(q_i^2)}$ and the external meson lines trasform into the closed loops, to each of them there corresponds the factor $f(q_i^2)\ln s$.

Second peculiarity of our consideration is the restriction on the Regge parameter $\alpha_0 < 0$, the reason for which is that for the virtual "meson-meson" block the Veneziano amplitude was employed. In describing this block by the sum of the ladder diagrams the parameter α_0 , in principle, might be any positive number /6,7/, if the coupling constant is large enough. In this case however the Regge eikonal model, even in scalar theory $\mathcal{L}_{int} = g\varphi^3$, gives the rising asymptotics /14/ instead of the falling one for the amplitude (as it takes place in the conventional eikonal model). But if $\alpha_0 < 0$, as in our consideration; then when $s \to \infty$ the amplitude decreases as the negative degree of s in the Regge eikonal model, too.

From the results obtained (formulae (5) and (10)) one can draw the following conclusion. The discussed modification of the exchange mechanism in the high-energy particle scattering does not change essentially the Regge eikonal representation for the scattering amplitude. As $s \to \infty$ the virtual exchange block proves to contribute to the asymptotic region where the Veneziano amplitude gives rise to the Regge behaviour.

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