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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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TRANSFORMATION PROPERTIES OF STATES AT INFINITE MOMENTUM II

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Преобразование состояния бесконечного импульса

Известный результат, состоящий в том, что состояния спиральности массовых частиц с бесконечным импульсом обладают теми же самыми свойствами преобразования что и безмассовые частицы, обобщается на состояния с $p^2<0$.

В случае $p^2>0$ и $p^2<0$ определяются канонические состояния. Рассматривается асимптотическое поведение поворота Вигнера для этих состояний. Устанавливается связь между поворотом Вигнера для бесконечного импульса и поворотом в соответствующем подпространстве, вытекающем из преобразования подходящего светового вектора.

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Transformation Properties of States at
Infinite Momentum II

The well known result, that helicity states of massive particles at infinite momentum satisfy the same transformation laws as the states of massless particles, is extended to the states with $p^2<0$.

In the cases $p^2>0$ and $p^2<0$ canonical states are defined. The asymptotic rproperties of the Wigner rotation for these states are considered.

A connection is given between the Wigner rotation in the infinite momentum limit and the rotation in the corresponding subspace induced by the transformation of an appropriate light-like vector.

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Dubna, 1971

1. Introduction

In this paper we extend our considerations of transformation properties of massive particle states/1/to light-like and space-like representations. We consider two types of states: the helicity states and the canonical states. The states are constructed using the states $|\hat{p}s\lambda\rangle$ in a standard system and applying an unitary transformation $\mathcal{U}(L_p)$:

$$|p\, L_p s\lambda\rangle = \mathcal{U}(L_p)\,|\hat{p}s\lambda\rangle \tag{1}$$

which depends on the boost matrix L_p. The boost matrices for helicity states with $p^2 \geq 0$ are well known/1,3/. For $p^2 < 0$ we define the helicity states in the same manner. The boost matrix for the canonical states is a rotation free Lorentz transformation/2/. However the most important property of this matrix is

$$L^{c\,-1}_{R_p} R L^c_p = R \tag{2}$$

if R is an element of the little group $R\hat{p} = \hat{p}$. Taking this property together with $|L_p| = 1$ as a definition of the canonical boost in general we obtain unique solutions for $p^2 > 0$ and $p^2 < 0$. For $p^2 = 0$ there is no solution with $|L_p| = 1$. The more restricted condition

$$L^{-1}_{R_p} R L_p = R \quad , R \in O(2) , \quad \begin{matrix} R t^0 = t^0 & t^0 = (1,0,0,1) \\ R \bar{t}^0 = \bar{t}^0 & \bar{t}^0 = (1,0,0,-1) \end{matrix} \tag{3}$$

which is suggested by the property of gauge invariance leads to an one-parametric set of solutions for the boost matrices in the case $p^2=0$.

All these considerations are done for the group $SO(3.1)$ and its subgroups. The discussion of transformation properties of states at infinite momentum leads to an inspection of the Wigner rotation in this limit. For helicity states we generalize the result that the Wigner rotation reduces to a rotation in the n_1^0-n_2^0- plane to the case $p^2<0$. It is explicitly shown, that the rotation angle is always the same for all three cases. The asymptotic properties of the Wigner rotations of canonical states are discussed. The results can be connected with the transformation properties of appropriate light-like momentum vectors. The projection of the transformation of this light-like vector into a three dimensional subspace (n_1^0, n_2^0, n_3^0 or $\bar{t}^0 \cdot t^0, n_1^0, n_2^0$) leads (together with one further condition) to a unique transformation in this subspace which just coincides with the corresponding Wigner rotation in the infinite momentum limit. This appears as a generalization of the well known treatment of the abberation of light, which is just a three dimensional description of the transformation of light-like four-vectors.

In the following part of this paper we consider helicity states and their asymptotic properties. The canonical states and their behaviour are treated in the last section.

2. Helicity States

a) Boost Matrices

The helicity states[3] are given by $|ps\lambda\rangle = U(L_p^H)|\hat{p}s\lambda\rangle$ with the additional property $(\vec{S}\vec{p})/p\,|ps\lambda\rangle = \lambda|ps\lambda\rangle$, where

$\vec{S}=(M_{23},M_{31},M_{12})$, $M_{\mu\nu}$ generators of the Lorentz group, $p=(p^\mu) = (p^0,\vec{p}) = (\tilde{p}, p^3)$, $\mathfrak{p}=|\vec{p}|$, $pk=p^0k^0-\vec{p}\vec{k}$, L_p is the boost matrix which transforms the four vectors of the standard system $(\hat{p}, n_1^0, n_2^0, n_3^0)$ into the vectors $(p, n_{1(p)}, n_{2(p)}, n_{3(p)})$ of an arbitrary system. In the standard system the operator $M_{12} = W(\hat{p})$ is diagonal. In the arbitrary system we demand

$$U(L_p)\, W(\hat{p})\, U(L_p^{-1}) = W(p) = (\vec{S}\vec{p})/\mathfrak{p} \,. \tag{4}$$

Writing this in a covariant manner

$$
\begin{aligned}
&p^2>0 \quad W(p) = \varepsilon_{\mu\nu\sigma\tau} M^{\nu\sigma} p^\mu n^{H\tau}_{3(p)} \frac{1}{m}\,, \quad \hat{p}=(m,0,0,0)\,, \quad n_3^0=(0,0,0,1)\\
&p^2<0 \quad W(p) = -\varepsilon_{\mu\nu\sigma\tau} M^{\nu\sigma} p^\mu n^{H\tau}_{3(p)} \frac{1}{\mu}\,, \quad \hat{p}=(0,0,0,\mu)\,, \quad n_3^0=(1,0,0,0)\\
&p^2=0 \quad W(p) = \frac{1}{4}\varepsilon_{\mu\nu\sigma\tau} M^{\nu\sigma} \bar{t}^\tau_H p^\mu\,, \quad \hat{p}=t^0\,, \quad n_1^0=(0,1,0,0)\,, \quad n_2^0=(0,0,1,0)\,,
\end{aligned}
\tag{5}
$$

we determine $n^H_{3(p)}$, $\bar{t}^H_{(p)}$ in agreement with eq. (4). The boost matrices are:

$$
L^H_p = \begin{pmatrix}
p^0 m^{-1} & & & \mathfrak{p}\, m^{-1}\\
p^1 m^{-1} & n^H_{1(p)} & n^H_{2(p)} & p^1 p^0 (m\mathfrak{p})^{-1}\\
p^2 m^{-1} & & & p^2 p^0 (m\mathfrak{p})^{-1}\\
p^3 m^{-1} & & & p^3 p^0 (m\mathfrak{p})^{-1}
\end{pmatrix}, \quad p^2=m^2>0 \tag{6a}
$$

$$
L^H_p = \begin{pmatrix}
\mathfrak{p}\,\mu^{-1} & & & p^0\mu^{-1}\\
p^1 p^0 (\mu\mathfrak{p})^{-1} & & & p^1\mu^{-1}\\
p^2 p^0 (\mu\mathfrak{p})^{-1} & n^H_{1(p)} & n^H_{2(p)} & p^2\mu^{-1}\\
p^3 p^0 (\mu\mathfrak{p})^{-1} & & & p^3\mu^{-1}
\end{pmatrix}, \quad p^2=\mu^2<0 \tag{6b}
$$

$$L^H_p = \left[\begin{array}{cccc} \frac{p^{o2}+1}{2p^{o2}}\, p^o & & & \frac{p^{o2}-1}{2p^{o2}}\, p^o \\ \frac{p^{o2}-1}{2p^{o2}} \begin{pmatrix} p^1 \\ p^2 \\ p^3 \end{pmatrix} & n^H_{1(p)} & n^H_{2(p)} & \frac{p^{o2}+1}{2p^{o2}} \begin{pmatrix} p^1 \\ p^2 \\ p^3 \end{pmatrix} \end{array}\right] , \quad p^2 = 0 \qquad (6c)$$

$n^H_{1(p)}$ and $n^H_{2(p)}$ are chosen to be

$$n^H_{1(p)} = \left(0, 1 + \frac{p^3 - p}{p_\perp^2 p}\, p_1^2, \frac{p^3-p}{p_\perp^2 p}\, p_1 p_2, -\frac{p^1}{p}\right), n^H_{2(p)} = \left(0, \frac{p^3-p}{p_\perp^2 p}\, p_1 p_2, 1 + \frac{p^3-p}{p_\perp^2 p}\, p_2^2, -\frac{p^2}{p}\right), \qquad (7)$$

where $p_\perp^2 = p_1^2 + p_2^2$.

At infinite momentum we expect relations between the different boost matrices. According to $t^o = \frac{1}{2}[(2,0,0,0) + (0,0,0,2)]$ the boost matrices satisfy

$$\underset{p^2=0}{L^H_p} = \frac{1}{2}\left(\underset{p^2=4}{L^H_p} + \underset{p^2=-4}{L^H_p} \right) + O\left(\frac{1}{p^3}\right). \qquad (8)$$

This corresponds to the fact that at infinite momentum both space-like and time-like vectors approach the light cone, but from opposite directions.

b) Asymptotic Properties and Rotation Angles

Doing the same calculations as in I we can show

$$\lim_{p\to\infty} R^H_w(\Lambda, p)\, n^o_3 = \lim_{p\to\infty} L^{H\,-1}_{\Lambda p}\, \Lambda\, L^H_p\, n^o_3 = n^o_3 \quad , \quad p^2 \le 0 \qquad (9)$$

which means: In the limit $p\to\infty$ the general Wigner rotation reduces in any case to a rotation in the n^o_1-n^o_2-plane. For the representations we have $\lim_{p\to\infty} D^j_{\lambda\lambda'}(R^H_w) = \delta_{\lambda\lambda'} e^{i\lambda\varphi}$. This agrees with the transformation properties of massless particles (corresponding to $p^2=0$ and the discrete series) and includes the vanishing of the transitions between different helicity states induced by finite Lorentz transformations/1,5/. With our choice of $n^H_{1(p)}$ and $n^H_{2(p)}$ the rotation angle is in all cases the same and given by

$$\cos\varphi = (n_1^0, R^H_w n_1^0) = (n_1^0, L^{H-1}_{\Lambda p} \Lambda L^H_p n_1^0) = (n^H_1(\Lambda p), \Lambda n^H_1(p)) . \tag{10}$$

It depends only on the choice of $n^H_{1(p)}$ and $n^H_{2(p)}$. This means that at very high momenta states and matrix elements show the same transformation properties for all values of p^2 .

3. Canonical States

a) Boost Matrices

The canonical boost introduced by Joos [2] is defined as a rotation free Lorentz transformation which transforms $\hat{p}=(m,0,0,0)$ into p . This boost has the important property

$$R^c_{W(R,p)} = L^{c-1}_{Rp} R L^c_p = R \quad \forall\, R \text{ is an element of the little group } R\hat{p}=\hat{p}. \tag{11}$$

To get a simple generalization of this boost to the cases $p^2 \leq 0$ we take this property together with $|L^c_p|=1$ as a definition of the canonical boost. It is then possible to show, that there exists only one solution in the cases $p^2>0$ and $p^2<0$ and no solution in the case $p^2=0$. The proof goes as follows. Starting from

$$L^{c-1}_{Rp} R L^c_p p = Rp \quad \forall\, R\hat{p}=\hat{p}, \quad \begin{array}{ll} \hat{p}=(m,0,0,0), & p^2>0 \\ \hat{p}=(0,0,0,\mu), & p^2<0 \\ \hat{p}=(1,0,0,1), & p^2=0 \end{array} \tag{12}$$

and using the notation $\alpha(p)=L^c_p p$ we can write

$$R\,\alpha(p) = \alpha(Rp) \qquad \text{for all } R \text{ with } R\hat{p}=\hat{p} . \tag{13}$$

The general solutions of this equation are

$$p^2>0: \quad \alpha(p) = \left(\frac{p^2+p_0^2}{m}, \frac{2p^0}{m}\vec{p}\right) \quad \text{and} \quad \alpha(p) = \hat{p}$$

$$p^2<0: \quad \alpha(p) = \left(\frac{2p^3}{\mu}\tilde{p},\ \frac{\tilde{p}^2+p_3^2}{\mu}\right) \quad \text{and} \quad \alpha(p) = \hat{p}$$

$$p^2=0: \quad \alpha(p) = \lambda \cdot p \qquad\qquad \text{and} \quad \alpha(p) = \hat{p}\,, \tag{14}$$

where $\tilde{p} = (p^0, p^1, p^2)$,

The solutions $\alpha(p)=\hat{p}$ lead to matrices L_p with $|L_p| = -1$. For $p^2=0$ the solution $\alpha(p)=\lambda p$ is in contradiction with the transformation property $\hat{p}\to p$. The other two solutions give the canonical boost for $p^2>0$ /1,2/ and the following boost for $p^2<0$ /4/

$$L^{\tilde{c}}_p = \begin{pmatrix} \frac{1}{\mu(p^3+\mu)}p_0^2+1 & \frac{-1}{\mu(p^3+\mu)}p^0p^1 & \frac{-1}{\mu(p^3+\mu)}p^0p^2 & \frac{p^0}{\mu} \\ \frac{1}{\mu(p^3+\mu)}p_0p^1 & 1-\frac{1}{\mu(p^3+\mu)}p_1^2 & \frac{-1}{\mu(p^3+\mu)}p^1p^2 & \frac{p^1}{\mu} \\ \frac{1}{\mu(p^3+\mu)}p^0p^2 & \frac{-1}{\mu(p^3+\mu)}p^1p^2 & 1-\frac{1}{\mu(p^3+\mu)}p_2^2 & \frac{p^2}{\mu} \\ \frac{p^0}{\mu} & -\frac{p^1}{\mu} & -\frac{p^2}{\mu} & \frac{p^3}{\mu} \end{pmatrix}. \tag{15}$$

For the massless physical particles - belonging to the discrete representations - the translations of the $E(2)$ group are represented by the identity (gauge invariance). This suggests us to use a weaker condition and look for solutions of

$$L^{-1}_{R_p} R L_p = R \ \text{ if } \ \begin{matrix} Rt^0 = t^0 \\ R\bar{t}^0 = \bar{t}^0 \end{matrix} \qquad \text{i.e. for pure } SO(2) \text{ rotations} \tag{16}$$

in the case $p^2=0$. Here we shall obtain a set of one parametric solutions. If we denote the vectors n_1 and n_2 by n_a^i (a labels the vectors, i the components) then from eq. (16) it follows

$$n^i{}_{b(R_p)} R_{ba} = R^i{}_j\, n^j{}_{a(p)} \tag{17}$$

with the solutions

$$n^j_c = \lambda \delta^j_c + \mu p^j p_c \quad , j=1,2 \, ,$$
$$n^j_c = \nu^j p_c \quad , j=0,3 \, . \tag{18}$$

Taking into account the conservation of scalar products, normconditions and the regularity of the boost for $p^0 = p^3$ we obtain for this boost:

$$\begin{aligned}
t^0 &\rightarrow p \\
n_1^0 &\rightarrow n_1^0 + p^1 \frac{1}{p^0+p^3} \bar{t}_0 + bp \\
n_2^0 &\rightarrow n_2^0 + p^2 \frac{1}{p^0+p^3} \bar{t}_0 + bp \\
\bar{t}_0 &\rightarrow \left(\frac{2}{(p^0+p^3)} + 2b(p^0-p^3)\right)\bar{t}_0 + b^2 p_\perp^2 p + 2b(n_1^0 p^1 + n_2^0 p^2) \, ,
\end{aligned} \tag{19}$$

where b is an arbitrary function. This boost contains several well known boosts as special cases e.g. for $b = -\frac{1}{p^0+p^3}\frac{1}{p^0}$ the helicity boost and for $b=0$ the boost /4/

$$L_p = \begin{pmatrix}
\frac{1}{2}\left(p^0 + \frac{2}{(p^0+p^3)}\right) & \frac{p^1}{p^0+p^3} & \frac{p^2}{p^0+p^3} & \frac{1}{2}\left(p^0 - \frac{2}{p^0+p^3}\right) \\
\frac{p^1}{2} & 1 & 0 & \frac{p^1}{2} \\
\frac{p^2}{2} & 0 & 1 & \frac{p^2}{2} \\
\frac{1}{2}\left(p^3 - \frac{2}{p^0+p^3}\right) & -\frac{p^1}{p^0+p^3} & -\frac{p^2}{p^0+p^3} & \frac{1}{2}\left(p^3 + \frac{2}{p^0+p^3}\right)
\end{pmatrix} \tag{20}$$

with the important property $\bar{t}^0 \rightarrow \frac{2}{p^0+p^3}\bar{t}^0$ This boost does not contain generators of the group $E(2)$.

b) Asymptotic Properties

In 1 we have studied the limit $p \to \infty$ for the Wigner rotation of canonical states with $p^2 > 0$. It has been shown

$$\lim_{p\to\infty} R^c_{w(\Lambda,p)} n_p = \lim_{p\to\infty} L^{c-1}_{\Lambda p} \Lambda L^c_p n_p = \lim_{p\to\infty} n_{\Lambda p} \quad , \tag{21}$$

where $n_p = (0, \frac{\vec{p}}{p})$, $n_{\Lambda p} = (0, \frac{\vec{\Lambda p}}{|\Lambda p|})$, Λ was a finite Lorentz transformation. Using essentially the same methods similar result can be shown in the case $p^2 < 0$. Denoting $\tilde{p} = (p^0, p^1, p^2)$ then we have

$$\lim_{|\tilde{p}|\to\infty} R^{\tilde{c}}_{w(\Lambda,p)} m_p = \lim_{|\tilde{p}|\to\infty} m_{\Lambda p} \quad , \tag{22}$$

where $m_p = (\frac{\tilde{p}}{|\tilde{p}|}, 0)$, $m_{\Lambda p} = (\frac{\widetilde{\Lambda p}}{|\widetilde{\Lambda p}|}, 0)$, if Λ does not change $|\tilde{p}|$ essentially (i.e. if $|\tilde{p}|$ goes to infinity, then $|\widetilde{\Lambda p}|$ should also go to infinity). This means, that the result is valid for all states with $|p_3| \to \infty$ ($\tilde{p}^2 - p_3^2 = -\mu^2$, μ^2 finite). In the case $p^2 = 0$ the result depends on the type of boost, which is given by the choice of b. For $b = 0$ (eq. 20) and the helicity boost $\lim_{|\tilde{p}|\to\infty} L^{-1}_{\Lambda p} \Lambda L_p t^0 = t^0$ is valid. But this relation depends on b. For another asymptotic behaviour of b, we expect another asymptotic behaviour of the Wigner rotation.

c) Relations between the Transformation Properties of Light-Like Vectors and the Wigner Rotation in the Infinite Momentum Limit

The relations (21), (22) can be connected with the transformation properties of appropriate light-like momentum vectors. The transformation $p_L' = \Lambda p_L$ of a light-like momentum vector $p_L = (p, \vec{p})$ can be projected onto a rotation of normed vectors $\vec{u}_p = \frac{\vec{p}}{p}$, $u_{p'} = \frac{\vec{p}'}{p'}$ in the space spanned by the vectors n_1^0, n_2^0, n_3^0 as

$$\vec{n}_{p'} = \tau_\Lambda \vec{n}_p \tag{23}$$

where τ_Λ is a pure rotation matrix. To define τ_Λ uniquely we represent Λ in the form $\Lambda = L_\kappa \cdot R$ (L_κ pure Lorentz transformation, R pure rotation). The corresponding matrix τ_Λ can be represented in the same way

$$\tau_\Lambda = \tau_{L_\kappa} \cdot \tau_R \tag{24}$$

Clearly we have $\tau_R = R$. To determine τ_{L_κ} we remark:

a) τ_{L_κ} is a rotation matrix with the property $\vec{n}_{L_\kappa p} = \tau_{L_\kappa} n_p$. Possible invariant vectors (axis of rotations) are $\vec{\omega} = \alpha(\vec{n}_{L_\kappa p} + \vec{n}_p) + \beta(\vec{n}_{L_\kappa p} \times \vec{n}_p)$.

b) The Lorentz transformation L_κ has $v = a(0, -k^2, k^1, 0) + b\left(0, k^1, k^2, -\frac{k_\perp^2}{k^3}\right)$ as invariant vectors.

The condition that there is one common invariant vector both for the rotation τ_{L_κ} and the Lorentz transformation L_κ leads to $\alpha = 0$ and gives a unique defined matrix τ_{L_κ}. On the other hand the Wigner rotation for canonical states with $p = (\sqrt{p^2 + m^2}, \vec{p})$ satisfies for pure rotations R $R^c_{w(R,p)} = R$ and therefore

$$R^c_{w(R,p)} = R = \tau_R \quad . \tag{25}$$

In 1 we have obtained the results

a) $R^c_{w(L^c_\kappa, p)}$ is a rotation around the axis $\vec{k} \times \vec{p}$ or $\vec{L^c_\kappa p} \times \vec{p}$,

b) $\lim\limits_{p \to \infty} R^c_{w(L^c_\kappa, p)} \vec{n}_p = \lim\limits_{p \to \infty} \vec{n}_{L_\kappa p}$, $n_p = (0, \vec{n}_p)$

for pure Lorentz transformations. If the former light-like vector p_L has the same space-like directions as p then the normed vectors $\lim\limits_{p\to\infty} \vec{n}_{\Lambda p}$ and $\lim\limits_{p\to\infty} \vec{n}_{\Lambda p_L}$ coincide and we are able to conclude

$$\lim_{p\to\infty} R^{c}_{w}(L_{\kappa}, p) = \tau_{L_{\kappa}} . \tag{26}$$

Summarizing the foregoing we see that the transformation of an approrpiate light-like vector induces a full rotation (in the space spanned by n_1^0, n_2^0, n_3^0) which describes the abberation of light. Furthermore this rotation and the Wigner rotation in the infinite momentum limit coincide. This is a more complete description of the connection between the abberation of light and the Wigner rotation in the infinite momentum limit as considered in 1.

In the same way it is possible to consider the projection of $p_L' = \Lambda p_L$ with $p_L = (\tilde{p}, |\tilde{p}|)$, $\tilde{p} = (p^0, p^1, p^2)$ onto the space spanned by the vectors $(1,0,0,0), n_1^0, n_2^0$. This transformation is a $SO(2.1)$ rotation

$$\tilde{m}_{p'} = s_{\Lambda(p)} \tilde{m}_p \tag{27}$$

of the normed vectors $\tilde{m}_p = \frac{\tilde{p}}{|\tilde{p}|}$, $\tilde{m}_{p'} = \frac{\tilde{p}'}{|\tilde{p}'|}$ (as long as $|\tilde{p}| = |\tilde{p}_3| \neq 0$). To get a definition of s_Λ we proceed in the same manner. Representating $\Lambda = \tilde{L}^c_\kappa \tilde{R}$ ($\tilde{L}^c_\kappa$ is a canonical boost matrix with $k^2 < 0$, $\tilde{R}$ is a $SO(2.1)$ rotation matrix) and $s_\Lambda = s_{L_\kappa} s_R$ we have $s_R = \tilde{R}.s_{L_\kappa}$ is again uniquely defined by the condition that there is one common invariant vector for the Lorentz transformation and the $SO(2.1)$ rotation matrix. The resulting invariant vector is $(\tilde{p} \times \tilde{L}_\kappa p)$.

On the other hand the Wigner rotation for canonical states with $p = (\tilde{p}, \sqrt{\tilde{p}^2 + \mu^2})$ satisfies for the rotation $R^{\tilde{c}}_{w(\tilde{R},p)} = \tilde{R}$ and therefore

$$\tilde{R} = R^{\tilde{c}}_{w(\tilde{R},p)} = s_R . \tag{28}$$

For the transformation $\tilde{L}^c_\kappa$ the Wigner rotation has quite similar properties:

a) $R^{\tilde{c}}_w(L^{\tilde{c}}_\kappa, p)$ is a $SO(2.1)$ rotation with $\tilde{p}\times\tilde{k}$ or $\tilde{p}\times\widetilde{L^c_\kappa p}$ as invariant vector,

b) $\lim\limits_{|\tilde{p}|\to\infty} R^{\tilde{c}}_w(L^{\tilde{c}}_\kappa, p)\,\tilde{m}_p = \lim\limits_{|\tilde{p}|\to\infty} \tilde{m}_{L^{\tilde{c}}_\kappa p}$.

An appropriate light-like vector has the property $\tilde{p}_L = \mu\tilde{p}$, $\mu > 0$. Proceeding in the same manner we conclude

$$\lim_{|\tilde{p}|\to\infty} R^{\tilde{c}}_w(L^{\tilde{c}}_\kappa, p) = s_{L_\kappa} . \tag{29}$$

In other words: In the limit $|\tilde{p}|\to\infty$ and $p^2<0$ the Wigner rotation is Just the $SO(2.1)$ rotation (in the space spanned by $(1,0,0,0)$, n^0_1, n^0_2) induced by the Lorentz transformation Λ applied to the corresponding light-like vector.

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