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QUASIPOTENTIAL FORMALISM FOR THE TWO-PARTICLE SYSTEM WITH SPINS 0 AND 1/2

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Квазипотенциальный формализм для системы двух частиц со спином 0 и 1/2

Развит квазипотенциальный формализм для частиц со спинами 0 и 1/2. Получены квазипотенциальные уравнения для волновой функции и амплитуды рассеяния.

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Garsevanishvili V.R., Goloskokov S.V., Matveev V.A. Slepchenko L.A. E2-6046

Quasipotential Formalism for the Two-Particle System with Spin 0 and 1/2

The quasipotential formalism for the particles with spins 0 and 1/2 has been developed. The quasipotential equations for the scattering amplitude and the wave function have been derived.

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1. Introduction

Recently the Logunov-Tavkhelidze quasipotential approach[1] has been developed successfully. This approach allows one to solve effectively a number of problems concerning the two-particle system, e.g. a determination of the energy spectrum, an investigation of analytic and asymptotic properties of the scattering amplitude (see for example the review papers[2-5] and the references cited therein).

In a number of papers[6-9] generalizations of the quasipotential equations for the particles with unequal masses and for the many-particle systems have been considered.

Starting from the Hamiltonian formalism in quantum field theory an equation[10] has been derived which turned out to be an effective tool for the construction of an original version of the relativistic configuration representation in the two-particle system[11].

Quasipotential equations for spin particles have been considered in refs.[12,14].

In the present paper we consider the quasipotential approach for the particles with spins 0 and 1/2 in more detail than in ref.[4].

To make things more clear in the second section the case of the free particles is considered.

In the third section we derive the quasipotential equations for the wave function and for the scattering amplitude.

In the fourth section equations obtained are considered in the Foldy-Wouthuysen representation.

2. Non-Interacting Particles

As is known the two-particle system in quantum field theory can be described by the Bethe-Salpeter amplitude[13] in our case defined as

$$\chi_P(x_1, x_2) = \langle 0 | T(\psi(x_1)\phi(x_2)) | P \rangle, \tag{2.1}$$

where $\psi(x_1)$ and $\phi(x_2)$ are the Heisenberg operators of the fields with spins $\frac{1}{2}$ and 0, respectively, $|P\rangle$ is the state vector with the total 4-momentum P.

In the case of the free particles the amplitude $\chi_P(x_1,x_2)$ obeys the following system of equations

$$(i\gamma\partial_{x_1} - M)\chi_P(x_1,x_2) = 0, \tag{2.2a}$$

$$(\Box_{x_2} - \mu^2)\chi_P(x_1,x_2) = 0. \tag{2.2b}$$

Using the translation invariance one can write $\chi_P(x_1,x_2)$ as follows

$$\chi_P(x_1,x_2) = e^{-iPX}\chi_P(x). \tag{2.3}$$

Here

$$X = \frac{x_1 + x_2}{2}, \qquad x = x_1 - x_2.$$

Proceeding to the momentum representation

$$\chi_P(x) = \int d^4p\, e^{-ipx}\chi_P(p) \tag{2.4}$$

we get from Eq. (2.2)

$$[\gamma(\frac{P}{2} + p) - M]\chi_P(p) = 0, \tag{2.5a}$$

$$[(\frac{P}{2} - p)^2 - \mu^2]\chi_P(p) = 0. \tag{2.5b}$$

In the c.m.s., where $P = (E,0)$ the equations (2.5) take the form

$$[\gamma_0(\frac{E}{2} + p_0) - \vec{\gamma}\vec{p} - M]\chi_{\vec{P}=0}(p_0, \vec{p}) = 0, \tag{2.6a}$$

$$[(\frac{E}{2} - p_0)^2 - \vec{p}^2 - \mu^2]\chi_{\vec{P}=0}(p_0, \vec{p}) = 0. \tag{2.6b}$$

It follows from Eq. (2.6) that

$$2Ep_0 = M^2 - \mu^2 \equiv \Delta^2 \tag{2.7}$$

and the solution $\chi_{\vec{P}=0}(p_0,\vec{p})$ may be written in the following form:

$$\chi_{\vec{P}=0}(p_0,\vec{p})=\delta\left(p_0-\frac{\Delta}{2E}\right)\Phi_E(\vec{p}). \qquad (2.8)$$

The function $\Phi_E(\vec{p})$ obeys the equation

$$\left[E-\left(1+\frac{\omega(\vec{p})}{W(\vec{p})}\right)(\vec{\alpha}\vec{p}+\beta M)\right]\Phi_E(\vec{p})=0, \qquad (2.9)$$

where

$$\omega(\vec{p})=\sqrt{\mu^2+\vec{p}^{\,2}}\ ; \quad W(\vec{p})=\sqrt{M^2+\vec{p}^{\,2}}. \qquad (2.10)$$

Let us consider the equal time Bethe-Salpeter amplitude $\chi_{\vec{P}=0}(x_1,x_2)$ in the c.m.s.

$$\chi_{\vec{P}=0}(t,\vec{x}_1\ ;t,\vec{x}_2)=e^{-iEt}\chi_{\vec{P}=0}(0,\vec{x}_1-\vec{x}_2). \qquad (2.11)$$

In the momentum representation we obtain

$$\chi_{\vec{P}=0}(0,\vec{x})=\int dp_0 d\vec{p}\, e^{i\vec{p}\vec{x}}\chi_{\vec{P}=0}(p_0,\vec{p})=\int d\vec{p}\, e^{i\vec{p}\vec{x}}\Phi_E(\vec{p}). \qquad (2.12)$$

Thus the function $\Phi_E(\vec{p})$ is the 3-dimensional Fourier transform of the equal-time Bethe-Salpeter amplitude in the c.m. system.

Eqs (2.6) possess four linearly independent solutions, which correspond to the four signs of energies of individual particles. This is due to the two-time description of the two-particle system.

In the framework of the one-time formalism, which is the basis of the quasipotential approach, only two solutions have the physical meaning.

The first solution describes the two-particle system with the positive total energy $E=\omega+W$. The second solution can be expressed through the amplitude which describes the system of two antiparticles using the charge conjugation and the time-reversal operators.

3. Equations for the Wave Function and Scattering Amplitude

In this section the equations for the one-time wave function and scattering amplitude of two interacting particles will be obtained.

The two-particle Green function is defined by the following expressions:

$$G(x_1,x_2;x_3,x_4)=\langle 0|T(\psi(x_1)\phi(x_2)\bar{\psi}(x_3)\phi(x_4))|0\rangle. \qquad (3.1)$$

The equation for the Fourier transform of the function (3.1) looks as follows

$$G_P(p,k)=F_P(p,k)+\int d^4k_1d^4k_2\ F_P(p,k_1)K_P(k_1,k_2)\times \qquad (3.2)$$
$$\times G_P(k_2,k),$$

where

$$F_P(p,k) = -\frac{\delta^4(p-k)}{[\gamma(\frac{P}{2}+p)-M+i0][(\frac{P}{2}-k)^2-\mu^2+i0]} \qquad (3.3)$$

$K_P(k_1,k_2)$ is the kernel of the equation and is defined as a sum of irreducible graphs in the two-particle system. In Eq. (3.2) the translation invariance has been taken into account.

Let us define now the Fourier transform of the two-time Green function in the c.m.s. /14/

$$\tilde{G}(E;\vec{p},\vec{k}) = \int dp_0\, dk_0\, G_{\vec{P}=0}(p_0,\vec{p};k_0,\vec{k}) \qquad (3.4)$$

and find the Green function of two free particles, having integrated over the relative energies:

$$\tilde{F}(E;\vec{p},\vec{k}) = -\delta^3(\vec{p}-\vec{k})\int \frac{dp_0}{[\gamma_0(\frac{E}{2}+p_0)-\vec{\gamma}\vec{p}-M+i0][(\frac{E}{2}-p_0)^2-\vec{p}^{\,2}-\mu^2+i0]} =$$

$$= \frac{i\pi}{\omega(\vec{p})}\,\frac{\delta^3(\vec{p}-\vec{k})}{E\gamma_0-(1+\frac{\omega(\vec{p})}{W(\vec{p})})(\vec{\gamma}\vec{p}+M)+i0} = \tilde{F}(E;\vec{p})\,\delta^3(\vec{p}-\vec{k}). \qquad (3.5)$$

Define now the quasipotential $V(E;\vec{p},\vec{k})$ by the following relation

$$\tilde{G}^{-1}(E;\vec{p},\vec{k}) = \tilde{F}^{-1}(E;\vec{p})\delta^3(\vec{p}-\vec{k}) - \frac{1}{i\pi}V(E;\vec{p},\vec{k}). \qquad (3.6)$$

Proceeding as usually /2/ we obtain the following equations for the one-time wave function

$$[E\gamma_0-(1+\frac{\omega(\vec{p})}{W(\vec{p})})(\vec{\gamma}\vec{p}+M)]\psi(\vec{p}) = \frac{1}{\omega(\vec{p})}\int d\vec{k}\, V(E;\vec{p},\vec{k})\,\psi(\vec{k}) \qquad (3.7)$$

and for the scattering amplitude

$$T(\vec{p},\vec{k}) = V(E;\vec{p},\vec{k}) + \int\frac{d\vec{q}}{\omega(\vec{q})}V(E;\vec{p},\vec{q})\,\frac{1}{E\gamma_0-(1+\frac{\omega(\vec{q})}{W(\vec{q})})(\vec{\gamma}\vec{q}+M)+i0}\,T(\vec{q},\vec{k}). \qquad (3.8)$$

The normalization condition for the wave function reads /15/

$$\int d\vec{p}\, d\vec{k}\, \bar{\psi}(\vec{p})\,\frac{\partial}{\partial E}\tilde{G}^{-1}(E;\vec{p},\vec{k})\,\psi(\vec{k}) = 1. \qquad (3.9)$$

It can be shown that the scattering amplitude satisfies the unitarity condition.

The unitarity condition in the quasipotential approach has been considered in detail in ref. /16/.

4. The Foldy-Wouthuysen Representation and the Two-Component Description

The equations obtained in the previous sections take a simple form in the Foldy-Wouthuysen representation. We pass to this representation by means of the following unitary operator

$$U(\vec{p}) = \frac{\vec{\gamma}\vec{p}+M+W}{\sqrt{2W(M+W)}}. \qquad (4.1)$$

In this representation Eq. (3.7) looks as follows

$$[E\gamma_0 - \omega(\vec{p}) - W(\vec{p})]\psi_F(\vec{p}) = \frac{1}{\omega(\vec{p})}\int d\vec{k}\, V_F(E;\vec{p},\vec{k})\psi_F(\vec{k}), \tag{4.2}$$

where

$$\psi_F(\vec{p}) = U(\vec{p})\psi(\vec{p}); \quad V_F(E;\vec{p},\vec{k}) = U(\vec{p})V(E;\vec{p},\vec{k})U^+(\vec{k}).$$

The equation for the scattering amplitude takes the form:

$$T_F(\vec{p},\vec{k}) = V_F(E;\vec{p},\vec{k}) + \int \frac{d\vec{q}}{\omega(\vec{q})} V_F(E;\vec{p},\vec{q}) \frac{1}{E\gamma_0 - \omega(\vec{q}) - W(\vec{q}) + i0} T_F(\vec{q},\vec{k}). \tag{4.3}$$

From (4.3) one can obtain the scattering amplitude as a 4x4 - matrix:

$$T_F(\vec{p},\vec{k}) = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}. \tag{4.4}$$

Here T_{ll} are 2x2 matrices.

However in the Foldy-Wouthuysen representation all the information about the scattering is contained in the amplitude $T_{11} \equiv T^{(+)}$.

If the quasipotential is choosen in the following standard form /14/

$$V_F(E;\vec{p},\vec{k}) = \begin{pmatrix} V_F^{(+)} & 0 \\ 0 & V_F^{(-)} \end{pmatrix} \tag{4.5}$$

we obtain the simple equation for the physical part of the scattering amplitude:

$$T_F^{(+)}(\vec{p},\vec{k}) = V_F^{(+)}(E;\vec{p},\vec{k}) + \int \frac{d\vec{q}}{\omega(\vec{q})} \frac{V_F^{(+)}(E;\vec{p},\vec{q})\, T_F^{(+)}(\vec{q},\vec{k})}{E - \omega(\vec{q}) - W(\vec{q}) + i0}. \qquad (4.6)$$

Expanding the quasipotential and the scattering amplitude in a complete set of the 2x2 matrices:

$$V_F^{(+)}(E;\vec{p},\vec{k}) = V_1^{(+)}(E;\vec{p},\vec{k}) + i\vec{\sigma}\vec{n}_{pk}\, V_2^{(+)}(E;\vec{p},\vec{k}), \qquad (4.7a)$$

$$T_F^{(+)}(\vec{p},\vec{k}) + T_1^{(+)}(\vec{p},\vec{k}) + i\vec{\sigma}\vec{n}_{pk}\, T_2^{(+)}(\vec{p},\vec{k}), \qquad (4.7b)$$

where

$$\vec{n}_{pk} = \frac{[\vec{p}\times\vec{k}]}{|[\vec{p}\times\vec{k}]|},$$

one can easily obtain the system of equations for the amplitudes $T_1^{(+)}$ and $T_2^{(+)}$.

In the weak coupling case, using method developed in ref. /1/, one can construct the local quasipotentials

$$V_{1,2}^{(+)}(E;\vec{p},\vec{k}) = V_{1,2}^{(+)}(E;(\vec{p}-\vec{k})^2). \qquad (4.8)$$

In the case of strong interactions at high energies an assumption of smooth behaviour of the local quasipotential /17/ allows the main features of high-energy hadron scattering to be reproduced.

The formalism developed in this paper may serve as a convenient tool for the field-theoretic as well as for the phenomenological analysis of the πN and KN systems.

In particular, an application of the equations considered to the scattering of particles with spin 0 and 1/2 can be found in ref. /18/.

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