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DEEP INELASTIC ELECTROPRODUCTION OF HADRONS ON A PHOTON TARGET

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DEEP INELASTIC ELECTROPRODUCTION OF HADRONS ON A PHOTON TARGET

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Глубоко неупругое электророждение адронов на фотонной мишени

Показано, что исследование реакции $ee \to eeN$ в режиме, когда один из конечных электронов детектируется под большим углом, позволяет наблюдать процесс глубоко-неупругого электророждения адронов на фотонной мишени.

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Kunszt Z., Ter-Antonyan V.M. E2-6036

Deep Inelastic Electroproduction of Hadrond on a Photon Target

We discuss the inelastic scattering $ee \to ee N$ hadrons, on the condition that one of the final state electrons is detected at large angles. A simple estimate shows that this process may make possible the observation of the deep inelastic electroproduction of hadrons on photon target.

Preprint. Joint Institute for Nuclear Research. Dubna, 1971

1. In the last year the importance of the two-photon mechanism of hadron production in electron-positron (electron) colliding beam experiments at high energies was stressed by several authors [1,2,6] (Fig.1).

The two-phonon mechanism means the following

$$e+e \rightarrow e+\gamma^{*}+e+\gamma^{*} \rightarrow e+e+\mathcal{H} \quad , \tag{1}$$

where γ^{*} denotes virtual photons with space-like four-momentum and $\mathcal{H}$ is a neutral C-even state of hadrons (e.g. $\pi^{0}, \eta, \pi^{+}\pi^{-}, K^{+}K^{-},$ etc). In connection with this mechanism we can study the annihilation of two real photons into hadrons $\gamma+\gamma \rightarrow \mathcal{H}$.

Since there is a rapid falloff in the amplitude of the process (1) with increasing the mass of the virtual photon, we might think at first, that it is hopeless to obtain information for the process $\gamma+e \rightarrow ee\mathcal{H}$ using colliding electron-electron beams. Brodsky et al. [2] however have noticed, that the reaction (1) may be feasible at higher values of $q^{2}, (q'^{2})$ too if we sum up all the possible final states of hadrons. This would mean, on the other hand, that the reaction (1) in the deep inelastic region, where one of the final state electrons is detected at large angles, may be observed, supplying us with experimental data on the deep inelastic electro-production of hadrons on photons target.

2. The reaction under investigation

$$e + \gamma \longrightarrow e + \mathcal{H} \qquad (2)$$

represents the t-channel process for the annihilation $e^+ + e^- \to \gamma + \mathcal{H}$ studied by us in a previous paper [3]. These two processes together give a complete characterization for the absorptive part of the forward photon-virtual-photon scattering.

In lowest order in electromagnetism, the reaction (2) can be represented by two types of Feynman's diagrams (see Figure 2). This classification comes naturally from the C-parity conservation: the two types of diagrams' cannot interfere.

The diagrams of the first type involve C-odd hadron states. The corresponding differential cross section is given in the o.m. system as

$$\frac{d^2\sigma^-}{dE_2\, d\Omega} = \frac{\alpha}{16\pi^2} \frac{E_2}{E^3 (E - E_2 \cos^2\frac{\theta}{2})} \Big\{ E^2 + (E - E_2 \cos^2\frac{\theta}{2})^2 -$$

$$- 2E_2 (2E - E_2) \sin^2\frac{\theta}{2} \Big\} \sigma_{tot} (e^+e^- \to \text{Hadrons}),$$

where

$$\sigma_{tot} (e^+e^- \to \text{Hadrons}) = \frac{8\pi^2\alpha^2}{q^2+2\nu}\, \rho(q^2+2\nu).$$

At higher energies, the quantity $q^2+2\nu$ for the process (1) becomes independent of the energy of the final electron ($q^2+2\nu = 4E(E-E_2)$) so that the principle of automodelity [5] can be applied and gives the simple asymptotic expression

$$\sigma^{(-)}(E) = \mathrm{const}\ \frac{\pi\alpha^3(2\ln 2-1)}{2E^2}.$$

So measuring only the electron scattered into large angle, the background contributions of the diagrams(Fig.2a)can be evaluated.

In the following we restrict our considerations to the contributions of the second-type diagram. Here we have a fourth-rank tensor

$$\rho_{\mu\nu;\lambda\sigma} = \sum_N (2\pi)^4\delta(q-q'-P_n)\,R_{\mu\lambda}(N,0)\,R^*_{\nu\sigma}(N,0),$$

where

$$R_{\mu\nu} = i\int d^4x\, e^{-iqx}\langle N|T(J_\mu(x)J_\nu(0))|0\rangle$$

and the cross section is given by it as follows

$$d\sigma^{(+)} = \frac{1}{4k_1q'}L^{\mu\nu}\ell^{\lambda\sigma}\rho_{\mu\nu;\lambda\sigma}\frac{(4\pi\alpha)^3}{q^4}\cdot\frac{d\vec{k}_2}{(2\pi)^3 2k_0} \qquad (3)$$

here $L^{\mu\nu}$ and $\ell^{\lambda\sigma}$ denote the density matrices of the lepton pair and the photon respectively. Current conservation, P,T and Lorentz-invariance imply that the tensor $\rho_{\mu\nu;\lambda\sigma}$ can be decomposed into four structure functions.
A convenient decomposition is proposed in the paper [3]

$$\rho_{\mu\nu;\lambda\sigma} = \rho_1(q^2,\nu)\, G_{\mu\nu} G'_{\lambda\sigma} + \rho_2(q^2,\nu)\, Q_{\mu\nu} G'_{\lambda\sigma} +$$
$$+ \rho_3(q^2,\nu)\left(G''_{\mu\lambda} G''_{\nu\sigma} + G''_{\mu\sigma} G''_{\nu\lambda} \right) + \tag{4}$$
$$+ \rho_4(q^2,\nu)\left(G'_{\mu\lambda} G''_{\nu\sigma} - G'_{\mu\sigma} G''_{\nu\lambda} \right),$$

where

$$G_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}, \qquad Q_{\mu\nu} = Q_\mu \cdot Q_\nu$$
$$G'_{\lambda\sigma} = -g_{\lambda\sigma} + \frac{q_\lambda q'_\sigma + q_\sigma q'_\lambda}{qq'}, \qquad Q_\mu = q_\mu - \frac{q^2}{qq'} q'_\mu$$
$$G''_{\mu\nu} = -g_{\mu\nu} + \frac{q'_\mu q_\nu}{qq'}.$$

If we want to determine all the structure functions $\rho_i(q^2,\nu)$ we must design experiments with polarized electron beam and photon target. The corresponding cross section in the c.m. system is given as

$$\frac{d^2\sigma^{(+)}}{dE_2\, d\Omega_2} = \frac{\alpha^3}{16E^3} \frac{1}{\sin^2\frac{\Theta}{2}} \Big\{ \rho_1 + \rho_3 +$$
$$+ \frac{2EE_2 \cos^2\frac{\Theta}{2}}{(E - E_2 \cos^2\frac{\Theta}{2})} \left(4EE_2 \sin^2\tfrac{\Theta}{2}\, \rho_2 + (1-\xi_3)\rho_3 \right) \tag{5}$$
$$- \frac{E - E_2 \sin^2\frac{\Theta}{2}}{2E_2 \sin^2\frac{\Theta}{2}}\, \xi_2\, \rho_4 \Big\},$$

where the helicity of the incoming electron is equal to −1 and ξ_1, ξ_2 denotes the Stokes parameters of the density matrix of the photon. Further specifications can be given similarly to that we have done for the crossed reaction [3].

3. In colliding beam experiments where the photon target consists of the equivalent photon spectrum of the electron beam all the particles which are involved are unpolarized.

Averaging over initial photon spin we obtain the usual second-rank tensor

$$W_{\mu\nu} \equiv \rho_{\mu\nu\lambda\sigma}\,\ell^{\lambda\sigma} = W_1^{(\gamma)} g_{\mu\nu} + W_2^{(\gamma)} \left(q'_\mu - \frac{\nu}{q^2} q_\mu\right)\left(q'_\nu - \frac{\nu}{q^2} q_\nu\right)\frac{q^2}{\nu} \tag{6}$$

$$W_1^{(\gamma)}(q^2,\nu) = \rho_1(q^2,\nu) + \rho_3(q^2,\nu)$$

$$W_2^{(\gamma)}(q^2,\nu) = q^2 \rho_2(q^2,\nu) - \rho_3(q^2,\nu)$$

and the cross section can be written as follows

$$\frac{d^2\sigma^{(+)}}{dq^2 dE_2} = \frac{\pi\alpha^3}{q^4}\,\frac{1}{q'_0 E^2}\Big[-q^2 W_1^{(\gamma)}(q^2,\nu) + \tag{7}$$
$$+ 2(k_1 q')(k_2 q')\,\frac{q^2}{\nu^2}\,W_2^{(\gamma)}(q^2,\nu)\Big].$$

For the estimation of the structure function $W_i^{(\gamma)}$ we have to look for some models. As in the previous work [3] we give an estimate using vector meson dominance, automodelity and the parton model.

In the automodel limit, where

$$q^2 \to \infty, \quad \nu \to \infty, \quad \omega = \frac{q^2}{2\nu} = \text{fixed} \tag{8}$$

the VMD can be used only for the real photon. Applying the VMD, only the ρ meson contributions are taken into account.

The cross section for the prooess $e + \rho \to e + \mathcal{H}$ is determined by the tensor

$$W_{\mu\nu}^{(\rho)} = \frac{1}{2\pi m_\rho} \sum_N (2\pi)^4 \delta(q - q' - P_N) \langle \rho | \mathcal{J}_\mu^{(\rho)}(0) | N \rangle \langle N | \mathcal{J}_\nu^{(\rho)}(0) | \rho \rangle$$

$$= W_1^{(\rho)}(q^2, \nu)\, g_{\mu\nu} + \frac{1}{m_\rho^2} \left(q'_\mu - \frac{\nu}{q^2} q_\mu\right)\left(q'_\nu - \frac{\nu}{q^2} q_\nu\right) W_2^{(\rho)}(q^2, \nu). \quad (9)$$

Comparing the relations (6) and (9), and using the field--current identity

$$\mathcal{J}_\mu(x) = -\frac{m_\rho^2}{2\gamma_\rho} V_\mu^{(\rho)}(x)$$

We can find the following relations between the corresponding structure functions

$$W_1^{(\gamma)}(q^2, \nu) = \frac{3 m_\rho \pi}{2\gamma_\rho^2} W_1^{(\rho)}(q^2, \nu)$$

$$W_2^{(\gamma)}(q^2, \nu) = \frac{3\pi}{2\gamma_\rho^2} \cdot \frac{\nu}{m_\rho} \frac{1}{2\omega} W_2^{(\rho)}(q^2, \nu) \quad (10)$$

From the automodelity it follows that as $q^2 \to -\infty$, $\nu \to +\infty$ $\omega = \frac{-q^2}{2\nu}$ fixed

$$\frac{\nu}{m_\rho} W_2^{(\rho)}(q^2, \nu) = F_2^{(\rho)}(\omega) \quad , \quad m_\rho W_1^{(\rho)}(q^2, \nu) = F_1^{(\rho)}(\omega) .$$

Assuming that the proton consists mainly of spin 1/2 constituents, we obtain the Callan-Gross relation $F_1(\omega) = \frac{1}{2\omega} F_2(\omega)$.

Finally we arrive at the following formula

$$\frac{d^2\sigma^{(+)}}{dq^2 dE_2} = \frac{\tilde{\lambda}\alpha^3}{q^2}\cdot\frac{1}{q_0' E^2}\cdot\frac{3\sigma}{4\gamma_\rho^2}\,\frac{1}{\omega}F_2(\omega)\left[-1+\frac{2k_1q'\cdot k_2q'}{\nu^2}\right]. \qquad (11)$$

In order to estimate whether this process is feasible in a real experiment, we have to connect this cross section with the observed cross section of the process (1). For this purpose we apply the equivalent photon method in its simplest (and roughest) version where the correlation between the q and q' region is neglected:

$$\frac{d^2\sigma_{ee}}{dq^2 dE_2} = \int_{q'_{0min}}^{q'_{0max}} dq_0'\, n(q_0')\,\frac{d^2\sigma_{e\gamma}}{dq^2 dE_2}$$

$$n(q_0') = \frac{\alpha}{\pi}\,\frac{1}{q_0'}\,\frac{E^2+(E-q_0')^2}{E^2}\ln\frac{E}{m_e}\,. \qquad (12)$$

The connection of q_0' with ω is given as

$$q_0' = \frac{2E}{\omega\beta}\,, \quad \text{where} \quad \beta = -1 + \frac{4E(E-E_2)}{q^2}\,. \qquad (13)$$

In the following we shall assume that at the values of ω which corresponds to the interval $q_0 \in (q'_{0min}, q'_{0max})$ the asymptotic structure function $F_2(\omega)$ is approximately constant. Since the factor $\frac{(k_1q')(k_2q')}{\nu^2}$ is independent of q_0' too, we obtain for the cross section of the process (1)

$$\frac{d^2\sigma_{ee}}{dq^2 dE_2} = \frac{4\pi}{\alpha} \ln\frac{E}{m_e}\left(\ln\frac{E}{m_e} - 0.75\right)\frac{d^2\sigma_{e\gamma}}{dq^2 dE_2} .$$

As to the calculation of $\frac{d^2\sigma_{ee}}{dq^2}$ we notice, that performing the integral over E_2 the factor of $F_2(\omega)$ can be decomposed into three terms

$$\beta\left(-1 + \frac{2(k_1q')(k_2q')}{\nu^2}\right) = -\beta - \frac{8E^2}{q^2} + \frac{32E^4}{q^4\beta} \tag{14}$$

and the main contributions some from the last one.

So we can write

$$\frac{d\sigma_{ee}}{dq^2} \simeq \frac{48\alpha^4}{q^4} \ln\frac{E}{m_e}\left(\ln\frac{E}{m_e} - 0{,}75\right)\left(\ln\frac{8E^2}{-q^2}\right)\frac{F_2}{\gamma_\rho^2} . \tag{15}$$

Taking into account that the factor $\ln\frac{8E^2}{-q^2}$ is a smooth function of q^2 in the interval $|q^2| \in (q^2_{min}, 4E^2)$, the following expression is obtained for the total cross section:

$$\sigma_{ee} \approx \frac{48\alpha^4}{|q^2_{min}|} \ln\frac{E}{m_e}\left(\ln\frac{E}{m_e} - 0{,}75\right)\left(\ln\frac{8E^2}{|q^2_{min}|}\right)\frac{F_2}{\gamma_\rho^2} . \tag{16}$$

From this formula we can see that the cross section increases with the energy. Recently Csonka and Rees [5] have proposed an experimental device by use of which we can produce colliding beams with very high energy. E.g. directing the 20 GeV's electron beam from SLAC to a storage ring of 2 GeV we can produce colliding beams with energy 12.6 GeV in the c.m. system; the expected luminosity however is small (2×10^{-29} cm^{-2} sec^{-1}) . Therefore keeping in mind the real possibilities, a relevant estimation should have been done with energy $E \simeq 3$ GeV and luminosity $\mathcal{L} \simeq$ $L = 10^{33}$ cm^{-2} sec^{-1} . Then using $|q^2_{min}| = 1$ GeV^2 we obtain $\sigma_{ee} \approx 10^{-34}$ cm^2 .

After completing this work we have received a preprint by S.J.Brodsky, T.Kinoshita and H.Terazawa (CLNS-153, 1971) and we have learned that they and T.F.Walsh (DESY 71/15,1971) investigated the same subject as ours.

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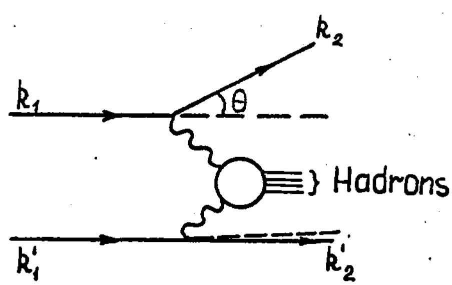

Fig.1. Diagram, exhibiting the two photon mechanism for the process $e + e \to e + e +$ Hadrons

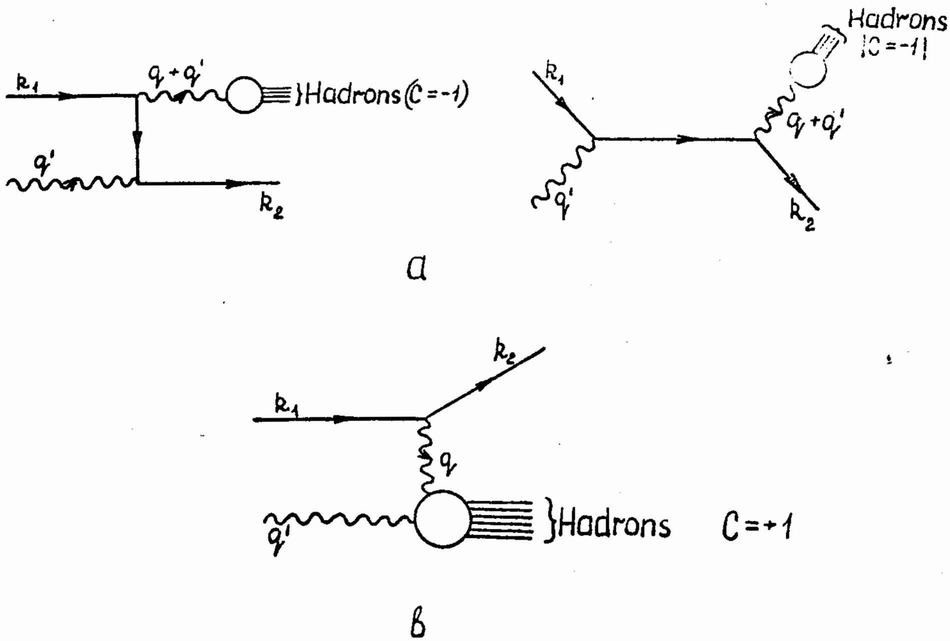

Fig.2. Electroproduction of (a) neutral C = – state and (b) neutral C = + state of hadrons on photon target.

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