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E2 - 6025

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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1971

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In the theoretical investigations of the high energy hadron scattering there exists now a tendency to regard hadrons in such collisions as composite systems with internal degrees of freedom. There are quark, droplet(1), parton(2) models, each of them has a certain success in the description of the high energy hadron scattering. That is why the results of V.Matveev and A.Tavkhelidze, paper(3) based on the assumption about the coherent nature of the interacting hadron excited states, are of great interest. In that paper the processes of the hadron scattering are described by means of the four-dimensional relativistic oscillator coherent wave functions.

Here we shall try to show how it is possible to connect the quark models and the coherent state method for the high energy hadron scattering. This paper is a brief version of the JINR preprint (4).

Consider first the case of mesons. We start from the equation

$$\left\{ p_1^2 + p_2^2 + V\left((x_1 - x_2)^2\right) \right\} \psi(x_1, x_2) = 0 . \qquad (1)$$

This equation describes the motion of the quark (1) and antiquark (2) in a meson and corresponds to the limit in which the violation of spin and U_3 symmetry is neglected. The metric is

$$g_{00} = 1 \quad , \quad g_{\alpha\alpha} = -1 \quad , \quad \alpha = 1, 2, 3 .$$

$$p_{j,\alpha} = i g_{\alpha\alpha} \frac{\partial}{\partial x_{j,\alpha}} \quad , \quad \alpha = 0, 1, 2, 3 .$$

To separate the motion of the meson as a whole and the relative motion of the quark and antiquark we make the substitution

$$x_1 = x + \xi \quad , \quad x_2 = x - \xi \ .$$

Hence $$p_1 = \frac{1}{2}\hat{p} + \frac{1}{2}\eta \quad , \quad p_2 = \frac{1}{2}\hat{p} - \frac{1}{2}\eta \, ,$$

where*) $$\hat{p}_\alpha = i g_{\alpha\alpha} \frac{\partial}{\partial x_\alpha} \quad , \quad \eta_\alpha = i g_{\alpha\alpha} \frac{\partial}{\partial \xi_\alpha} \ .$$

Consider the motion of a meson as a whole and put

$$\psi(x_1, x_2) = e^{-i(px)} \psi(\xi) \ .$$

Then $$\left\{ M^2 + \eta^2 + 2V(4\xi^2) \right\} \psi(\xi) = 0 \, ,$$

where $M^2 = p^2$ is the squared meson mass. There exists an essential difficulty in solving such equations. For instance, consider the simple case when the potential function V corresponds to the harmonic oscillator

$$2V(4\xi^2) = \omega^2 \xi^2 + c \ .$$

For this potential

$$\left\{ M^2 + c + \left(-\frac{\partial^2}{\partial \xi_0^2} + \omega^2 \xi_0^2 \right) - \sum_{\alpha=1}^{3} \left(-\frac{\partial^2}{\partial \xi_\alpha^2} + \omega^2 \xi_\alpha^2 \right) \right\} \psi(\xi) = 0 \ . \quad (2)$$

It is clear that the solution will be either noncovariant or unnormalizable in the usual sense. Bearing in mind this situati-

*) Symbol $\hat{p}$ differs operator $i g_{\alpha\alpha} \frac{\partial}{\partial x_\alpha}$ from fixed momentum of the meson, which we denote by p.

on we put

$$\xi_0 = i\,\xi_4 \tag{3}$$

considering that ξ_4 is a real variable and define the norm $\langle\Psi,\Psi\rangle$ as an integral

$$\int_{-\infty}^{\infty}\!\!\cdots\!\int \Psi^*(\xi)\,\Psi(\xi)\,d\xi_1\,d\xi_2\,d\xi_3\,d\xi_4 .$$

With such a definition of the norm the solution is normalizable,

$$\langle\Psi,\Psi\rangle = \left(\int_{-\infty}^{\infty} e^{-\omega\xi^2}\,d\xi\right)^4 = \frac{\pi^2}{\omega^2} .$$

Having done the substitution (3) in eq. (2) we obtain

$$\left\{M^2 + c - \sum_{\alpha=1}^{4}\left(-\frac{\partial^2}{\partial\xi_\alpha^2} + \omega^2\xi_\alpha^2\right)\right\}\Psi(\xi) = 0 . \tag{4}$$

In this case we obtain the covariant form of the wave function and the equidistant positive spectrum for M^2. Introduce the quantum amplitudes for the harmonic oscillator

$$a_\alpha = \frac{\omega\xi_\alpha + i\eta_\alpha}{\sqrt{2\omega}} , \quad a_\alpha^+ = \frac{\omega\xi_\alpha - i\eta_\alpha}{\sqrt{2\omega}} \tag{5}$$

and put $4c = 4\omega - M_0^2$ Eq. (4) will be as follows

$$\left(M^2 + 2\omega\sum_{\alpha=0}^{3} g_{\alpha\alpha}\,a_\alpha^+ a_\alpha - M_0^2\right)\Psi = 0 . \tag{6}$$

It can be seen that the ground state of the system $\Psi = |0\rangle$ is the state with the norm equal to unity.

$$\langle 0|0\rangle = 1 .$$

and satisfies the relation

$$a_\alpha |0> = 0 \quad , \quad \alpha = 0,1,2,3.$$

Having considered the general excited state, which contains n quanta

$$\psi = \sum_{\substack{(j_1, j_2 .. j_n \\ j=0,1,2,3)}} \epsilon_{j_1, j_2 .. j_n} \overset{+}{a}_{j_1} \overset{+}{a}_{j_2} .. \overset{+}{a}_{j_n} |0> \tag{7}$$

it is easy to see that the norm can be negative, for instance, the state with one time quantum

$$\psi = \overset{+}{a}_0 |0>$$

$$<\psi, \psi> = <0| a_0 \overset{+}{a}_0 |0> = <0| \overset{+}{a}_0 a_0 | 0> - <0,0> = -1.$$

Integration over the imaginary axis ξ_0 which ensures covariance and positivity of the M^2 spectrum, causes negativity of the norm. Thus it is clear that the norm of the states (7) will be positive if there are no creation operators $\overset{+}{a}_0$ of the time quanta in these states.

In this way we get the condition

$$\sum_{\alpha=0}^{3} e_\alpha \epsilon_{\alpha, j_2 .. j_n} = 0 ; \tag{8}$$

where e is a time-like vector, for instance, $e = p$. Condition (8) ensures the absence of time quanta in the expression ψ so that the norm will be positive.

It was noticed in paper [3] that the transversality condition is a criterion for choosing the physically permissable states.

To derive the formula for the high energy two-meson scattering amplitude we chose the potential which describes that scattering in the form (3)

$$W^{(I,II)} = i \sum_{j=1}^{2} \sum_{\ell=1}^{2} G_{j,\ell}^{(I,II)} V_{j,\ell} \; ; \; V_{j,\ell} = \sum_{\alpha=0}^{3} g_{\alpha\alpha} V_{j,\ell}^{(\alpha)}$$

$$\begin{aligned} V_{j,\ell}^{(\alpha)} &= \hat{p}_\alpha (\hat{p}_\alpha' \delta(x_j - x_\ell') + \delta(x_j - x_\ell')\hat{p}_\alpha') + \\ &+ (\hat{p}_\alpha' \delta(x_j - x_\ell') + \delta(x_j - x_\ell')\hat{p}_\alpha')\hat{p}_\alpha = \\ &= \hat{p}_\alpha' (\hat{p}_\alpha \delta(x_j - x_\ell') + \delta(x_j - x_\ell')\hat{p}_\alpha) + \\ &+ (\hat{p}_\alpha \delta(x_j - x_\ell') + \delta(x_j - x_\ell')\hat{p}_\alpha)\hat{p}_\alpha' . \end{aligned} \tag{9}$$

Here $\hat{p}$, x_j corresponds to the first meson (I), $\hat{p}'$, x_ℓ' to the second meson (II). $G_{j,\ell}^{(I,II)}$ are the constants characterizing the interaction between quark (j = 1) or antiquark (j = 2) in the first meson (I) and quark ($\ell = 1$) or antiquark ($\ell = 2$) in the second meson (II). The Born scattering amplitude T is defined by the matrix element $W^{(I,II)}$ of the interaction potential between the states

$$\psi_I \psi_{II} = e^{-i(px+p'x')} |0,0'\rangle$$

$$\psi_{II}^* \psi_I^* = \langle 0',0| e^{i(qx+q'x')} .$$

namely
$$(2\pi^4)\, \delta(q+q'-p-p')\, T(s,t) =$$

$$= \int \langle 0',0| e^{i(qx+q'x')} W^{(I,II)} e^{-i(px+p'x')} |0,0'\rangle \, dx\, dx',$$

where
$$s = (p+p')^2 \; , \; t = (p-q)^2 .$$

Substituting the integral representation of the δ- funotion into (9), after simple calculations, we obtain the Matveev and Tavkhelidze formula

$$T(s,t) = i(s-u)\, G\, e^{t/2\omega},$$

where

$$G = \sum_{j=1}^{2} \sum_{\ell=1}^{2} G_{j,\ell}^{(I,II)}.$$

This result can be generalized to the baryon oase. In this case we shall start from the following equation

$$\left\{ \sum_{j=1}^{3} p_j^2 + V\left(\frac{(x_1-x_2)^2 + (x_1-x_3)^2 + (x_2-x_3)^2}{3} \right) \right\} \psi(x_1,x_2,x_3) = 0. \tag{10}$$

To separate the motion of the baryons as a whole and the relative motion we introduce the variables :

$$\begin{aligned} & x_j = x + \xi_j \quad , \quad \sum_{j=1}^{3} \xi_j = 0 \\ & \xi_1 = \sqrt{\tfrac{2}{3}}\, y \quad , \quad \xi_2 = -\sqrt{\tfrac{1}{6}}\, y + \sqrt{\tfrac{1}{2}}\, z \\ & \xi_3 = -\sqrt{\tfrac{1}{6}}\, y - \sqrt{\tfrac{1}{2}}\, z \end{aligned} \tag{11}$$

and the corresponding momenta

$$\hat{p}_\alpha = i g_{\alpha\alpha} \frac{\partial}{\partial x_\alpha} \quad , \quad \eta_\alpha = i g_{\alpha\alpha} \frac{\partial}{\partial y_\alpha} \quad , \quad \zeta_\alpha = i g_{\alpha\alpha} \frac{\partial}{\partial z_\alpha} .$$

In this way equation (10) can be written in the form

$$\left\{\frac{1}{3}\hat{p}^2+\eta^2+\zeta^2+V(y^2+z^2)\right\}\psi=0 \,. \tag{12}$$

As in the case of mesons we put here

$$y_0 = i y_4 \;, \quad z_0 = i z_4$$

with real y_4, z_4 and define the norm by the integral

$$\int_{-\infty}^{\infty}\!\!\cdots\!\int \psi^*(y,z)\,\psi(y,z)\,d_4y\,d_4z \,.$$

Consider the particular case of the oscillator potential

$$V(y^2+z^2)=\frac{\omega^2}{4}(y^2+z^2)+c \;; \qquad \omega, c = \text{const.}$$

and introduce the corresponding quantum amplitudes

$$a_\alpha=\frac{1}{\sqrt{\omega}}\left(\frac{\omega}{2}y_\alpha+i\eta_\alpha\right), \quad \overset{+}{a}_\alpha=\frac{1}{\sqrt{\omega}}\left(\frac{\omega}{2}y_\alpha-i\eta_\alpha\right)$$

$$b_\alpha=\frac{1}{\sqrt{\omega}}\left(\frac{\omega}{2}z_\alpha+i\zeta_\alpha\right), \quad \overset{+}{b}_\alpha=\frac{1}{\sqrt{\omega}}\left(\frac{\omega}{2}z_\alpha-i\zeta_\alpha\right).$$

By means of these amplitudes eq. (12) takes the form

$$\left\{M^2-M_0^2+3\omega\sum_{\alpha=0}^{3} g_{\alpha\alpha}\left(\overset{+}{a}_\alpha a_\alpha+\overset{+}{b}_\alpha b_\alpha\right)\right\}\varphi=0. \tag{13}$$

It is seen that the general form of the eigenfunctions for this equation is

$$\varphi=\sum_{(j,\mu=0,1,2,3)} \epsilon_{j_1\cdots j_r,\mu_1\cdots\mu_s}\,\overset{+}{a}_{j_1}\cdots\overset{+}{a}_{j_r}\,\overset{+}{b}_{\mu_1}\cdots\overset{+}{b}_{\mu_s}|0\rangle \tag{14}$$

and the corresponding mass square values

$$M^2 = 3\omega(r+s) + M_0^2 .$$

To ensure the abcense of the time quanta in (13) the correspon-ding transversality conditions can be written as in the meson case.

In the general case of the potential $V(y^2+z^2)$ to obtain two-baryon elastic amplitude we assume that before collision both particles have the lowest mass in their spectrum, i.e. their states are

$$e^{-ipx}|0_I\rangle \quad , \quad e^{-ip'x'}|0_{II}\rangle$$

and after collision

$$e^{-iqx}|0_I\rangle \quad , \quad e^{-iq'x'}|0_{II}\rangle .$$

The shape of the potential W is chosen the same as in the meson case.

$\hat{p}, x_j$ is related to the first particle (I) and $\hat{p}', x'_\ell$ to the second one (II). If the first particle is a meson, j = 1,2 and

$$x_j = x + \xi_j \quad ; \quad \xi_1 = \xi \quad , \quad \xi_2 = -\xi .$$

of the first particle is a baryon, j = 1,2,3 and

$$x_j = x + \xi_j ,$$

where ξ_j are expressed through the variables y, z in (11). The second particle is always considered as a baryon. In this case for the Born amplitude we obtain

$$T(s,t) = i(s-u) \sum_{(j,\ell)} G_{j,\ell}^{(I,II)} \cdot F_I(t) F_{II}(t) , \tag{15}$$

where $\quad F(\kappa^2) = \langle 0 | e^{i\kappa \xi_j} | 0 \rangle .$

Comparing this formula with that for mesons one can see that it concerns both meson-meson and meson-baryon and baryon-baryon scattering. As was indicated in paper (3) this Born amplitude oould be used to oonstruct the corresponding quasipotential and to obtain more precise expression for the scattering amplitude by means of the quasipotential equation. However, it is possible to use formula (15) direotly, regarding T as the scattering amplitude i.e. neglecting the corrections to the Born aproximation. Then, taking into account that

$$F_I(0) = F_{II}(0) = 1$$

we obtain, as in paper(3), the following expression for the total cross-section at high energy

$$\sigma_{tot} = 2 \sum_{j,\ell} G_{j,\ell}^{(I,II)} .$$

One oan see that here it is possible to interpret $2 G_{j,\ell}^{(I,II)}$ as a total cross section of j-quark interaction with ℓ-quark. In this interpretation we obtain the additivity rule

$$\sigma_{tot} = \sum_{j,\ell} \sigma_{tot}^{(j,\ell)} \tag{16}$$

which was considered formerly in a number of papers, and we shall discuss here neither the limits of application of this rule nor its comparison with experiment.

However, we must notice that the necessity of making (15) and (16) more preoise is immediately seen as the principal equations contain neither symmetry violations oaused by spin and isotopic structures nor the difference between the effeotive λ and p, n quark masses. Nevertheless, if we use (15) then as the elastic differential cross section, when $s \to \infty$, is proportional to

$$\left| \sum_{j,l} G_{j,l}^{(I,II)} F_I(t)\, F_{II}(t) \right|^2$$

we obtain

$$\frac{d\sigma_{el}}{dt} = \left(\frac{d\sigma_{el}}{dt} \right)_{t=0} F_I^2(t)\, F_{II}^2(t). \tag{17}$$

Here, as follows from (5),(6) the functions $F_I(t)$, $F_{II}(t)$ correspond to the electromagnetic formfactors of interaoting particles (I), (II). Formula (17) obtained from other considerations is adduced in (7). In the same paper there are formulas analogous to (15) in which the functions $g_{j,l}(t)$ are instead of constants $G_{j,l}$. It is easy to obtain such a generalization in our scheme. It is sufficient to use non-local interaotion instead of δ-interaction :

$$W = i \sum_{j,l,\alpha} g_{\alpha\alpha} V_{j,l}^{(\alpha)}; \quad V_{j,l}^{(\alpha)} = \hat{p}_\alpha \left(\hat{p}'_\alpha \Phi_{j,l}(x_j - x'_l) + \Phi_{j,l}(x_j - x'_l) \hat{p}'_\alpha \right) +$$

$$+ \left(\hat{p}'_\alpha \Phi_{j,l}(x_j - x'_l) + \Phi_{j,l}(x_j - x'_l) \hat{p}'_\alpha \right) \hat{p}_\alpha ,$$

where

$$\Phi_{j,l}(x) = \frac{1}{(2\pi)^4} \int g_{j,l}(k^2)\, e^{ikx}\, dk .$$

Then repeating literally our calculations we obtain instead of (15)

$$T(s,t) = i(s-u)\sum_{j,l} g_{j,l}(t)\,F_{I}(t)\,F_{II}(t).$$

Consider one more operator potential W which leads to the same result (15) as the potential (9). The suggested modification is to change the total momenta of particles p, p' by single quark momenta p_j, p'_l.

$$\widetilde{W}^{(I,II)} = i\sum_{j,l} G_{j,l}^{(I,II)} N^{(I)} N^{(II)} \widetilde{V}_{j,l}$$

$$\widetilde{V}_{j,l} = \sum_{\alpha=0}^{3} g_{\alpha\alpha} V_{j,l}^{(\alpha)} \tag{18}$$

$$V_{j,l}^{(\alpha)} = p_{j,\alpha}\left(p'_{l,\alpha}\,\delta(x_j - x'_l) + \delta(x_j - x'_l)\,p'_{l,\alpha}\right) + $$

$$+ \left(p'_{l,\alpha}\,\delta(x_j - x'_l) + \delta(x_j - x'_l)\,p'_{l,\alpha}\right) p_{j,\alpha}\,.$$

where N = 2 if the particle is a meson and N = 3 if the particle is a baryon. It is easy to show that potential (18) leads to the same Born elastic amplitude as the potential (9).

Notice, that it is possible to consider the potential W as an interaction through the quark currents. Really putting into (18) the four-dimensional δ-function

$$\delta(x_j - x'_l) = \frac{1}{(2\pi)^4}\int e^{i\kappa(x_j - x'_l)}\,d\kappa$$

we obtain

$$\widetilde{W} = \frac{i}{(2\pi)^4}\sum_{j,l} G_{j,l}^{(I,II)} N^{(I)} N^{(II)} \int \left(p_j e^{i\kappa x_j} + e^{i\kappa x_j} p_j\right)\left(p'_l e^{-i\kappa x'_l} + e^{-i\kappa x'_l} p'_l\right) d\kappa. \tag{19}$$

Introduce the four-dimensional current densities of j-quark

$$I_j(X)=\frac{N}{2}\{p_j\,\delta(X-x_j)+\delta(X-x_j)\,p_j\}. \tag{20}$$

Then

$$\int I_j(X)\,dX = N p_j .$$

In the meson case

$$N\langle 0|p_j|0\rangle = 2\langle 0|p_j|0\rangle = 2\langle 0|p_1+p_2|0\rangle = \hat{p}$$

and in the baryon case

$$N\langle 0|p_j|0\rangle = 3\langle 0|p_j|0\rangle = 3\langle 0|p_1+p_2+p_3|0\rangle = \hat{p} .$$

It is seen that the coefficient N is chosen so that

$$\langle 0|\int I(X)\,dX\,|0\rangle = \hat{p} .$$

Consider the four-dimensional current density Fourier - component

$$\mathcal{J}_j^{(I)}(\kappa)=\int I_j^{(I)}(X)\,e^{i\kappa X}\,dX=\frac{N^{(I)}}{2}(p_j\,e^{i\kappa x_j}+e^{i\kappa x_j}\,p_j). \tag{21}$$

Then (19) can be written in the form

$$\widetilde{W}^{(I,II)}=i\sum_{j,e}G_{j,e}^{(I,II)}\int\mathcal{J}_j^{(I)}(\kappa)\,\mathcal{J}_e^{(II)}(-\kappa)\,d\kappa .$$

But as follows from (21) $\mathcal{J}_j^{(I)}(\kappa)$ and $\mathcal{J}_e^{(II)}(-\kappa)$ are proportional accordingly to $e^{i\kappa x}$ and $e^{-i\kappa x'}$. On the other hand, the Born elastic amplitude of the particles (I) and (II) which had momenta p, p' before interaction and q, q' after it is defined by the matrix component of the Fourier form $\widetilde{W}^{(I,II)}$. Therefore the corresponding operator is

$$\frac{4i}{(2\pi)^4}\sum_{j,l} G_{j,l}^{(I,II)} \mathcal{J}_j^{(I)}(k)\, \mathcal{J}_l^{(II)}(-k). \tag{22}$$

In the special reference system, when $k^0 = 0$ (23) operator (22) takes the following form

$$\frac{4i}{(2\pi)^4}\sum_{j,l} G_{j,l}^{(I,II)} \mathcal{J}_j^{(I)}(\vec{k})\, \mathcal{J}_l^{(II)}(-\vec{k}), \tag{24}$$

where

$$\mathcal{J}_j^{(I)}(\vec{k}) = \mathcal{J}_j^{(I)}(0,\vec{k}) = \frac{\mathcal{N}^{(I)}}{2}\left(p_j\, e^{-i\vec{k}\vec{x}_j} + e^{-i\vec{k}\vec{x}_j}\, p_j\right)$$

$$\mathcal{J}_l^{(II)}(-\vec{k}) = \mathcal{J}_l^{(II)}(0,-\vec{k}) = \frac{\mathcal{N}^{(II)}}{2}\left(p'_l\, e^{i\vec{k}\vec{x}'_l} + e^{i\vec{k}\vec{x}'_l}\, p'_l\right)$$

or

$$\mathcal{J}_j^{(A)}(\vec{k}) = \int e^{-i\vec{k}\vec{X}}\, I_j^{(A)}(X)\, dX \quad , \quad A = I, II.$$

and

$$I_j^{(I)}(X) = \frac{\mathcal{N}^{(I)}}{2}\left\{p_j\, \delta(\vec{X}-\vec{x}_j) + \delta(\vec{X}-\vec{x}_j)\, p_j\right\}$$

$$I_l^{(II)}(X) = \frac{\mathcal{N}^{(II)}}{2}\left\{p'_l\, \delta(\vec{X}-\vec{x}'_l) + \delta(\vec{X}-\vec{x}'_l)\, p'_l\right\}. \tag{25}$$

As is seen, the operators (25) can be considered as usual three-dimensional quark current densities. Thus, in the special reference system (23) the amplitude T is defined by the corresponding operator's (24) matrix element which is a linear combination of the Fourier-component products of the three-dimensional quark current densities.

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Received by Publishing Department
on September 7, 1971.