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THE $K_{\ell 3}$ DECAYS AND CHIRAL SYMMETRY-BREAKING

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1. It is well-known that the form factors in the semi-leptonic decays of kaons are very sensitive to the choice of the chiral symmetry breaking. It has been shown [1] that the most common scheme of $SU(3) \times SU(3)$ symmetry-breaking through the $I = Y = 0$ components of a tensor transforming as the $(3,3^*) + (3^*,3)$ representation does not lead to an adequate description of $K_{\ell 3}$ form factors from the latest experiments. So, the ξ parameter calculated with this scheme is near to zero in comparison with the experimental values: $-1 \leqslant \xi \leqslant -0.5$.
Then, it appears necessary to try other models of symmetry-breaking.

In this note we consider a model of chiral $SU(3) \times SU(3)$ symmetry-breaking where a term transforming as $(1.8) + (8.1)$ representation is added to the $(3.3^*) + (3^*,3)$ breaking.

We calculate the $K_{\ell 3}$ form factors using an effective Lagrangian which appears in the non-linear realization of the $SU(3) \times SU(3)$ group with the linearization on the $SU(2) \times Y$ subgroup [2,3], and we show that such a model gives a realistic description of $K_{\ell 3}$ form factors.

2. We start with the effective Lagrangian density [2,3]

$$\mathcal{L}_{eff} = \mathcal{L}_{inv} + \mathcal{L}_{s.b.} \quad . \tag{1}$$

Here $\mathcal{L}_{inv}$ is the $SU(3) \times SU(3)$ invariant part and $\mathcal{L}_{s.b.}$ is the breaking term which is assumed to be of the form

$$\mathcal{L}_{s.b.} = u_0 + c\, u_8 + d\, g_8 \quad , \tag{2}$$

where $u_{0,8}$, g_8 are the scalar densities transforming respectively as $(3,3^*) + (3^*, 3)$ and $(1,8) + (8,1)$ representations and c, d are breaking parameters. $\mathcal{L}$ s.b. (as the full effective Lagrangian density) is an essentially non-linear function of the preferred pseudoscalar π, K, η and scalar $\varkappa (I=\frac{1}{2}, Y=\pm 1)$ meson fields.

The Lagrangian vector $J_\mu^{V,m}$ and pseudovector $J_\mu^{A,m}$ currents satisfy the current-field identities and their divergences have the form :

$$\partial_\mu J_\mu^{V,i} = 0 \qquad , \qquad i=1,2,3 \quad ,$$

$$\partial_\mu J_\mu^{V,a} = \frac{3}{4}\left(c\langle u_8\rangle + d\langle g_8\rangle\right)\frac{1}{F_\varkappa}\varkappa^a = m_\varkappa^2 F_\varkappa \varkappa^a \ , \quad a=4,5,6,7 \ , \tag{3}$$

$$\partial_\mu J_\mu^{A,i} = \frac{1}{3}(\sqrt{2}+c)(\sqrt{2}\langle u_0\rangle + \langle u_8\rangle)\frac{1}{F_\pi}\pi^i = m_\pi^2 F_\pi \pi^i \ , \quad i=1,2,3 \ ,$$

$$\partial_\mu J_\mu^{A,a} = \left[\frac{1}{3}(\sqrt{2}-\frac{c}{2})(\sqrt{2}\langle u_0\rangle - \frac{\langle u_8\rangle}{2}) + \frac{3}{4}d\langle g_8\rangle\right]\frac{1}{F_K}K^a = m_K^2 F_K K^a \ , \quad a=4,5,6,7 \ ,$$

where $m_\varkappa$, m_π, m_K are the masses, $F_\varkappa$, F_π, F_K - the weak decay constants and $\varkappa^a$, π^i, K^a - the renormalized fields of the scalar and pseudoscalar mesons, and $\langle u_0\rangle$, $\langle u_8\rangle$, $\langle g_8\rangle$ - the vacuum expectation values.

Following [3] we shall take

$$\frac{F_K^2}{F_\pi^2} = 1.33 \qquad , \qquad \frac{F_\varkappa^2}{F_\pi^2} = 0.25 \qquad , \tag{4}$$

and from [4]

$$m_{\varkappa}^2 = 1.2 \text{ GeV}^2 . \tag{5}$$

Then from (3) we obtain the magnitude of the $(1.8)+(8.1)$ breaking term as following :

$$d\langle g_8\rangle = \frac{4}{3}\,\frac{\sqrt{2}-\frac{c}{2}}{\sqrt{2}+c}\,m_{\varkappa}^2 F_{\varkappa}^2 - 2c\,\frac{\sqrt{2}-\frac{c}{2}}{\sqrt{2}+c}\left[\frac{F_\pi^2 m_\pi^2}{\sqrt{2}+c} - \frac{F_K^2 m_K^2}{\sqrt{2}-\frac{c}{2}}\right] . \tag{6}$$

3. We define the $K_{\ell 3}$ form factors as usually :

$$\langle \pi^i(q)|\,J_\mu^{V,a}(0)\,|K^b(p)\rangle = \frac{i f_{iab}}{(2\pi)^3\sqrt{4p_0 q_0}}\left[(p+q)_\mu f_+(t) + (p-q)_\mu f_-(t)\right] , \tag{7}$$

where : $t=(p-q)^2$ and $J_\mu^{V,a}$ – the strangeness changing vector current, which in this model is proportional to the $K^*(1^-, 890)$ and $\varkappa(0^+, 1100)$ meson fields:

$$J_\mu^{V,a} = G_{K^*} K_\mu^{*a} - F_{\varkappa}\,\partial_\mu \varkappa^a . \tag{8}$$

To calculate the matrix element in (7) we need the effective vertices for $\varkappa K\pi$ and $K^* K\pi$ interactions, which from (1) turn out to be

$$\mathcal{L}_{\varkappa K\pi} = \frac{F_K^2 - F_\pi^2}{F_\pi F_K F_\varkappa} f_{aib}\,\varkappa^a \pi^i K^b + \frac{F_\varkappa^2}{2F_\pi F_K F_\varkappa} f_{aib}\left(\varkappa^a \pi^i \Box K^b - \varkappa^a \Box \pi^i K^b\right) +$$

$$+ \frac{1}{F_\pi F_K F_\varkappa}\left[\frac{1}{6}\left(1+\frac{4\sqrt{3}}{c}\right)m_\varkappa^2 F_\varkappa^2 - \frac{1}{2}\left(\frac{\sqrt{2}}{c}+1\right)d\langle g_8\rangle\right] f_{aib}\,\varkappa^a \pi^i K^b , \tag{9}$$

$$\mathcal{L}_{K^*K\pi} = \frac{1}{F_\pi F_K}\,\frac{m_{K^*}^2}{G_{K^*}} f_{aib} K_\mu^{*a}\left[\left(F_\pi^2 - \frac{F_K^2}{2}\right)\partial_\mu \pi^i K^b - \left(F_K^2 - \frac{F_\varkappa^2}{2}\right)\pi^i \partial_\mu K^b\right] ,$$

where, the fact that the $K^*K\pi$ interaction contains no more than one derivative [3] has been used. Consequently, the $f_\pm(t)$ form factors have the form :

$$f_+(t) = \frac{1}{2F_\pi F_K}\left(F_\pi^2 + F_K^2 - F_\varkappa^2\right)\frac{m_{K^*}^2}{m_{K^*}^2 - t} \quad , \tag{10}$$

$$\begin{aligned} f_-(t) = {} & \frac{1}{2F_\pi F_K}\left(F_K^2 - F_\pi^2\right) - \frac{p^2-q^2}{m_{K^*}^2} f_+(t) + \\ & + \frac{1}{2F_\pi F_K}\,\frac{(q^2-p^2)F_\varkappa^2 + (t-p^2-q^2)(F_K^2-F_\pi^2) - \frac{1}{3}\left(\frac{4\sqrt{2}}{c}+1\right)m_\varkappa^2 F_\varkappa^2}{m_\varkappa^2 - t} + \\ & + \frac{1}{2F_\pi F_K}\,\frac{d\langle g_8\rangle\left(\frac{\sqrt{2}}{c}+1\right)}{m_\varkappa^2 - t} \, . \end{aligned} \tag{11}$$

Using (8) we can calculate the $\varkappa \to K\pi$ decay width :

$$\begin{aligned} \Gamma(\varkappa\to K\pi) = \frac{3}{32\pi F_K^2 F_\varkappa^2 m_\varkappa^2} \Big[& \frac{F_K^2 - F_\pi^2}{2}\left(m_\varkappa^2 - m_\pi^2 - m_K^2\right) + \\ & + \frac{F_\varkappa^2}{2}\left(m_\pi^2 - m_K^2\right) - \frac{1}{6}\left(1 + \frac{4\sqrt{2}}{c}\right) m_\varkappa^2 F_\varkappa^2 + \\ & + \frac{1}{2}\left(1 + \frac{\sqrt{2}}{c}\right) d\langle g_8\rangle \Big] \cdot p_{c.m.} \quad , \end{aligned} \tag{12}$$

where :

$$p_{c.m.} = \frac{1}{2m_\varkappa}\left[\left(m_\varkappa^2 + m_\pi^2 - m_K^2\right)^2 - 4m_\varkappa^2 m_\pi^2\right]^{\frac{1}{2}} = 0.43\ \text{Гэв} \, .$$

4. The $K_{\ell 3}$ decay analysis uses the parameter $\xi(t) \equiv f_-(t)/f_+(t)$. The comparative analysis of different measurements gives the world-average [1] :

$$\xi(0)_{exp.} = -0.85 \pm 0.20 \, .$$

From (11) we see that $f_-(t)$ and consequently the parameter ξ

depend on the values of symmetry breakings: c, $d\langle g_8\rangle$.
Using (6) for the fixed $\varkappa$- meson mass we can obtain the ξ- parameter dependence on c or $d\langle g_8\rangle$. The same we can do for $\Gamma(\varkappa\to K\pi)$ when considering (12). It's easy to find the intervals for c and $d\langle g_8\rangle$ for which $\xi(0)$ and $\Gamma(\varkappa\to K\pi)$ take reasonable values in agreement with exprimental data (see Table 1).

It is interesting to observe that the values for c are near the exact SU(2) $\times$ SU(2) limit (i.e. $c=-\sqrt{2}$).

Concerning the parameter $d\langle g_8\rangle$ it must be said that it is connected with the scalar components of the meson nonet (when we add the corresponding relations to(3))and a treatment similar to the c-parameter one can be achieved when conclusive experimental information about η-η' meson mixing will be available. The results we obtained depend also on the modified Glashow-Weinberg values [3] for (4) (using the $K^*K\pi$ decay width instead of the second Weinberg sum rule) and on the uncertain value of $\varkappa$- meson mass (5).

Finally, let us remark that similar results can be obtained in the context of the current algebra (when the corresponding symmetry-breaking model is used) [5,6] or still with other models [7].

Table 1

C	$d\langle g_8\rangle$ GeV^4	$\xi(0)$	$\Gamma(\varkappa\to K\pi)$ GeV
-1.345	0.16	-0.45	0.014
-1.35	0.23	-0.52	0.025
-1.355	0.25	-0.56	0.035
-1.36	0.31	-0.61	0.05
-1.365	0.37	-0.66	0.07
-1.37	0.46	-0.74	0.12
-1.375	0.59	-0.83	0.18
-1.38	0.79	-0.94	0.32
-1.385	1.09	-1.1	0.62

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